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The exponent $\lambda\,(x, Q^2)$ of the proton structure function $F_2\,(x,\ Q^2)$ at low x

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Abstract. The exponent λ of the structure function $F_2 \sim x^{-\lambda}$ is calculated using the solution of the DGLAP equation for gluon at low x reported recently by the present authors. The quantity λ is calculated both as a function of x at fixed Q^2 and as a function of Q^2 at fixed x and compared with the most recent data from H1.

Keywords. DGLAP equation for gluon; method of characteristics; low x; exponent of proton structure function.

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In a recent paper [1] we have obtained an approximate analytical solution of the DGLAP equation [2] for the gluon at low x in the leading order. Our solution has the form

$$G(x,t) = G(\tau) x^{-\{1-(t_0/t)^{\gamma^2}\}} \left(\frac{t_0}{t}\right)^{\gamma^2 n_f/18} \exp\left[-\frac{11}{12}\left\{1-\left(\frac{t_0}{t}\right)^{\gamma^2}\right\}\right]. \qquad (1)$$

Here $\gamma = \sqrt{12/\beta_0}, \beta_0 = \frac{1}{3}(33-2n_f)$, n_f being the number of flavours and $t = \ln(Q^2/\Lambda^2)$, $t_0 = \ln(Q_0^2/\Lambda^2)$, where Λ is the QCD cut-off parameter and Q_0^2 is the starting scale. $G(x,t) = xg(x,Q^2)$ is the gluon momentum density and

$$\tau(x,t) = \exp\left[\left(-\ln\frac{1}{x} + \frac{11}{12}\right)\left(\frac{t_0}{t}\right)^{\gamma^2} - \frac{11}{12}\right]. \qquad (2)$$

In eq. (1), $G(\tau)$ is the input gluon distribution which is obtained from any specific non-perturbative input available in the literature by the formal replacement $x \to \tau$. The form of the input has changed because in deriving eq. (1) we used the method of characteristics [3] to solve the partial differential equation for the gluon momentum density without any assumption of the factorizability of x and t dependence of the gluon as was done earlier [4,5].

At low x the gluon being the dominant parton, the scaling violation of F_2 arises mainly from the gluon $(g \to q\bar{q})$ and so the contribution from the quark can be neglected. In the DGLAP formalism an approximate relationship can be obtained between the gluon momentum density $G(x,Q^2)$ and the logarithmic slope of the structure function $F_2(x,Q^2)$. There are several such relations [6–8] available in the literature. The most general one [8] in the LO reads

$$\frac{\partial F_2(x,Q^2)}{\partial \ln Q^2} \approx \frac{10\alpha_s}{27\pi} G\left[\frac{x}{1-\alpha}\left(\frac{3}{2}-\alpha\right)\right], \tag{3}$$

where $\alpha(<1)$ is a parameter that determines the suitable point of expansion of the gluon momentum density under the integral in the DGLAP equation for F_2 and α_s is the strong coupling constant in the LO. In our analysis we showed that our result was fairly successful in explaining the data [9] if α is chosen around $\simeq 0.7$. This value of α has an important physical interpretation: it implies that the longitudinal momentum x_g of the gluon is about three times the longitudinal momentum of the probed quark (or antiquark) in DIS [10].

In this short note we calculate the exponent $\lambda(x,Q^2)$ given as the derivative

$$\lambda(x,Q^2) = \left.\frac{\partial \ln F_2(x,Q^2)}{\partial \ln(1/x)}\right|_{Q^2} \tag{4}$$

and compare the prediction with the most recent H1 data [11] where the measurement of the exponent in a large kinematical domain at low x, $3\cdot 10^{-5} \leq x \leq 0.2$ and $1.5 \leq Q^2 \leq 150$ GeV^2 has been reported. The exponent $\lambda(x,Q^2)$ being directly measurable from the structure function data can give us helpful insight into the behaviour of the structure function specially at low x. Theoretical justification of the use of the exponent for such a study has also been reported in the literature [12] on the basis of the j-plane singularity of the Mellin transform of the structure function.

To obtain an expression for $\lambda(x,Q^2)$ we first differentiate eq. (3) with respect to $\ln(1/x)$ and then integrate from Q_0^2 to Q^2. Finally we get

$$\lambda(x,Q^2) = \lambda(x,Q_0^2)\frac{F_2(x,Q_0^2)}{F_2(x,Q^2)} + \frac{1}{F_2(x,Q^2)} \times \frac{10}{27\pi}\int_{Q_0^2}^{Q^2} \alpha_s(Q^2)\frac{\partial G(x',Q^2)}{\partial \ln(1/x)}\,\mathrm{d}\ln Q^2, \tag{5}$$

where

$$x' = \left(\frac{1.5-\alpha}{1-\alpha}\right)x \tag{6}$$

with $\alpha < 1$ and $\alpha_s(Q^2)$ is the strong coupling constant given in the LO as

$$\alpha_s(Q^2) = \frac{12\pi}{(33-2n_f)\ln(Q^2/\Lambda^2)}. \tag{7}$$

In eq. (5), $\lambda(x,Q_0^2) = \partial \ln F_2(x,Q_0^2)/\partial \ln(1/x)$ is the exponent at the starting scale Q_0^2 while $F_2(x,Q_0^2)$ is the input structure function. $F_2(x,Q^2)$ is obtained from eq. (3). $G(x',Q^2)$ is

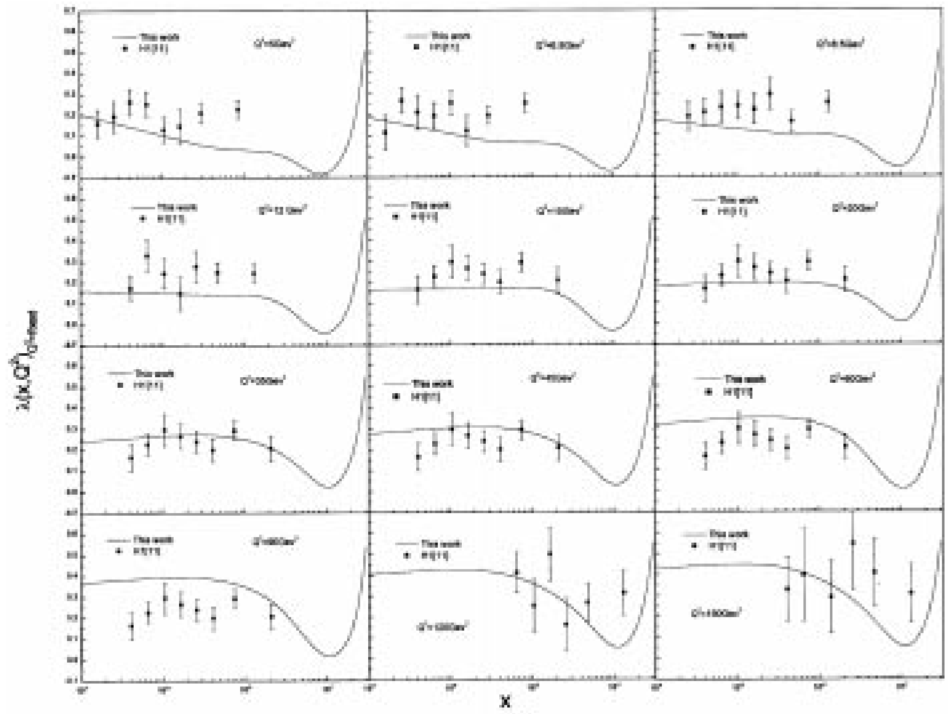

Figure 1. Exponent $\lambda(x,Q^2)$ plotted against x at several fixed Q^2 values and compared with data from H1 [11]. The error bars represent the statistical and systematic uncertainties added in quadrature.

taken from our solution (1) with x replaced by x' because the gluon appears here only through the scaling violation relation. In our analysis we take the value of α to be 0.7 because it gave good phenomenological test as discussed earlier [1]. We take all our inputs from MRS98LO [13].

In figure 1 we show $\lambda(x,Q^2)$ calculated from eq. (5) as a function of x at twelve different fixed Q^2 values from 5 GeV^2 to 150 GeV^2. We observe that at low $x \lesssim 10^{-2}$, and for 8.5 GeV$^2 < Q^2 \leq 150$ GeV^2 the derivative $\lambda(x,Q^2)$ is almost independent of x consistent with the H1 [11] data explored in this range. As x increases above 10^{-2}, λ falls sharply with increasing x reaching a minimum at $x \approx 0.1$ and then increases rapidly. It presumably indicates the breakdown of the low x assumption and transition to the valence quark region as noted in ref. [11]. In figure 2 we compare our prediction with H1 data for $\lambda(x,Q^2)$ as a function of Q^2 at twelve different fixed x $(0.00017 \leq x \leq 0.05)$ values. For $x \lesssim 0.0011$ we notice that there is a slow fall of λ logarithmically with Q^2 up to about $Q^2 \approx 8$ GeV^2 but above this value the exponent rises almost linearly with $\ln Q^2$ consistent with the H1 observation. However, the maximum value reached by λ which is about $\sim$0.4 for $x \lesssim 0.019$ gradually falls as the value of the fixed x is increased so that when $x \approx 0.029$–0.05 it rises only up to a maximum value of ≈ 0.2. Thus in this explored kinematic range it might suggest a form $\lambda(x,Q^2) \sim a(x)\ln(Q^2/\Lambda^2)$ rather than a constant $a(=0.0481)$ as suggested in ref. [11]. However, we must be cautious. This deviation may be due to the approximate

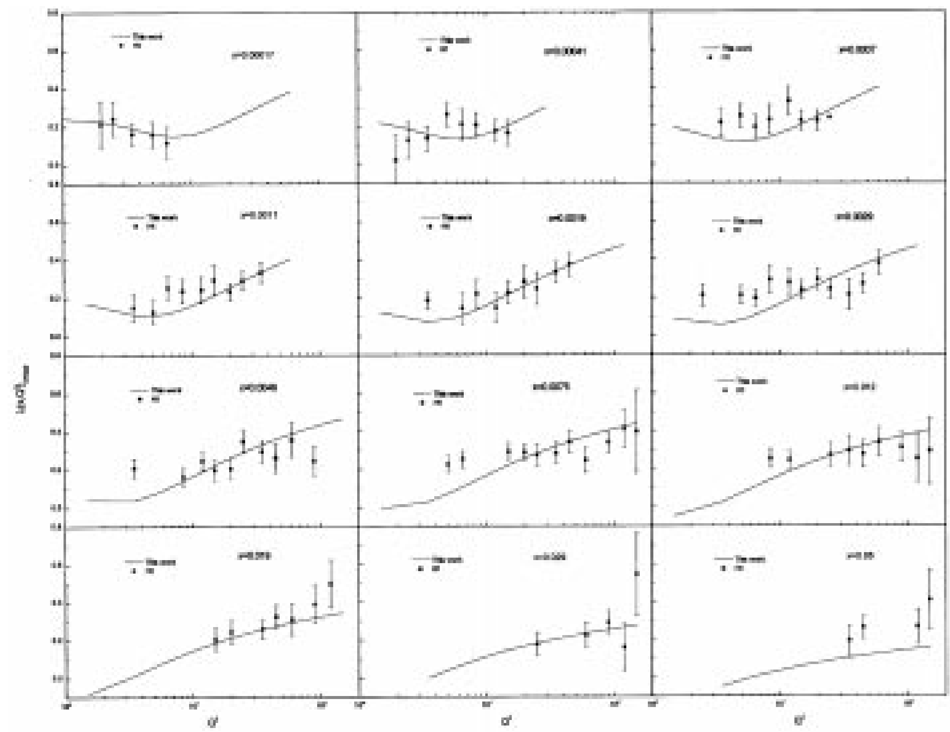

Figure 2. Exponent $\lambda(x, Q^2)$ plotted against Q^2 at different fixed x and compared with H1 data from ref. [11]. The error bars represent systematic and statistical uncertainties added in quadrature.

nature of the calculational technique done in LO only which might have large corrections at small x and large Q^2. But it is also to be noted that Desgrolard *et al* [14] have also indicated such x-dependence of the derivative $\lambda(x, Q^2)$ based on various Regge-type models [15,16]. To see quantitatively the agreement of our prediction with experiment, we quote in table 1 some χ^2 [17,18] in different ranges of the kinematic variables. In calculating χ^2, the systematic and the statistical errors are treated in quadrature. As is evident, the agreement of our prediction with the H1 data [11] is not very satisfactory if we consider the entire range of $x - Q^2$ explored in ref. [11]. This high value of χ^2/d.o.f. is mainly due to large deviations of our predictions in the lower $Q^2 (\lesssim 5\text{–}8.5\ \text{GeV}^2)$ region. It is also to be noted that there are some large fluctuations of the data. But if we squeeze the domain in both x and Q^2 then in a limited kinematic range our prediction is comfortable with the experimental data.

To conclude, the exponent $\lambda(x, Q^2)$ computed from the LO gluon distribution proposed in ref. [1] conforms to the qualitative features of the recent H1 [11] data for low x ($x \lesssim 10^{-2}$) and high Q^2 ($Q^2 \gtrsim 8.5\ \text{GeV}^2$) region. However, a simple parametrization like $F_2 = c(Q^2)x^{-\lambda(Q^2)}$ in the entire $x - Q^2$ range explored in ref. [11] seems to be not possible in our formalism which we have carried out only at the leading order. The NLO effect in our formalism is presently under consideration which we will communicate in future.

Table 1. Comparison of the prediction for $\lambda(x,Q^2)$ with H1 data [11].

For $\lambda(x,Q^2)$ vs. x with Q^2 fixed		
Q^2 (GeV^2)	x	χ^2/d.o.f.
$3.5 \leq Q^2 \leq 150$	$0.000105 \leq x \leq 0.132$	1.858
$3.5 \leq Q^2 \leq 150$	$0.000105 \leq x \leq 0.05$	1.327
$5.0 \leq Q^2 \leq 150$	$0.000165 \leq x \leq 0.132$	1.768
$5.0 \leq Q^2 \leq 150$	$0.000165 \leq x \leq 0.05$	1.195
$8.5 \leq Q^2 \leq 90$	$0.00026 \leq x \leq 0.05$	1.039
For $\lambda(x,Q^2)$ vs. Q^2 with x fixed		
x	Q^2 (GeV^2)	χ^2/d.o.f.
$0.00017 \leq x \leq 0.05$	$2.0 \leq Q^2 \leq 150$	1.678
$0.00017 \leq x \leq 0.05$	$5.0 \leq Q^2 \leq 150$	1.410
$0.00041 \leq x \leq 0.05$	$8.5 \leq Q^2 \leq 150$	0.985
$0.00041 \leq x \leq 0.05$	$8.5 \leq Q^2 \leq 90$	1.009

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