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On unitarity relations and their application to meson–antimeson mixing

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Abstract. In view of the recent observation of nonexponential decay features for a certain quantum-mechanical system, we revisit our earlier study of the small-time behaviour of the meson–antimeson complex wherein the commonly employed Weisskopf–Wigner approximation could be tested, in principle. We find that the experiments for this testing are difficult because of (i) the smallness of the times required for this study, (ii) the high accuracy required for probing the nonleading terms (which distinguish this approximation from the general theory) in the small-time behaviour of the relevant probabilities and (iii) the crude knowledge of the required 'flavour-tagging' procedures, as available at present.

Keywords. Small-time-behaviour; meson–antimeson complex; Weisskopf–Wigner approximation.

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1. Introduction

In a previous paper [1] with the above-given title, the small-time behaviour of the structural amplitudes $(a, b, \overline{a}, \overline{b})$ of the meson–antimeson complex was studied on the basis of the unitarity of the time-development operator, as arising from the hermiticity of the full Hamiltonian of the system. The corresponding behaviour obtained in the usually employed Weisskopf–Wigner approximation (abbreviated henceforth as WWA) was compared with that of the general theory, thus providing tests of the WWA. One ingredient of the WWA is the exponential decay law. Recently, nonexponential decay features have been observed in a quantum-mechanical system [2]. This may encourage a future experimental study of the small-time behaviour of the meson–antimeson complex. However, we show that the experiments required for our WWA tests are hard at present; see the three observations I, II and III in §2. As previously [1], no symmetry assumption is made; CP-, T- and CPT-noninvariances are allowed. For simplicity, we use the earlier notation in the discussion below.

2. The three observations I, II and III

(I) *The range of times required for the experimental study is very small*: For the meson–antimeson complex, no departures from the WWA have been observed for the investigated times which go down to $\approx 10^{-10}$ s for neutral kaons and $\approx 10^{-12}$ s for $f = D^0$, $B^0_{d,s}$. The nonexponential decay features were observed [2] in some system by performing measurements at an accuracy at the microsecond level. To investigate meson systems, one obviously needs a much higher accuracy. Theoretical estimates of the times where a departure from the exponential law is expected are even smaller [3].

(II) *The required accuracy of the study of the time-dependence is high*: The reason is the need to probe, in this time-dependence, nonleading terms which are the only ones distinguishing the WWA from the general theory, as shown below. In ref. [1], the small-time behaviour of the one-time asymmetries $A_{1,2}$ and the two-time asymmetries $A_{3,4}$ was given. In order to see that the WWA differs from the general theory only through nonleading terms, the behaviour of the denominator (or, equivalently, numerator) occurring in the definition of the asymmetry needs to be considered along with the behaviour of the asymmetry itself. The behaviour of the denominator also helps one to see how difficult/easy a particular measurement might be. The behaviour of the denominator was not given (except for A_1) in ref. [1], and the point about nonleading terms being crucial for testing the WWA was not brought out.

Furthermore, we shall consider eight new asymmetries, for completeness; these are the remaining [4] one-time asymmetries $B_{1,2,3,4}$ and the remaining two-time asymmetries $B_{5,6,7,8}$, all defined here. It is worth noting that the consideration of these new asymmetries B_j ($j = 1$ to 8) is important. The reason [4] is that out of the four probabilities $|a|^2, |b|^2, |\bar{b}|^2$ and $|\bar{a}|^2$, one can construct three independent asymmetries. Having already considred$A_{1,2}$, one must consider at least one more (say, B_1) in order to check whether the claim of the WWA being distinguishable only by nonleading terms is based on the full set of (future) experimental data. Then, the consideration of the remaining three one-time asymmetries $B_{2,3,4}$ is not crucial; in fact, the small-time behaviour of $B_{2,3,4}$ and their respective numerators and denominators is qualitatively the same as for the B_1 case. The same remarks hold also for the two-time asymmetries constructed out of $N_{ff}, N_{\bar{f}\bar{f}}, N_{f\bar{f}}$ and $N_{\bar{f}f}$: it is important to consider at least one more (say, B_5) two-time asymmetry, and not crucial to consider the remaining three ($B_{6,7,8}$) for which all the relevant features are qualitatively the same as for the B_5 case.

Our study of the new asymmetries B_j confirms the need to investigate nonleading terms for the purpose of testing the WWA. For a given asymmetry A_i, the WWA is distinguished from the general theory by the behaviour of the numerator (and the asymmetry), but not the behaviour of the denominator. For B_j, the distinction arises from the behaviour of the numerator (and the denominator), but not from the behaviour of the asymmetry. In all cases, the distinction is provided by nonleading terms in the behaviour of the eight probabilities out of which the A_i and the B_j are constructed.

(A) *The one-time asymmetries* – The small-time behaviour of the amplitudes $a, b, \bar{a}, \bar{b}$ given in the table in ref. [1] can be conveniently written as

$$a(t) = 1 + i(\gamma_1 + \alpha_1)t + (\gamma_2 + \alpha_2)t^2 + O(t^3)$$
$$\bar{a}(t) = 1 + i(\gamma_1 - \alpha_1)t + (\gamma_2 - \alpha_2)t^2 + O(t^3)$$

$$b(t) = (i\delta_1 + \beta_1)t + (\delta_2 + i\beta_2)t^2 + O(t^3)$$
$$\bar{b}(t) = (i\delta_1 - \beta_1)t + (\delta_2 - i\beta_2)t^2 + O(t^3) \quad (1)$$

where t is the proper time, and $\alpha_{1,2}, \beta_{1,2}, \gamma_{1,2}, \delta_{1,2}$ are real constants. This leads to the small-t behaviour of $(|b|^2 - |\bar{b}|^2)$, $(|b|^2 + |\bar{b}|^2)$, $(|a|^2 - |\bar{a}|^2)$ and $(|a|^2 + |\bar{a}|^2)$ as t^3, t^2, t^2 and 1 respectively in the general theory, and t^2, t^2, t and 1 respectively in the WWA. The WWA result easily follows from the expressions for $a, \bar{a}, b$ and $\bar{b}$ given [1] in eqs (36)–(39). Thus only the CP-odd differences $(|b|^2 - |\bar{b}|^2)$ and $(|a|^2 - |\bar{a}|^2)$ distinguish the WWA. To leading order, the behaviour of $|a^2|$ is the same in the WWA and the general theory; that holds also for $|b^2|$, for $|\bar{b}|^2$, and for $|\bar{a}|^2$. The above, of course directly concerns $A_{1,2}$.

We now come to the new [4] four asymmetries $B_{1,2,3,4}$ formed out of $|a|^2$, $|b|^2$, $|\bar{b}|^2$ and $|\bar{a}|^2$ as

$$B_1 = \frac{|a|^2 - |b|^2}{|a|^2 + |b|^2}, \quad B_2 = \frac{|\bar{a}|^2 - |b|^2}{|\bar{a}|^2 + |b|^2}, \quad B_3 = \frac{|a|^2 - |\bar{b}|^2}{|a|^2 + |b|^2}, \quad B_4 = \frac{|\bar{a}|^2 - |\bar{b}|^2}{|\bar{a}|^2 + |b|^2} \quad (2)$$

which, in contrast to $A_{1,2}$, do not have a definite behaviour under CP. One can see that in the general theory, the numerators and denominators of $B_{1,2,3,4}$ and the asymmetries $B_{1,2,3,4}$ themselves go as $[1 + O(t^2)]$. However, in the WWA, the numerators and denominators go as [1+ (a term linear in t) + $O(t^2)$], while the asymmetries themselves go as $[1 + O(t^2)]$. Thus these asymmetries do not distinguish the WWA from the general theory, but the denominators (and numerators) have the distinctive feature of the nonleading linear term being present in only the WWA.

(B) *The two-time asymmetries* – First, the old [1] asymmetries $A_{3,4}$. The small-time behaviour of the denominator occurring in the two-time asymmetry A_3 is $(t_1 \pm t_2)^2$ for $\varepsilon = \pm 1$. It is some time-independent constant for A_4 for $\varepsilon = \pm 1$. These statements hold for both the general theory and the WWA. Using the behaviours of $A_{3,4}$ given earlier [1], one sees that only the CP-odd differences occurring as numerators (and not the CP-even sums occurring as denominators) of $A_{3,4}$ distinguish the WWA from the general theory.

We now come to the new four asymmetries $B_{5,6,7,8}$ formed out of $N_{ff}, N_{\bar{f}\bar{f}}, N_{f\bar{f}}$ and $N_{\bar{f}f}$ as

$$B_5 = \frac{N_{f\bar{f}} - N_{ff}}{N_{f\bar{f}} + N_{ff}}, \quad B_6 = \frac{N_{f\bar{f}} - N_{\bar{f}\bar{f}}}{N_{f\bar{f}} + N_{\bar{f}\bar{f}}},$$
$$B_7 = \frac{N_{\bar{f}f} - N_{ff}}{N_{\bar{f}f} + N_{ff}}, \quad B_8 = \frac{N_{\bar{f}f} - N_{\bar{f}\bar{f}}}{N_{\bar{f}f} + N_{\bar{f}\bar{f}}}. \quad (3)$$

These, in contrast to $A_{3,4}$, do not have a definite behaviour under CP. One can see that in the general theory, the numerators and denominators of $B_{5,6,7,8}$ and the asymmetries $B_{5,6,7,8}$ themelves go as $[1 + Q(t^2)]$ where $Q(t^2)$ means terms of the type (t_1^2) or (t_2^2) or $(t_1 t_2)$. However, in the WWA, the numerators and denominators go as $[1 + (a$ term linear in $t_1) +$ (a term linear in $t_2) + Q(t^2)]$, while the asymmetries themselves go as $[1 + Q(t^2)]$. Thus the asymmetries do not distinguish the WWA from the general theory, but the denominators (and numerators) have the distinctive feature of the nonleading linear terms (varying as t_1 and t_2) being present in only the WWA. The above statements hold for $\varepsilon = \pm 1$ both. The behaviour of $B_{5,6,7,8}$ is thus analogous to that of the one-time-asymmetries $B_{1,2,3,4}$.

In summary, therefore, only the differences (and not the sums) of the pair of probabilities occurring in any of the four previously considered asymmetries A_i ($i = 1$ to 4) distinguish the WWA from the general theory. Since the predominant behaviour (indicated by the sums) is the same in the WWA and the general theory, testing the WWA for small times would mean looking for nonleading terms in the time-dependence of all the eight probabilities entering the asymmetries A_i; that calls for rather accurate measurements. Our consideration of the new asymmetries B_j and their numerators and denominators also shows that the WWA is distinguished from the general theory only by nonleading terms.

(III) *The available flavour-tagging techniques are quite approximate* – The amplitudes a, b, $\bar{a}$, and $\bar{b}$ are probability amplitudes. Therefore, the asymmetries A_i and B_j are between probabilities. For an experimental measurement of these asymmetries, one needs 'flavour-tagging' of f and $\bar{f}$ in the final state for $A_{3,4}$ and $B_{5,6,7,8}$ and, additionally, in the initial state for $A_{1,2}$ and $B_{1,2,3,4}$. Unfortunately, this is likely to make the experiments difficult for the following reason. Because of the importance of testing the WWA, one should make flavour-tagging as model-independent as possible; in particular, it should not be done through a WWA-based analysis. Since weak interaction decay amplitudes usually necessitate further assumptions (see, e.g., refs [4–6] for the case of the CPLEAR experiment corresponding to A_1 for the presently investigated times), it is better to utilize strong interactions having the property of flavour conservation for the purpose of tagging. We shall restrict our comments to only the flavour-tagging procedures already known. For the case of $A_{1,2}$ and $B_{1,2,3,4}$, reactions (16) and (17) used by CPLEAR [6] provide the initial-state flavour-tagging for $f = K^0$; the final-state tagging could be provided (also for $A_{3,4}$ and $B_{5,6,7,8}$) through [7] the reactions $\bar{K}^0 p \to \pi^+ \Lambda$ and $K^0 p \to K^+ n$, as already mentioned [1]. Unfortunately, the experimental knowledge of the cross-sections of these two-body reaction is not [7] accurate, thus making the tagging procedure difficult; even at the (much larger) times presently investigated, one avoided such strong-interaction tagging and utilised [6] tagging by weak decays of the neutral kaon; see ref. [8] for some similar (not accurate, again) possibilities. For $A_{3,4}$, for $f = B^0_{d,s}$, one could use the empirical 'jet-charge' method wherein, briefly speaking, one decides the flavour of the parent-flavoured quark (antiquark) by estimating (the sign of) its charge. This estimation is done by performing suitable weighted averages over the charges of the particle tracks in the jet produced by the flavoured quark (antiquark); here, efficiencies typically of the order of 70% are quoted for different versions of the jet-charge method; this is quite approximate; see ref. [9] for a review.

3. Conclusion

Thus the experiments required for testing the small-time behaviour of the WWA are difficult because they need very accurate measurements for quite small times, using flavour-tagging techniques which are crude at present.

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