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Thermal condensation mode in a dusty plasma

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Abstract. In the present work, the radiative condensation instability is investigated in the presence of dust charge fluctuations. We find that the charge variability of the grain reduces the growth rate of radiative mode only for fluctuation wavelength smaller or of the order of the Debye length and this reduction is not very large. Far from the Debye sphere, radiative mode can damp due to thermal conduction of electrons and ions.

Keywords. Dusty plasma; radiative condensation; charge fluctuations.

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1. Introduction

The structure formation in subcritical Jeans mass regions in the interstellar medium may be initiated by the radiative cooling of the plasma particles. The radiative instabilities have been invoked to explain the cloudiness of the diffuse interstellar and intergalactic medium, the formation of prominences in the solar corona, etc. [1–3]. Furthermore, the radiative mode can play the dual role of first triggering the gravitational collapse of the matter, and then, when the matter is sufficiently compressed, during the burning process in the core, heavier elements (e.g., C, O_2, Fe, etc.) will be cooked and transported to the envelope where they may cause the cooling of the plasma particles.

It is well known [4] that the interstellar grains composed mainly of graphite, silicate and metallic compounds, comprise about 1% of the mass of the interstellar medium, whose main constituent is the neutral hydrogen, with 10% helium abundance and a significant fraction of these grains are charged [5]. Due to the low ionization fraction (10^{-7}), these grains are generally embedded in a weakly ionized plasma background, and may acquire ± 1, and 0 electronic charges. It is not unusual to consider the grains of different sizes (micron or less) and compositions, but only with $\pm 2e, \pm e$ or 0 electronic charges [4–6]. Umebayashi and Nakano [7] indicated that electrically charged grains are rare in dense molecular clouds as these clouds are shielded from UV radiation. However, in HII regions, which are formed when a significant fraction of radiation from a new born star, or an associated emission nebula, escapes without being absorbed by the in-falling matter from the accretion disk, the dust grain, depending upon the thermal velocities of the background plasma particles, may pick up between 10 to 100 electronic charges from the ionized gas [5].

Interstellar dust is heated by the absorption of energy from the ambient radiation field and plasma. The most observable effect of the plasma–grain interaction is the thermal infrared (IR) emission from the collisionally-heated dust, and the gradual release of refractory elements, locked up in the grain, back into the gas phase. If the dust is sufficiently small so that the energy deposited by the incident particle (or photon) in the dust is much larger than the internal energy of the dust, the dust will experience a surge in its energy due to heating. Further, dust grain cools significantly (evaporatively or radiatively) between the successive collisons, and the dust experiences a thermal fluctuation in time. Therefore, it is important to investigate the effect of radiative condensation modes in a dusty plasma. Present analysis is intended to investigate the effect of charged dust on the radiative plasma mode.

In the present work, the effect of the charged dust grains on the radiatively cooling plasma is considerd. The model can be of relevance in the aftermath of extreme compression of the matter when the heavy elements are transported to the envelope and radiative condensation takes place. It is required to consider also a partially ionized plasma model in the presence of the dust dynamics. The investigation of radiative condensation mode for a partially ionized plasma in the absence of dust grain has recently been carried by Birk and his group [10,11]. The overlap of self-gravitational mode with the radiative condensation mode for a partially and fully ionized plasma has been investigated recently [12,13].

The presence of the dust grain in the plasma introduces new spatial and temporal scales. As a result, not only the collective behavior of the plasma is altered, but also, new modes are excited in the plasma. For example, the large difference in masses (grain mass $\sim 10^{-15}$–10^{-5} g, $m_e \sim 10^{-27}$ g, $m_i \sim 10^{-24}$ g), the dusty plasma dynamics can be studied in two different limits: (i) the dust particle is so heavy that it provides a stationary background for the perturbations propagating in the much lighter electron–ion plasma and (ii) the perturbations are of the order of or less than the typical plasma frequencies of the dusty fluid (of the order of Hz) and wavelengths are visible to the bare eyes. Although large dust grains undergo temperature fluctuations due to collision, the mean square fluctuation in their temperature is much less than the equilibrium temperature. Therefore, thermal modes may not be important for massive grains.

The collisional charging of a dust grain can be regarded as the most fundamental interaction it undergoes with the ambient plasma, since the acquired charge determines the grain cross section for all collisions with the ionized gas. In hot plasma ($\sim$10 eV), the efficiency of grain charging is limited by secondary electron emission, and, at higher temperature ($\sim$100 eV), by transparency of the dust to the impinging electrons. The variation of the dust charge introduces additional temporal scale in a dusty plasma. The grain charge variation is described by kinetic processes which are usually high-frequency (of the order of mega-Hertz) processes and may not be dynamically important for dusty plasma mode ($\sim$ few Hz). Therefore, dusty plasma introduces two widely separated time scales, namely dust acoustic (slow) and charge fluctuation (fast) time scales [8]. Hence, for low-frequency modes, charge can be considered to have some average value related to the balance between ion and electron fluxes [9]. However, the charge variation should be of importance to the normal plasma modes whose frequencies are not so separated from the charging frequency of the grains. Therefore, as the present model will investigate the radiative condensation modes of a purely ionized plasma, the dust dynamics will be included only via the charge dynamics. The dust momentum equation will be ignored in view of the preceding discussion. The grains will always exist in a hot thermal plasma soup, though evaporation of molecular size grains cannot be ruled out. Generally, a charged grain embedded in a hot plasma soup with temperatures greater than a million degree will not evaporate as the equilibrium grain temperature will be less than 100 K [14]. This is due to the fact that the ambient plasma density of the interstellar medium is very low ($\sim 10^3$ m^{-3}) [5].

2. Basic equations

The multi-fluid system consists of electrons, ions and charged grains. The size, charge and mass distribution of the grains is ignored for simplicity. It is known that grains can be charged via several competing processes, viz., electron and ion collisions, photoemission, secondary emission due to electron or ion impact and electric field emission, etc. We assume that the grain charging is primarily collisional.

In some environments, interstellar grains may be irregularly shaped [5]. In the present work we restrict our formulation to spherical grains and assume that all grains are of radius a. The governing equations are

$$\frac{\partial n_\alpha}{\partial t} + \nabla\cdot(n_\alpha \vec{v}_\alpha) = 0, \tag{1}$$

$$m_\alpha n_\alpha \left[\frac{\partial \vec{v}_\alpha}{\partial t} + \vec{v}_\alpha\cdot\nabla\vec{v}_\alpha\right] = -\nabla P_\alpha - q_\alpha n_\alpha \nabla\phi - m_\alpha n_\alpha \nabla\psi, \tag{2}$$

$$\frac{3}{2} n_\alpha \frac{\partial T_\alpha}{\partial t} + P_\alpha \nabla\cdot\vec{v}_\alpha = \chi_\alpha \nabla^2 T_\alpha - n_\alpha L(n_\alpha, T_\alpha), \tag{3}$$

$$P_\alpha = n_\alpha T_\alpha, \tag{4}$$

where $\alpha = e, i$. n_α is electron and ion number densities, v_α is the velocity of the αth particle, P_α is the pressure, ϕ and ψ are the electrostatic and gravitational potentials and q_α, m_α, T_α and χ_α are the charge, mass, temperature and thermal conductivity respectively of the plasma particles. The energy gain–loss function (per unit mass and per unit time) $L(n_\alpha, T_\alpha)$ accounts for the heating H and cooling C, $L = C - H$. In the above equation, it is assumed that L only depends on the density and temperature, and we consider the radiation for energy loss term. Here χ_α is the thermal conductivity of the plasma particles [15].

The charge residing on the dust particles is a dynamical variable. For a negatively charged particle, the fluctuation in the grain charge number Z_d can be given as [16]

$$\left(\frac{\partial}{\partial t} + \vec{v}_d\cdot\nabla\right) Z_d = \frac{\nu_{\rm ed}}{n_{d0}}\left[n_{\rm e} - n_{\rm e0} - \frac{\nu_{\rm id}}{\nu_{\rm ed}}(n_i - n_{i0})\right], \tag{5}$$

where $\nu_{\rm ed,\,id}$ are the electron–dust, and ion–dust collision frequencies given by

$$v_{\rm ed} = P\Omega_c \frac{\tau+z}{1+\tau+z}, \quad v_{\rm id} = \sqrt{\frac{m_e}{m_i}}\,\tau v_{\rm ed}\ \exp(z).$$

We use the notation $\tau = T_i/T_e$, the charge fluctuation frequency is denoted as $\Omega_c = \omega_{pi}^2 a(1+\tau+z)/(v_{Ti}\sqrt{2\pi})$, the ion thermal velocity is $v_{Ti}^2 = T_i/m_i$, $z = Z_d e^2/aT_e$, $\omega_{pi}^2 = 4\pi e_i^2 n_{i0}/m_i$, $P = an_{d0}T_e/n_{e0}e^2$. The potential field is defined by the following Poisson's equation,

$$\nabla^2\phi = -4\pi\left[e(n_i - n_e) - Z_d e n_d\right]. \tag{6}$$

Here for definiteness, the grain is assumed to have negative charge $-Z_d e$.

3. Stability analysis

We assume an infinite, homogeneous, quasineutral plasma without any equilibrium flow. In a steady state without flow it follows from eq. (5) that $\left(\partial/\partial t + v_d\cdot\nabla\right)Z_d = 0$. We linearize equations for all the species, denote the first order quantities by the subscript 1 and assume a plane wave space and time dependence as $\exp(i\vec{k}\cdot\vec{x} - i\omega t)$ for them. From the continuity equations, one can write

$$\vec{k}\cdot\vec{v}_{\alpha 1} = \omega\frac{n_{\alpha 1}}{n_{\alpha 0}}. \tag{7}$$

Ignoring electron inertia one gets from eq. (2)

$$\frac{n_{e1}}{n_{e0}} = \left(\frac{e\phi_1}{T_{e0}} - \frac{T_{e1}}{T_{e0}}\right), \tag{8}$$

where the isothermal equation of state $P_e = n_e T_e$ has been used. For ions we shall retain the inertia term. Then from eq. (2)

$$\frac{\omega}{k}\frac{v_{i1}}{v_{Ti}^2} = \left(\frac{e\phi_1}{T_{i0}} + \frac{T_{i1}}{T_{i0}}\right) + \frac{n_{i1}}{n_{i0}}. \tag{9}$$

The first order energy equation becomes

$$\left[\frac{3}{2}\omega + i\left(\hat{\chi}_\alpha k^2 - n_{\alpha 0}L_{T_\alpha}\right)\right]\frac{T_{\alpha 1}}{T_{\alpha 0}} = \left(\omega - i n_{\alpha 0}L_{n_\alpha}\right)\frac{n_{\alpha 1}}{n_{\alpha 0}}. \tag{10}$$

Here, $L_{T_\alpha} = -\partial L/n_{\alpha 0}\partial T_{\alpha 0}$ and $L_{n_\alpha} = \partial L/T_{\alpha 0}\partial n_{\alpha 0}$, and $\hat{\chi}_\alpha = \chi_\alpha/n_{\alpha 0}$ is the thermal diffusivity of the plasma particles.

The Poisson equation for the electrostatic potential is

$$\phi_1 = \frac{4\pi e}{k^2}\left[(n_{i1} - n_{e1}) - Z_{d1}n_{d0}\right]. \tag{11}$$

The charge fluctuation equation is

$$-i\omega Z_{d1} = \frac{eP\Omega_c}{n_{d0}}\left(n_{e1} - \varepsilon n_{i1}\right), \quad \varepsilon = \sqrt{\frac{m_e}{m_i}}\,\tau\exp(z). \tag{12}$$

Now it is a simple task to carry out the standard elimination process in order to arrive at the following dispersion relation

$$k^2\lambda_{\mathrm{D}e}^2 + \left(\frac{\Omega_e}{\Omega_e+\omega-in_{e0}L_{ne}}\right)\left(\frac{\omega+i\nu_{\mathrm{ed}}}{\omega}\right) - \left(\frac{\lambda_{\mathrm{D}e}}{\lambda_{\mathrm{D}i}}\right)^2\left(\frac{k^2 v_{Ti}^2\Omega_i}{\Omega_i\,\omega^2-k^2 v_{Ti}^2\left(\omega-i\,n_{i0}\,L_{ni}\right)}\right)\left(\frac{\omega+i\,\varepsilon\,\nu_{\mathrm{ed}}}{\omega}\right)=0. \tag{13}$$

Here, $\Omega_\alpha=(3/2)\omega+i(\hat{\chi}_\alpha k^2-n_{\alpha0}\,L_{T\alpha})$ with subscript α corresponding to electrons and ions. $\lambda_{\mathrm{D}e,i}=v_{Te,i}/\omega_{pe,i}$ is the Debye length of plasma particles. Before analyzing dispersion relation eq. (13) numerically for a wide range of parameters, we present an analytical result for a limiting case.

For $k\lambda_{\mathrm{D}e}\ll 1$ (which is tantamount to assuming the plasma fluctuations to be quasi-neutral), we see from eq. (13) that for $\varepsilon=1$, the charge fluctuation factor gets cancelled. This implies that there is no influence of the dust grains on the dynamics of the radiative mode. This can also be seen from analyzing the following dispersion relation, which we obtain from eq. (13),

$$\left(\frac{\Omega_e}{\Omega_e+\omega-i\,n_{e0}\,L_{ne}}\right)=\left(\frac{\lambda_{\mathrm{D}e}}{\lambda_{\mathrm{D}i}}\right)^2\left[\frac{k^2 v_{Ti}^2\Omega_i}{\Omega_i\,\omega^2-k^2 v_{Ti}^2\left(\omega-i\,n_{i0}\,L_{ni}\right)}\right]. \tag{14}$$

Taking the frequency as complex $\omega=\omega_r+i\omega_i$, and solving the above dispersion relation perturbatively for $\omega_i\ll\omega_r$, we recover the usual radiative condensation mode

$$\omega_i=\frac{2\,nL_n-k^2\,\hat{\chi}}{15}. \tag{15}$$

Here, it has been assumed that $L_{n\alpha}\approx L_{T\alpha}\approx L_n$, $n_{e0}\approx n_{i0}=n$. Since the radiative loss rate in plasma grows with the density, the term L_n is destabilizing. The growth rate is almost independent of the wave number. We can see the origin of the condensational instability from eq. (3). In the absence of thermal conduction term, free energy released in the process of plasma cooling, drives the thermal fluctuation, giving rise to the instability. However, when the plasma particles can conduct the heat (a process effective at short wavelengths), the released radiative energy is quickly transported away by the conducting electrons and there is no free energy available to drive the thermal fluctuations. The mode stabilizes. Thus, in the short wavelength regime, thermal conductivity has stabilizing effect suppressing perturbations of wavelength shorter than the conductive wavelength.

Next, we solve eq. (13) numerically in its totality and plot the normalized growth rate ω_i/nL_T against $k\lambda_{\mathrm{D}e}$. First, we assume that there is no thermal conduction and look for the purely growing radiative mode in the absence of charge fluctuation. In figure 1, we plot the root of eq. (13) for $L_{Ti}/L_{Te}=0.1, L_{ni}/L_{Te}=0.1, \lambda_{\mathrm{D}e}=\lambda_{\mathrm{D}i}$, $\omega_{pi}/nL_{Te}=1$, and $P=1$, $\varepsilon=1$. Here on *x*-axis $k\lambda_{\mathrm{D}}$ and on *y*-axis, normalize unstable root, ω_i/nL_{T_e} is given. We see that the growth rate is linear for $k\lambda_{\mathrm{D}}\le 1$ and then the mode saturates. The result at first glance is surprising. One would have expected that the growth rate will keep increasing linearly. If one does an analysis of eq. (13), then for a cold ion, one gets the growth rate $\gamma\sim 2nL/(1+k\lambda_{\mathrm{D}}^2)$. This fact suggests that, when the wavelength of

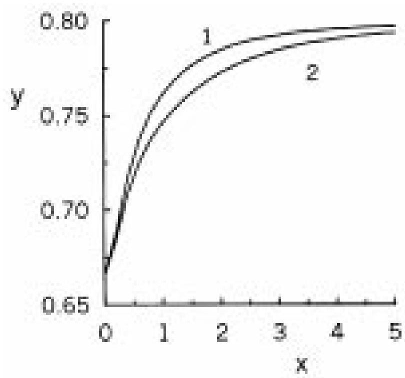

Figure 1. The unstable root ($y = \omega_i/nL_{Te}$) of dispersion relation (13) is plotted against $x = k\lambda_{De}$ for $L_{Ti}/L_{Te} = 0.1$, $L_{ni}/L_{Te} = 0.1$, $\lambda_{De} = \lambda_{Di}$, $\omega_{pi}/nL_{Te} = 1$, and $P = 1$, $\varepsilon = 1$. The instability keeps growing which can be attributed to the radiative cooling of electrons. When charge fluctuation is present $\Omega_c/nL_{Te} = 1$, the radiative mode is affected only in the vicinity of $k\lambda_{De} \sim O(1)$. The growth rate is reduced by about 3 to 4% (the dashed curve).

fluctuation is smaller than the Debye length, the fluctuation electric field will oppose the runaway growth. In case when ions are not cold, it will further reduce the growth rate. Therefore, it is not surprising that the linear growth rate of radiative instability is arrested around $k\lambda_D \sim O(1)$. The saturation of the growth curve beyond $k\lambda_D \sim O(1)$ is indicative of the fact that local space charge field cannot completely suppress the instability and it will keep growing at a constant rate, in spite of the existence of localized field and thermal effects. We attribute this condensation mode to the radiative cooling of electrons. The free energy, made available due to the cooling of electrons (via, e.g., bremsstrahlung), causes the fluctuations to grow. If $L_{Ti}/L_{Te} = 1$ and $L_{ni}/L_{Te} = 1$, then two identical unstable root appears as the electrons as well as ions, both contribute in equal measure in destabilizing the plasma. When the charge fluctuation is present $\Omega_c/nL_{Te} = 1$, the radiative mode is affected only in the vicinity of $k\lambda_{De} \sim O(1)$. The growth rate is reduced by about 3 to 4% (the dashed curve in figure 1). The charge on the grain fluctuates due to the electron–ion collision. Therefore, when the wavelength of the fluctuation is of the order of Debye length, the radiating electron feels the electric field of the dust. Thus, charge fluctuation will affect the radiative condensation mode in $k\lambda_D \sim O(1)$ vicinity. When $k\lambda_D > 1$, then the radiating electron is exposed to the space-charge field and behaviour of the the mode is similar to the previous case. In figure 2 we plot the radiative mode in the presence of thermal conduction of plasma particles. The curve 1 (figure 2) corresponds to the case when the charge fluctuation is absent, and $L_{Ti}/L_{Te} = 0.1$, $L_{ni}/L_{Te} = 0.1$, $\lambda_{De} = \lambda_{Di}$, $\omega_{pi}/nL_{Te} = 1$, $P = 1$, $\varepsilon = 1$, $\hat{\chi}_e k^2/nL_{Te} = \hat{\chi}_i k^2/nL_{Te} = 0.1$. When the charge fluctuation is switched on $\Omega_c/nL_{Te} = 1$ (the curve 2, figure 2), the growth rate is somewhat smaller for $k\lambda_{De} \leq 3$ than in the absence of charge dynamics. Therefore, we can conclude that the charge fluctuation on the dust grain may not affect radiative mode significantly, except, probably reducing the growth rate when the wavelength of the fluctuation is comparable with the Debye length. The result could have been anticipated physically since

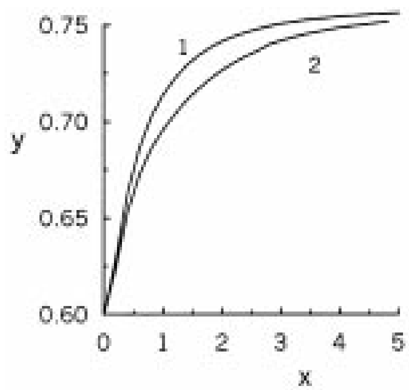

Figure 2. The plot is given for $x = k\lambda_{\rm D}$ against $y = \omega_i/nL_{T_e}$. Curve 1 corresponds to the case when charge fluctuation is absent and $L_{Ti}/L_{Te} = 0.1$, $L_{ni}/L_{Te} = 0.1$, $\lambda_{\rm De} = \lambda_{\rm Di}$, $\omega_{pi}/nL_{Te} = 1$, $P = 1$, $\varepsilon = 1$, $\hat{\chi}_e k^2/L_{Te} = \hat{\chi}_i k^2/L_{Te} = 0.1$. When the charge fluctuation is switched on $\Omega_c/nL_{Te} = 1$ (curve 2), the growth rate is somewhat smaller for $k\lambda_{\rm De} \leq 3$ than in the absence of charge dynamics.

the charge fluctuation model adopted in the present work is collisional (i.e., short range interaction) and binary collisions are dominant inside the Debye sphere. Therefore, the inclusion of charge fluctuation should affect the mode in the vicinity of Debye sphere.

4. Conclusions

In the present work, the radiative condensation mode for a dusty plasma has been studied. It has been pointed out that in a dusty plasma the effect of dust dynamics on the plasma modes can often enter through the charge fluctuation. We have studied the radiative instability by including only the charge dynamic equation of the grain. We find that the charge fluctuation on the dust may affect the growth rate for wavelengths of fluctuations which are comparable with the Debye length. The effect is towards reducing the growth rate. However, the effect is not very significant (about 3–4%). When the wavelength of perturbation is very large in comparison with the Debye length, the radiative mode is unaffected by the charge fluctuation. The presence of grains will have no role to play in the radiative cooling of plasma particles. The conclusion cannot be very different if the complete grain dynamics is retained, and the reason being that the momentum exchange of the grain takes place at a much longer time scales (due to large inertia of the grains).

There are several issues which needs to be addressed. Our model considers only collisional charging of the grain, though, in many circumstances, photoelectric charging of the grain plays a very important role [17]. Therefore, the role of grain size distribution (constrained by the interstellar extinction curve) along with the photoelectric model of charge fluctuation needs to be included in the present model.

Present model investigates the effect of grain charge fluctuation on the radiative plasma mode. The present model could be useful to a post-compression scenario where heavy element is cooked in the core of a collapsing cloud and is transported to the envelope. The presence of heavy element in the envelope may cause the cooling of the plasma particles.

Therefore, our investigation may be of relevance in the HII region of a newly formed gaseous nebula.

References

[1] G B Field, *Astrophys. J.* **142**, 531 (1965)
[2] S A Balbus, in *The physics of the interstellar medium and intergalactic medium*, *Astrophys. Soc. of the Pacific Conf. Series* edited by A Ferrara, C F McKee, C Heiles and P R Shapiro, California, **80**, 327 (1995)
[3] B Meerson, *Rev. Mod. Phys.* **68**, 215 (1996)
[4] B G Elmgreen, *Astrophys. J.* **232**, 729 (1979)
[5] L Spitzer, in *Physical processes in the interstellar medium* (John Wiley, New York, 1978)
[6] R Nishi, T Nakano and T Umebayashi, *Astrophys. J.* **368**, 181 (1991)
[7] T Umebayashi and T Nakano, *Publ. Astron. Soc. Japan* **32**, 405 (1980)
[8] J Vranješ, B P Pandey and S Poedts, *Phys. Plasmas* **9**, 1464 (2002)
[9] A V Ivlev and G Morfil, *Phys. Rev.* **E63**, 16409 (2001)
[10] G T Birk, *Phys. Plasmas* **7**, 3811 (2000)
[11] G T Birk and H Weichen, *Phys. Plasmas* **8**, 5057 (2001)
[12] B P Pandey and V Krishan, *IEEE Trans. Plasma Sci.* **29**, 307 (2001)
[13] B P Pandey, V Krishan and M Roy, *Pramana – J. Phys.* **56**, 95 (2001)
[14] A Dalgarno and R A McCray, *Ann. Rev. Astron. Astrophys.* **10**, 375 (1972)
[15] S I Braginskii, *Review of plasma physics* edited by M A Leontovich (Consultant Bureau, New York, 1965) vol. 1, p. 205
[16] J Vranješ, B P Pandey and S Poedts, *Phys. Rev.* **E64**, 066404 (2001)
[17] J C Weingartner and B T Draine, *Astrophys. J.* **134**, 263 (2001)