

Higher-dimensional string theory in Lyra geometry

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Abstract. In this paper, a study on string theory has been done in five-dimensional space-time based on Lyra geometry. Also a polynomial relation between the two scale factors is assumed. The equations of state for strings have been used for different solutions.

Keywords. String cosmology; Lyra geometry; higher dimension.

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1. Introduction

There has been considerable interest in solutions of Einstein's equations in higher dimensions in the context of physics of the early universe both for cosmologists and particle physicists. It is well-known that our universe was much small in its early stage than it is today. Indeed the present four-dimensional stage of the universe could have been preceded by a higher-dimensional stage, which at later times becomes effectively four-dimensional in the sense that the extra dimensions became unobservably small due to dynamical contraction [1]. Moreover, the detection of time variation of fundamental constraints may be a strong evidence for the existence of extra dimension [2]. In cosmology, this higher-dimensional theory might be useful at the very early stages of the evolution of the universe.

The concept of string theory was developed to describe events at the early stages of the evolution of the universe. It is believed that strings may be one of the sources of density perturbations that are required for the formation of large scale structures in the universe. According to Kibble [3] the strings are nothing but the important topologically stable defects due to the phase transition that occurs as the temperature lowers below some critical temperature of the early stages of the universe. The world sheets of the strings are two-dimensional time-like surfaces [3]. The present day configurations of the universe are not contradicted by large scale network of the strings in the early universe.

Also the vacuum strings can generate density fluctuations sufficient to explain the galaxy formation [4]. These strings have stress energy and they couple to the gravitational field so that it may be interesting to study the gravitational effects of it.

The general relativistic treatment of strings was initiated by Stachel and Letelier [5]. Cosmic strings as source of gravitational field in general relativity was discussed by many authors [6].

While attempting to unify gravitation and electromagnetism in a single space-time geometry, Weyl [7] showed how one can introduce a vector field with an intrinsic geometrical significance. But this theory was not accepted as it was based on non-integrability of length transfer. Lyra [8] proposed a new modification of Riemannian geometry by introducing a gauge function which removes the non-integrability condition of a vector under parallel transport.

In consecutive investigations Sen [9] and Sen and Dunn [10] proposed a new scalar tensor theory of gravitation and constructed an analog of Einstein field equation based on Lyra's geometry which in normal gauge may be written as

$$R_{ab} - \frac{1}{2}g_{ab}R + (3/2)\phi_a\phi_b - \frac{3}{4}g_{ab}\phi_c\phi^c = -8\pi GT_{ab}, \quad (1)$$

where ϕ_a is the displacement vector and other symbols have their usual meaning as in Riemannian geometry.

Halford [11] has pointed out that the constant displacement field ϕ_i in Lyra's geometry play the role of cosmological constant Λ in the normal general relativistic treatment. According to Halford the present theory predicts the same effects within observational limits, as far as the classical solar system tests are concerned, as well as tests based on the linearised form of field equations. Soleng [12] has pointed out that the constant displacement field in Lyra's geometry will either include a creation field and be equivalent to Hoyle's creation field cosmology or contain a special vacuum field which together with the gauge vector term may be considered as a cosmological term. Subsequent investigations were done by several authors in cosmology within the framework of Lyra geometry [13].

But as far as our knowledge goes there has not been any work in literature where Lyra's geometry has been considered for study of string cosmology in higher-dimensional space-time. As string concept is useful before the particle creation and can explain galaxy formation so it is interesting to study string cosmology in higher-dimensional theory. In this work we shall deal with string cosmology in higher dimensions with time dependent displacement vectors based on Lyra geometry in normal gauge, i.e., displacement vector

$$\phi_i = (\beta(t), 0, 0, 0, 0) \quad (2)$$

and look forward whether this study shows any significant properties due to the introduction of the gauge field in the Riemannian geometry.

2. Field equations

The five-dimensional line element is taken as

$$ds^2 = -dt^2 + R^2(t)(dx^2 + dy^2 + dz^2) + A^2(t)d\psi^2. \quad (3)$$

The fifth coordinate is taken to be space-like.

According to Letelier [5] the massive strings are nothing but the geometric strings (massless) with particles attached along its extension. So, the total energy momentum tensor for a cloud of massive strings can be written as [5,6]

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$$T_a^b = \rho V_a V^b - \lambda X_a X^b. \quad (4)$$

Here ρ is the rest energy for a cloud of strings with particles attached to them (P-strings).

So one can write

$$\rho = \rho_p + \lambda, \quad (5)$$

where ρ_p is the particle energy density and λ the tension density of the string. The five velocity V^a which has the components $V^a = (1, 0, 0, 0, 0)$ for the cloud of particles and the five vector $X^a = (0, 0, 0, 0, A^{-1})$ the direction of string will satisfy

$$V_a V^a = -1 = X_a X^a \quad \text{and} \quad V_a X^b = 0. \quad (6)$$

The field equation (1) for the metric (3) reduces to

$$3A'R'/AR + 3(R')^2/R^2 = \rho + \frac{3}{3}\beta^2, \quad (7)$$

$$3R''/R + 3(R')^2/R^2 = \lambda - \frac{3}{4}\beta^2, \quad (8)$$

$$2R''/R + A''/A + 2R'A'/RA + (R')^2/R^2 + \frac{3}{4}\beta^2 = 0 \quad (9)$$

(prime indicates differentiation with respect to t and we have chosen $8\pi G = 1$).

Now in view of all the three (strong, weak and dominant) energy conditions [6], one finds $\rho \geq 0$ and $\rho_p \geq 0$ together with the fact that the sign of λ is unrestricted, it may take values positive, negative or zero as well.

The different equations of state for string model be [6]

- (a) $\rho = \rho(\lambda)$ (Barotropic equation of state)
- (b) $\rho = \lambda$ (Geometric string)
- (c) $\rho = (1 + w)\lambda$ (Takabayasi string, i.e., P-string).

In the following section, we shall determine the exact solution of the field equations using the above equations of state for string model in Lyra geometry.

3. Solutions

Case I: Barotropic equation of state

In this case we take displacement vector a constant, i.e., $\beta = \text{constant}$. To solve the field equations one notes that there are three field equations connecting 4-unknowns. So one more relation connecting these variables is needed.

Here we assume the relation

$$A = \mu R^n \quad (\mu, n \text{ are constants}) \quad (10)$$

between the scale factors for unique solutions.

Using this relation, we get from eq. (9) as:

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$$R''/R + m(R')^2/R^2 = -b, \quad (11)$$

where $m = (n^2 + n + 1)/(n + 2)$; $b = [3/\{4(n + 2)\}]\beta^2$.

The above second-order differential has a first integral of the form:

$$(R')^2 = -[b/(m + 1)]R^2 + DR^{-2m}, \quad (12)$$

where D is an integration constant.

This differential equation can be written in the integral form:

$$\int \{-[b/(m + 1)]R^2 + DR^{-2m}\}^{-1/2} dR = \pm(t - t_0) \quad (13)$$

(t_0 is another integration constant).

Here the explicit form of R is

$$R = [C \sin E(t - t_0)]^{1/(m+1)}, \quad (14)$$

where $C^2 = (m + 1)D/b$, $E^2 = b(m + 1)$.

The other parameters have the following expressions:

$$A = \mu [C \sin E(t - t_0)]^{n/(m+1)}, \quad (15)$$

$$\rho = 3[E^2(n + 1)/(m + 1)^2] \cot^2 E(t - t_0) - \frac{3}{4}\beta^2, \quad (16)$$

$$\lambda = [3E^2/(m + 1)^2][\cot^2 E(t - t_0) - m \operatorname{cosec}^2 E(t - t_0) - 1] + \frac{3}{4}\beta^2. \quad (17)$$

The expansion scalar θ is given by

$$\theta = 3R'/R + A'/A = [E(n + 3)/(m + 1)] \cot E(t - t_0). \quad (18)$$

The proper volume

$$V^4 = AR^3 = \mu [C \sin E(t - t_0)]^{(n+3)/(m+1)}, \quad (19)$$

$$\rho_p = [3E^2/(m + 1)^2][n \cot^2 E(t - t_0) + m \operatorname{cosec}^2 E(t - t_0) + 1] - (3/2)\beta^2. \quad (20)$$

Case II: Geometric string ($\rho = \lambda$)

Here we also assume the same relation between the metric coefficients, i.e., $A = \mu R^n$, but the displacement vector is not constant.

In this case we get the following solution:

$$R = B[(t - t_0)]^{1/(n+3)}, \quad (21)$$

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$$A = \mu B^n [(t - t_0)]^{n/(n+3)}, \quad (22)$$

$$\frac{3}{4}\beta^2 = -[3/(n+3)^2][(t - t_0)]^{-2}, \quad (23)$$

where B is a constant and t_0 is an integration constant.

In this case, we see that $\beta^2 < 0$. This indicates that the manifold has imaginary connections. So under polynomial relationship between the metric coefficients, the geometric string model does not exist.

Case III: Takabayasi string (i.e. P-string)

Here the equation of state $\rho = (1 + w)\lambda$, where $w > 0$, a constant and it is small for string dominant era and large for particle dominant era.

Further using the polynomial relation $A = \mu R^n$, from the field equations, we get

$$R''/R + F(R')^2/R^2 = 0, \quad (24)$$

where $F = (2n^2 + wn^2 + 5n - 2w + wn + 2)/(2n + wn + 1 + w)$.

Solving eq. (24), we get

$$R = B[(t - t_0)]^{1/(F+1)}, \quad (25)$$

where B is a constant and t_0 an integration constant.

The other parameters are given by

$$A = \mu B^n [(t - t_0)]^{n/(F+1)}, \quad (26)$$

$$V^4 = \mu [B(t - t_0)]^{(n+3)/(F+1)}, \quad (27)$$

$$\theta = [n + 3]/(F + 1)(t - t_0)^{-1}, \quad (28)$$

$$\rho = [3(n + 2 - F)/(2 + w)(F + 1)^2](t - t_0)^{-2}, \quad (29)$$

$$\frac{3}{4}\beta^2 = [3\{F(w + 1) + n - 1\}/(2 + w)(F + 1)^2](t - t_0)^{-2}. \quad (30)$$

4. Discussions

Case I

For this solution, the universe starts at an initial epoch $t = t_0$ which is a point singularity. When $t - t_0 = \pi/2E$, then volume V^4 is maximum and after that the volume is decreasing with increasing time and when $t - t_0 = \pi/E$, then the volume will be zero.

Case III (For $w = 1$)

This solution will be interesting if $\sqrt{(19/12) - 3/2} < n < 0$. Then $t = t_0$ is the initial epoch of the universe. At this instant, $R \rightarrow 0$, $A \rightarrow \infty$, $V^4 \rightarrow 0$, $\theta \rightarrow \infty$. So this is a line singularity. Thus the universe starts with an infinite rate of expansion. Further, as t increases the scale factor A gradually decreases while the other one namely R increases. Thus the extra dimension becomes insignificant as time proceeds after the creation and we are left with real four-dimensional world.

In this model, the gauge function is large in the beginning but decreases with the evolution of the model.

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