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Sterile neutrino in a minimal three-generation see-saw model

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Abstract. We investigate symmetries in Dirac and Majorana mass matrices of neutrinos in a three-generation scenario. We show that if we invoke $L_e + L_\mu - L_\tau \times S_{2\mathrm{R}}$ symmetry, one combination of right-handed neutrino states remains massless which can be interpreted as a sterile neutrino. Next we consider a $SU(2)_\mathrm{L} \times U(1)_Y \times U(1)_\mathrm{R}$ gauge model and show how higher-dimensional operators can induce mixing between left- and right-handed states which explains solar, atmospheric and LSND experimental results.

Keywords. Sterile; neutrino; LSND.

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1. Introduction and brief summary

Solar [1], atmospheric [2] and LSND [3] oscillation experiments point towards the following neutrino mixing pattern [4]. Best fit values for $\Delta m^2_\odot, \Delta m^2_{\mathrm{Atm}}$ and $\Delta m^2_{\mathrm{LSND}}$ are given by $4.5 \times 10^{-5}, 3 \times 10^{-3}$ and 1 eV^2 respectively along with an approximate mixing pattern of

$$\begin{pmatrix} \nu_1 \\ \nu_2 \\ \nu_3 \\ \nu_4 \end{pmatrix} = \begin{pmatrix} 0.53 & 0.0 & -0.76 & 0.38 \\ 0.85 & 0.0 & 0.47 & -0.24 \\ 0.0 & 0.707 & 0.32 & 0.63 \\ 0.0 & 0.707 & -0.32 & -0.63 \end{pmatrix} \begin{pmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \\ \nu_s \end{pmatrix}. \tag{1}$$

Here ν_1 and ν_2 are light mass eigenstates whereas ν_3 and ν_4 are heavier eigenstates with mass $O(1)$ eV. Then in our notation $\Delta m^2_\odot = |m_1^2 - m_2^2|, \Delta m^2_{\mathrm{Atm}} = |m_3^2 - m_4^2|$ and $\Delta m^2_{\mathrm{LSND}} \approx |m_1^2 - m_3^2|$. The zeros of $e - \mu$ sector in eq. (1) should be filled by small entries for LSND transition to happen. KARMEN experiment [5] has narrowed a part of LSND parameter space and in the near future mini-boon experiment will settle the issue of $\overline{\nu_e} \leftrightarrow \overline{\nu_\mu}$ oscillations [6] reported by LSND.

2. Formalism and flavor symmetries

We use $(L_e + L_\mu - L_\tau) \times S_{2\mathrm{R}}$ where $S_{2\mathrm{R}}$ symmetry acts on e and μ generations. Then we can write Dirac and Majorana type mass matrices as,

$$M_{\mathrm{D}} = \begin{array}{c|ccc} & \nu_{\mathrm{R}}^e & \nu_{\mathrm{R}}^\mu & \nu_{\mathrm{R}}^\tau \\ \hline \overline{\nu_{\mathrm{L}}^e} & k & k & 0 \\ \overline{\nu_{\mathrm{L}}^\mu} & k' & k' & 0 \\ \overline{\nu_{\mathrm{L}}^\tau} & 0 & 0 & m_{33} \end{array}, \quad M_{\mathrm{R}} = \begin{array}{c|ccc} & \nu_{\mathrm{R}}^e & \nu_{\mathrm{R}}^\mu & \nu_{\mathrm{R}}^\tau \\ \hline \nu_{\mathrm{R}}^e & 0 & 0 & M \\ \nu_{\mathrm{R}}^\mu & 0 & 0 & M \\ \nu_{\mathrm{R}}^\tau & M & M & 0 \end{array}. \tag{2}$$

Now we can apply see-saw mechanism to these matrices and get light neutrino matrix

$$\mathscr{M}_\nu \; = \; -M_{\mathrm{D}}^T M_{\mathrm{R}}^{-1} M_{\mathrm{D}}. \tag{3}$$

This is the so-called type I see-saw formula [7]. On the other hand when M_{R} and M_{D} matrices have zero eigenvalues, one must 'take them out' of the matrix before using the see-saw formula. As was noted in [8], this turns out to be the case when there are leptonic symmetries such as the one we are considering. After see-saw light neutrino matrix is [9]

$$M = \begin{array}{c|cccc} & \nu'_e & \nu'_\mu & \nu_{\mathrm{L}}^\tau & \nu_s \\ \hline \nu'_e & 0 & 0 & m & 0 \\ \nu'_\mu & 0 & 0 & 0 & 0 \\ \nu_{\mathrm{L}}^\tau & m & 0 & 0 & 0 \\ \nu_s & 0 & 0 & 0 & 0 \end{array} \tag{4}$$

where we have defined

$$\nu'_e = \frac{k' \nu_{\mathrm{L}}^e - k \nu_{\mathrm{L}}^\mu}{\sqrt{k'^2 + k^2}}, \quad \nu'_\mu = \frac{k \nu_{\mathrm{L}}^e + k' \nu_{\mathrm{L}}^\mu}{\sqrt{k'^2 + k^2}}, \quad \nu_s = \frac{\nu_{\mathrm{R}}^e - \nu_{\mathrm{R}}^\mu}{\sqrt{2}}. \tag{5}$$

We see that for ranges of k and k', ν'_e and ν'_μ have different alignments with respect to ν_{L}^e and ν_{L}^μ. For example consider the limit $k' \gg k$. In this case we recover $\nu'_e \approx \nu_{\mathrm{L}}^e$ and $\nu'_\mu \approx \nu_{\mathrm{L}}^\mu$.

3. Gauge model for active-sterile mixing

Our model uses $SU(2)_{\mathrm{L}} \times U(1)_{I_{3\mathrm{R}}} \times U(1)_{\mathrm{B-L}}$ gauge group, even though it could easily be implemented in the context of the standard model gauge group as well. However, in this case the meaning of the see-saw scale will remain a mystery. Scalars required are listed in table 1. The challenge is to induce proper left–right mixing. These scalars allow for the following higher-dimensional operators.

$$\begin{aligned} &\frac{f_1}{M_p} \overline{L_e} \nu_s \langle \sigma_0 \rangle \langle \tilde{\phi} \rangle, \quad \frac{f_2}{M_p} \overline{L_\mu} \nu_s \langle \sigma_0 \rangle \langle \tilde{\phi} \rangle \\ &\frac{f_3}{M_p} \overline{L_\tau} \nu_s \langle \sigma_2 \rangle \langle \tilde{\phi} \rangle, \quad \frac{f_4}{M_p^2} \nu_s \nu_s \langle \Delta \rangle \langle \sigma_2 \rangle \langle \sigma_0 \rangle. \end{aligned} \tag{6}$$

Table 1. Relevant right-handed fermion and scalar fields and their transformation properties. Here we have defined $Y = I_{3R} +$ (B–L)/2.

	$SU(2)_L \times U(1)_{I_{3R}} \times U(1)_{B-L}$	$SU(2)_L \times U_Y(1)$	$L_e + L_\mu - L_\tau$	$S^{e\mu}_{2R}$
ν_{-R}	(1,1/2,−1)	(1,0)	1	−1
ν_{+R}	(1,1/2,−1)	(1,0)	1	1
$\nu_{\tau R}$	(1, 1/2,−1)	(1,0)	−1	1
Δ	(1,−1,+2)	(1,0)	0	1
ϕ	(2,1/2,0)	(2,1/2)	0	1
σ_2	(1,0,0)	(1,0)	+2	−1
σ_0	(1,0,0)	(1,0)	0	−1

Then we get the following neutrino mass texture in the original basis. Transforming back to original basis $(\nu'_e, \nu'_\mu) \to (\nu^e_L, \nu^\mu_L)$

$$M = \begin{array}{c|cccc} & \nu^e_L & \nu^\mu_L & \nu^\tau_L & \nu_s \\ \hline \nu^e_L & 0 & 0 & m & m_1 \\ \nu^\mu_L & 0 & 0 & m' & m_2 \\ \nu^\tau_L & m & m' & 0 & m_3 \\ \nu_s & m_1 & m_2 & m_3 & \delta \end{array} . \tag{7}$$

A similar texture for $m_3 = 0$ was found in ref. [10] where it was found that it is suitable for $2+2$ mixing scheme [11] between ordinary and sterile neutrinos.

4. Numerical fits of parameters

A possible set of parameters may be

$$m = \begin{pmatrix} 0 & 0 & 0.006 & 0.03 \\ 0 & 0 & 0.6 & 0.6 \\ 0.006 & 0.6 & 0 & 0.0006 \\ 0.03 & 0.6 & 0.0006 & 0.003 \end{pmatrix} \text{ eV}. \tag{8}$$

Then we get $\Delta m^2_\odot = 3 \times 10^{-5}$ eV^2, $\Delta m^2_{\text{Atm}} = 3.5 \times 10^{-3}$ eV^2, $\Delta m^2_{\text{LSND}} = 0.722134$ eV^2.

$$U = \begin{pmatrix} -0.0212239 & -0.706349 & -0.499125 & -0.501493 \\ -0.0212003 & -0.707226 & 0.500274 & 0.499107 \\ -0.697335 & 0.0221947 & 0.506895 & -0.50625 \\ -0.716117 & 0.0202588 & -0.493617 & 0.493059 \end{pmatrix} . \tag{9}$$

For LSND mixing we get

$$\sin^2 2\theta_{\text{LSND}} = 4|(U^*_{e3} U_{\mu 3} + U^*_{e4} U_{\mu 4})|^2 = 0.00359636. \tag{10}$$

5. Experimental tests of the constructed model

In terms of the mass and mixing angles following observable quantity is probed by the beta decay experiments is

$$m_{\nu_e}^2 = \sum_{i=1,4} m_i^2 |U_{ei}|^2. \tag{11}$$

For our choice of parameters we get $m_{\nu_e} \sim 0.028$. Thus we expect no signal for m_{ν_e} in KATRIN experiment [12] from our model. This is a way to test our model. Another experimental test is neutrinoless double beta decay [13].

$$|\langle m \rangle| = |\sum_{i=1,4} m_i U_{ei}^2| = m_{ee}. \tag{12}$$

If we get a lower bound from neutrinoless double beta decay at high confidence levels present scenario can be falsified.

6. Discussions and theoretical implications

Dirac-type neutrino mass of active neutrinos is $G \equiv SU(2)_{\mathrm{L}} \times U(1)_Y$ symmetry breaking. This makes the off-diagonal entry is of the order of m_Z. The diagonal entry, however, is G conserving and can be taken to be a large scale M. The matrix has two eigenvalues m_{D}^2/M and M. The first eigenvalue explains the smallness of the neutrino mass when $M \to \infty$. If on the other hand the off-diagonal entry is also G conserving, the mass eigenvalues will be of the order of $M^2/M = M$ and M. Obviously in this case see-saw mechanism cannot explain the smallness of neutrino mass. Sterile neutrino is a G singlet then its Dirac-type mass is G conserving. This is the reason why one needs to either look for special flavor symmetries for having a light sterile neutrino or let the sterile neutrino transform under a larger gauge symmetry not far above electroweak scale [14].

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