Brane world scenarios

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Abstract. We review proposals of brane world models which attempt to combine gauge theories with gravity at TeV scale by confining the gauge theory to a three-brane embedded in higher dimensional bulk. Gravity, however, propagates in the directions transverse to the brane as well.

Keywords. Randall–Sundrum model; string theory.

PACS Nos 11.27.+d; 11.25.Mj

Unification of all fundamental forces has always been the motivation of all extra dimension theories. Kaluza–Klein (KK) [1,2] compactification of higher dimensional, supergravity theories generated a lot of interest in early and mid-eighties of the last century.

The key feature of all these models was to evolve methods which hide extra dimensions from the four-dimensional observer. This means size of all extra dimensions should be smaller than 10^{-16} cm. Traditional KK mechanism typically used to take the compactification scale to be of the order of Planck scale, i.e., $\sim 10^{-33}$ cm. This is the scale at which gravity becomes strong and we can hope to unify gravity with other interactions.

In these models, Planck scale was treated as the fundamental scale and all other scales in the low energy theory are derived from the Planck scale. One of the relevant scales in the low energy, long distance (compared to the Planck scale) physics is the electroweak symmetry breaking scale $\sim 10^{-16}$ cm. This leads to the problem of obtaining a derived quantity which is 17 orders of magnitude bigger than the fundamental scale at our disposal. Therefore, by pushing the scale of new physics to the Planck scale we are led to the 'hierarchy problem'. All the models of unified theory suffer from this problem.

Recently, there are a couple of proposals in which this philosophy has been turned on its head. These models, known as 'brane world models', assume that ordinary matter, which includes all the field content of the standard model (SM) and possibly even the field content of the grand unified theory (GUT) gauge group, is trapped on a three-dimensional hypersurface (brane) embedded in a higher-dimensional space-time. These two proposals go under the name of the Arkani–Dimopoulos–Dvali (ADD) model [3] and the Randall–Sundrum (RS) model [4]. These new models have testable predictions at future colliders. Both the proposals also address the hierarchy problem $M_{ew}/M_{Pl} \sim 10^{-17}$ where M_{ew} is the electroweak symmetry breaking scale and M_{Pl} is the 4D Planck mass.

In a major departure from the KK philosophy, the brane world models propose that the four-dimensional Planck scale $\sim 10^{-33}$ cm is not a fundamental scale but is a derived

quantity and the fundamental length scale is of the order of the electroweak symmetry breaking scale. Both the ADD and the RS approach assume the fundamental mass scale to be of the order of a few TeV. Of course, this does not automatically get rid of the hierarchy problem as we now have to devise a mechanism to obtain the Planck scale from this fundamental scale. Another difference between these models and the KK models is that while in case of the latter, higher-dimensional Poincaré invariance is broken spontaneously by compactification, in the brane world models it is broken explicitly due to the presence of the brane.

Let us briefly discuss the ADD model. This model proposes that the scale of gravity is $\approx M_{\mathrm{ew}}$ in the bulk space-time. To make this proposal consistent with observations we need to change the nature of gravity so that this scale is related to 4D Planck mass in a natural way. This is achieved by assuming ordinary matter and gauge fields are localized on a 3-brane which is embedded in a higher-dimensional bulk. Only gravity propagates in higher dimensions and becomes strongly coupled at the TeV scale. This proposal ignores brane tension (mass per unit volume) and consequently backreaction of it on the geometry transverse to the brane. In the simplest form it considers compact extra dimensions of radius R. Only gravity becomes higher dimensional at length scales smaller than R. Since Newton's law in 4D is experimentally tested up to 0.2 mm, to be consistent with it, extra dimensions should be smaller than 0.2 mm but can be as large as, say, 0.1 mm. In other words, fundamental scale of higher-dimensional gravity is $M \sim 10^3$ GeV. One starts with a $4+d$ dimensional Einstein action,

$$S = -\frac{1}{16\pi G_{(4+d)}} \int \mathrm{d}^4 x \mathrm{d}^d y \sqrt{g_{(4+d)}} R_{(4+d)} \tag{1}$$

where $G_{(4+d)} \sim M^{-2-d}$. 4D gravity is obtained by simple dimension reduction and by ignoring massive KK modes. Since d-dimensional space is compactified on a flat torus, wave function of the zero mode is homogeneous in extra dimensions. 4D effective action for gravity

$$S_{\mathrm{eff}} = \frac{V_d}{16\pi G_{(4+d)}} \int \mathrm{d}^4 x \sqrt{g_{(4)}} R_{(4)}, \tag{2}$$

where $V_d \sim R^d$ is the volume of extra dimensions. We can now read off the 4D Newton's constant $G_{\mathrm{N}} = G^{(4+d)}/V_d = M^{-2-d}R^{-d}$, or equivalently, $M_{\mathrm{Pl}} = M(MR)^{d/2}$. Planck mass M_{Pl} is a derived quantity and can be made larger by having large extra dimensions. While this proposal removes $M_{\mathrm{ew}}/M_{\mathrm{Pl}}$ hierarchy, it resurfaces as a hierarchy of scales between the compactification scale $\sim 10^{-2}$ cm and the electroweak scale $\sim 10^{-16}$ cm. Alternatively, we have very light KK mass spectrum $m \sim meV$. Coupling of these to SM fields is very weak ($\approx$ 4D gravitational interaction). Low energy effective theory (SM+gravity) is valid upto $E \sim M$. New physics appears at energies $E \sim M$. If it is in the form of a string theory, then new physics involves massive stringy modes.

In ADD model, KK modes are uniformly distributed in the internal dimensions. We can get weak gravity coupling if we make them spend most of their time away from the visible brane thereby reducing their overlap with the physical brane. This precisely is the motivation of RS model. In fact, one automatically arrives at this result if one takes branes tension and its backreaction on the geometry into account.
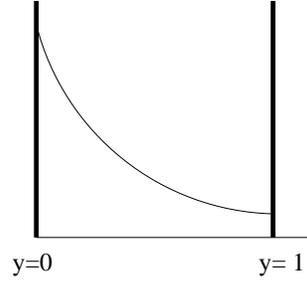
Figure 1. 3-branes at $y = 0$ and 1 with exponential warp factor.

RS model consists of 5D gravity with cosmological constant and 3-brane sources localized on a circle at $y = 0$ and 1. These two points on the circle are fixed point of Z_2 reflection symmetry

$$S = -\frac{1}{16\pi G_{(5)}} \int d^4x dy \sqrt{g_{(5)}} R_{(5)} - \Lambda \int d^4x dy \sqrt{g_{(5)}}$$

$$- \sum_{i=1}^{2} V_i \int d^4x \sqrt{g_{(4)i}} \tag{3}$$

where $16\pi G_{(5)} = 1/2M^3$ and V_0 and V_1 are brane tensions. Most general solution compatible with 4D Poincaré invariance and Z_2 reflection symmetry is

$$ds^2 = \exp(-2\pi kr|y|)\eta_{\mu\nu}dx^\mu dx^\nu + \pi^2 r^2 dy^2 \tag{4}$$

with $V_0 = -V_1 = 24M^3k$ and $\Lambda = -24M^3k^2$. Thus, 3-brane at $y = 0$ has positive tension (Planck brane), the one at $y = 1$ has negative tension (SM brane), and bulk cosmological constant is negative. Warp factor $\exp(-2\pi kr|y|)$ falls exponentially from the Planck brane to SM brane. Graviton zero mode is peaked near Planck brane and away from SM brane. 4D Planck mass can be determined by analysing normalized graviton zero mode, $M_{\text{Pl}}^2 = (M^3/k)(\exp(2kr\pi) - 1)$. Thus, $kr \approx 12$ gives the desired result, $M_{\text{Pl}} \sim 10^{19}$ GeV. This seems to solve the hierarchy problem without involving unnaturally large numbers. However, we have to tune $V_0 = -V_1 = 24M^3k$ and this is like fine tuning.

There are a couple of caveats in the RS model. Firstly, note $kr \sim 12$ implies $r \sim 12/k$, but fluctuation analysis shows there is no potential for the radius parameter (radion). A mechanism is needed to stabilise radion at $r \sim 12/k$. Secondly, negative tension branes are not stable solutions of field theory. In fact, it is possible to stabilise radion to its expected value without using unnatural parameters [5] by coupling RS model to bulk scalars. Even the second hurdle can be overcome if we embed RS model in string theory because stable objects with negative tension, e.g., orientifold planes, higher-dimensional branes wrapped on compact manifolds exist in string theory. It is therefore desirable to look for a stringy realisation of RS scenario. We will briefly review one of the attempts below.

Consider a Calabi–Yau compactification of type IIB string theory [6]. Bosonic sector of the massless spectrum of type IIB string theory consists of a metric g_{MN}, a complex scalar $\tau = C + i\exp(-\phi)$, a complex 2nd rank tensor $A_{MN} = B_{MN} + iC_{MN}$ and a self-dual 4th rank tensor D_{MNPQ}^+. Third and fifth rank field strength are defined as

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$$H_{MNP} = \partial_{[M} B_{NP]}, \quad F_{MNP} = \partial_{[M} C_{NP]},$$

$$G_{MNP} = H_{MNP} - \tau F_{MNP}, \quad F_{MNPQR} = \partial_{[M} D^{+}_{NPQR]}. \tag{5}$$

Most general ansatz for metric and fifth rank self-dual field strength compatible with 4D Poincaré invariance is (Our conventions are $\mu, \nu = 0, 1, 2, 3; m, n = 4, \dots, 9$.)

$$ds_{10}^2 = e^{2A(y)} \eta_{\mu\nu} dx^\mu dx^\nu + e^{-2A(y)} g_{mn}(y) dy^m dy^n \tag{6}$$

$$F_{(5)} = (1 + *)[d\alpha(y) \wedge dx^0 \wedge dx^1 \wedge dx^2 \wedge dx^3]. \tag{7}$$

In the conventional Calabi–Yau compactification, τ is not a function of internal coordinates. However in F-theory, τ is a function of internal coordinates y such that it solves

$$R_{mn} = \frac{\partial_m \tau \partial_n \bar\tau + \partial_n \tau \partial_m \bar\tau}{4(\text{Im } \tau)^2} + \left(T_{mn} - \frac{1}{8} g_{mn} T \right)_{D7} \tag{8}$$

$$\nabla^2 \tau = \frac{\nabla\tau \cdot \nabla\tau}{i \, \text{Im } \tau} - \frac{4(\text{Im } \tau)^2}{\sqrt{g}} \frac{\delta S_{D7}}{\delta\bar\tau}. \tag{9}$$

Using the ansatz (6), (7), 4D Einstein equation becomes

$$\nabla_y^2 e^{4A} = e^{2A} \frac{G_{mnp} \bar G^{mnp}}{12 \, \text{Im } \tau} e^{-6A} [\partial_m \alpha \partial^m \alpha + \partial_m e^{4A} \partial^m e^{4A}] + \frac{1}{2} e^{2A} (T_m^m - T_\mu^\mu)_{\text{loc}}. \tag{10}$$

We also need to check the Bianchi identity, $dF_5 = H_3 \wedge F_3 + 2T_3 \rho_3^{\text{loc}}$, where T_3 and ρ_3 are 3-brane tension and charge density respectively. Integrated Bianchi identity of F_5 gives the condition $\int_{M_6} H_3 \wedge F_3 + 2T_3 Q_{3,\text{loc}} = 0$. Since the first term is positive semidefinite, this condition is satisfied only when $Q_{3,\text{loc}}$ is negative [6]. Putting the ansatz (7), in the equation of motion/Bianchi identity

$$\nabla_y^2 \alpha = i e^{2A} \frac{G_{mnp}(*_6 \bar G^{mnp})}{12 \, \text{Im } \tau} + 2 e^{-6A} [\partial_m \alpha \partial^m e^{4A}] + 2 e^{2A} T_3 \rho_{\text{loc}}. \tag{11}$$

Combining (11) and (10), we get

$$\nabla_y^2 (e^{4A} - \alpha) = \frac{e^{2A}}{24 \, \text{Im } \tau} |i G_3 - *_6 G_3|^2 + e^{-6A} |\partial(e^{4A} - \alpha)|^2$$
$$+ 2 e^{2A} \left[\frac{1}{4} (T_m^m - T_\mu^\mu)_{\text{loc}} - T_3 \rho_{\text{loc}} \right]. \tag{12}$$

Suppose all the localised sources satisfy $(T_m^m - T_\mu^\mu)_{\text{loc}} = 4 T_3 \rho_{\text{loc}}$ (While this is true for D3-branes, O3 planes as well as D7-branes wrapped on a compact 4 manifold, it is not true for D5-brane wrapped on a compact 2 manifold.), then we we get the following solution:

$$i G_3 = *_6 G_3, \quad e^{4A} = \alpha. \tag{13}$$

Thus we see that warp factor is proportional to the flux of fifth rank field strength and third rank field strength is imaginary self-dual on the Calabi–Yau. It is worth pointing out here that so far we have not made any assumption about supersymmetry. It is also easy to check that all the equations are invariant under $g_{mn} \rightarrow l^2 g_{mn}$, which implies that the size of Calabi–Yau manifold is a free parameter.

Of course, loop corrections will destroy both these features of the non-supersymmetric solution. Therefore, it is desirable to look for a supersymmetric solution. The conditions for $N = 1$ supersymmetry (SUSY) in 4D, requires the compact manifold to be Kähler. The G_3 flux must have index structure $G_{ij\bar{k}}$ in terms of the complex coordinates. $G_{ij\bar{k}} g^{j\bar{k}} = 0$, this condition is equivalent to imaginary self-duality. Conventional Calabi–Yau compact-ification, typically give rise to sevaral massless scalar fields. Turning on VEVs for these fields correspond to deformations of the Calabi–Yau manifold. In the real world, no such massless scalars are known. Furthermore, in $N = 2$ field theories it is possible to break $N = 2$ SUSY down to $N = 1$ by adding $N = 2$ FI term which gives mass to the complex scalar. There must be a similar mechanism on the string theory side. Let us recall that $N = 1$ SUSY requires the index structure of G_3 as $G_{ij\bar{k}}$. Other possible index structures are G_{ijk}, $G_{\bar{i}\bar{j}\bar{k}}$ and $G_{i\bar{j}\bar{k}}$. Of these, first one is proportional to the analytic 3-form on the Calabi–Yau and the second one is proportional to the anti-analytic 3-form. Absence of these terms indicate that the superpotential is of the form $W = \int \Omega \wedge G$, where $\Omega = dz^1 \wedge dz^2 \wedge dz^3$ is the analytic 3-form. Let us consider a Calabi–Yau manifold with fixed size, i.e., Kähler parameters are frozen, and study only shape deformations [6]. Shape deformations are characterized by third rank tensors which is related to the complex structure of the Calabi–Yau manifold.

Suppose these shape deformations are denoted by τ_{ij} then SUSY vacuum is given by [6],

$$W = 0, \quad \frac{\partial W}{\partial \tau_{ij}} = 0, \quad \frac{\partial W}{\partial \tau} = 0. \tag{14}$$

These three equations imply G_3 is not $(0,3)$ type, $(1,2)$ type or $(3,0)$ type. This is consistent with the earlier result of $N = 1$ SUSY vacuum. Explicit superpotential can be determined by taking a specific Calabi–Yau manifold. Using singular limit of Calabi–Yau, it can be shown that most of the complex structure moduli can be frozen [6].

In summary, the ADD model proposes to solve the $M_{\text{ew}}/M_{\text{Pl}}$ hierarchy problem using the brane world model. It leads to a new kind of model but fails to solve hierarchy problem. The RS model improves upon the ADD model by taking brane tension into account. It involves warped/non-factorisable geometry. The warp factor helps relate M_{ew} to M_{Pl}. RS model also involves a subtle fine-tuning of brane tensions but is still a better solution of hierarchy problem. Negative tension objects make stringy realisation of RS model essential for their survival. In the string theory realisation, warp factor is proportional to the flux of fifth rank field strength. This model requires non-constant τ field in the internal direction. This signals existence of 7-branes wrapped on a four-dimensional compact submanifold of the Calabi–Yau. Non-trivial warp factor also requires non-zero flux of third rank field G_3. Clearly, more analysis is required for freezing Kähler parameters. It would be interesting to see if one can construct semi-realistic models using this kind of string theoretic embedding.

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