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# Expressions for third-order aberration theory for holographic images

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**Abstract.** Expressions for third-order aberration in the reconstructed wave front of point objects are established by Meier. But Smith, Neil Mohon, Sweatt independently reported that their results differ from that of Meier. We found that coefficients for spherical aberration, astigmatism, tally with Meier's while coefficients for distortion and coma differ.

**Keywords.** Spherical aberration coefficient; coma; distortion; astigmatism.

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## 1. Introduction

Tolerance limit for various types of aberrations has an important bearing in the design and fabrication of holographic microscope and other optical elements. There is a well established aberration theory in the case of conventional optical elements. It was Meier [1] who pioneered third-order aberration theory for applications in holography. There are reports in the literature contrary to Meier's results, e.g.,

(i) Smith [2] recalculated third-order aberration on the lines of Meier and tabulated his results. Some of his results differed from Meier's reported results.
(ii) Neil Mohon [3] also repeated Meier's method of calculation but left the results half done calculating up to spherical aberration only.
(iii) Sweatt [4] pictured the point hologram as a lens by attributing fictitious refractive indices to the hologram. Based on his theory he deduced aberration expressions. He concluded that some of his results agreed with Meier's and the rest with Smith's results.

We in this paper obtained the expressions for aberrations and found that the expressions for coefficients of spherical aberration, astigmatism fully agreed with those of Meier. Our expressions for distortion coefficient and coma do not tally with Meier's. The possible consequence of this disagreement is discussed in terms of aberration in holographic image.

## 2. Detailed analysis

A source of coherent radiation illuminates an object and the reflected (or transmitted) wave front propagates to the plane in which the hologram is to be recorded. A reference wave front derived from the same source interferes with the object wave front in the hologram plane. The intensity distribution recorded in the holographic plate is given by

$$\begin{aligned} I &= |\vec{\mathrm{O}}+\vec{\mathrm{R}}|^2 \\ &= |\vec{\mathrm{O}}| + |\vec{\mathrm{R}}| + |\vec{\mathrm{O}}^*|\,|\vec{\mathrm{R}}| + |\vec{\mathrm{O}}|\,|\vec{\mathrm{R}}^*| \end{aligned} \tag{1}$$

where $\vec{\mathrm{O}}$ and $\vec{\mathrm{R}}$ are the complex amplitudes of the object wave and reference wave respectively. After suitable exposure and processing, a hologram transparency may be obtained with the amplitude transmittance proportional to the intensity given by eq. (1).

When reconstructed with a coherent wave front of complex amplitude $\vec{\mathrm{C}}$, a wave front $\vec{\mathrm{H}}$ emerges from the hologram and it is given by

$$H = \vec{\mathrm{C}}|\vec{\mathrm{O}}|^2 + \vec{\mathrm{C}}|\vec{\mathrm{R}}|^2 + \vec{\mathrm{C}}\vec{\mathrm{O}}^*\vec{\mathrm{R}} + \vec{\mathrm{C}}\vec{\mathrm{O}}\vec{\mathrm{R}}^*. \tag{2}$$

Two terms of interest are given by

$$\begin{aligned} \vec{\mathrm{H}}_{\mathrm{real}} &= \vec{\mathrm{C}}\cdot\vec{\mathrm{O}}^*\cdot\vec{\mathrm{R}} \\ \vec{\mathrm{H}}_{\mathrm{v}} &= \vec{\mathrm{C}}\cdot\vec{\mathrm{O}}\cdot\vec{\mathrm{R}}^*. \end{aligned} \tag{3}$$

One of them is diverging and other is converging. The phase of the two emergent beams are

$$\phi_{\mathrm{v}} = \phi_{\mathrm{C}} + \phi_{\mathrm{O}} - \phi_{\mathrm{R}} \tag{4a}$$

$$\phi_{\mathrm{real}} = \phi_{\mathrm{C}} - \phi_{\mathrm{O}} + \phi_{\mathrm{R}} \tag{4b}$$

where $\phi_{\mathrm{C}}$, $\phi_{\mathrm{O}}$ and $\phi_{\mathrm{R}}$ are the phases due to the coherent wave, object wave and reference wave.

To find the actual phase of the beams in terms of experimental parameters, we consider a point object $\mathrm{P_o}$ $(x_{\mathrm{o}}, y_{\mathrm{o}}, z_{\mathrm{o}})$ in a coordinate system shown in figure 1 whose origin lies on the centre of the holographic plate with the $x$- and $y$-axes in the plane of the emulsion. Let the wavelength of light from $\mathrm{P_o}$ be $\lambda_{\mathrm{o}}$.

The phase of the spherical wave front $\mathrm{P_o}$ at Q in the plane of the hologram, relative to its phase at the origin is given by

$$\begin{aligned} \phi_{\mathrm{o}}(x,y) &= \frac{2\pi}{\lambda_{\mathrm{o}}}(\overline{\mathrm{P_oQ}} - \overline{\mathrm{P_oO}}) \\ &= \frac{2\pi}{\lambda_{\mathrm{o}}} z_{\mathrm{o}} \left[ \left\{ 1 + \frac{(x_{\mathrm{o}}-x)^2 + (y_{\mathrm{o}}-y)^2}{z_{\mathrm{o}}^2} \right\}^{1/2} \right. \\ &\qquad \left. - \left( 1 + \frac{x_{\mathrm{o}}^2 + y_{\mathrm{o}}^2}{z_{\mathrm{o}}^2} \right)^{1/2} \right]. \end{aligned} \tag{5}$$

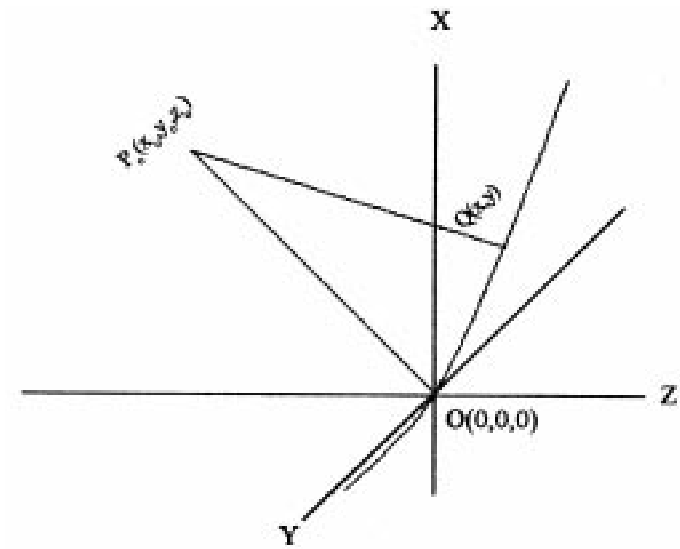


**Figure 1.** Schematic diagram illustrating the phases of spherical wave front in hologram plane.

In writing the above equation we have assumed that

$$z_o^2 \gg \{(x_o - x)^2 + (y_o - y)^2\} \quad \text{(see figure)}. \tag{6}$$

Considering up to third-order terms and neglecting higher order terms in $1/z_o, \phi_o(x,y)$ is given by

$$\begin{aligned}\phi_o(x,y) = &\left(\frac{2\pi}{\lambda_o}\right)\left(\frac{1}{2z_o}\right)(x^2 + y^2 - 2xx_o - 2yy_o) \\ &- \frac{2\pi}{\lambda_o}\left(\frac{1}{2z_o}\right)^3 (x^4 + y^4 + 2x^2y^2 - 4x^3x_o - 4x^3y_o \\ &- 4x^2yy_o - 4xy^2x_o + 6x^2x_o^2 + 6y^2y_o^2 + 2x^2y_o^2 + 2y^2x_o^2 \\ &+ 8xyx_oy_o - 4x_o^3x - 4y_o^3y - 4x_oy_o^2 - 4yx_o^2y_0).\end{aligned} \tag{7}$$

If a point source at $(x_r, y_r, z_r)$ supplies the reference wave, also $\lambda_o$, then a similar expression holds for the phase of reference wave $\phi_R$, with $(x_o, y_o, z_o)$ replaced by $(x_r, y_r, z_r)$.

Similarly if a point source at $(x_c, y_c, z_c)$ supplies reconstruction wave, its phase is given by the same expression with the substitution of $(x_c, y_c, z_c)$ for $(x_o, y_o, z_o)$ and $\lambda_c$, the reconstructing wavelength for $\lambda_o$. Thus the expression for $\phi_v$ can be written using eq. (4) as

$$\begin{aligned}\phi_v = \frac{\pi}{\lambda_c}\Bigg\{ & x^2\left(\frac{1}{z_c} - \frac{6x_c^2}{4z_c^3} - \frac{2y_c^2}{4z_c^3} + \frac{\mu}{z_o} - \frac{6\mu x_o^2}{4z_o^3} - \frac{\mu}{z_r} + \frac{\mu 6x_r^2}{4z_r^3} + \frac{2y_r^2}{4z_r^3}\right) \\ & + y^2\left(\frac{1}{z_c} - \frac{6y_c^2}{4z_c^3} - \frac{2x_c^2}{4z_c^3} + \frac{\mu}{z_o} - \frac{6\mu y_o^2}{4z_o^3} - \frac{2\mu x_c^2}{4z_o^3} - \frac{\mu}{z_r} + \frac{\mu 6y_r^2}{4z_r^3} + \frac{2x_r^2}{4z_r^3}\right)\end{aligned}$$

$$
\begin{aligned}
&-2x\left(\frac{x_{\rm c}}{z_{\rm c}}\frac{2x_{\rm c}^3}{4z_{\rm c}^3}-\frac{2x_{\rm c}y_{\rm c}^2}{4z_{\rm c}^3}+\frac{x_{\rm o}\mu}{z_{\rm o}}-\frac{2\mu x_{\rm o}^3}{4z_{\rm o}^3}\right.\\
&\qquad\left.-\frac{\mu^2x_{\rm o}y_{\rm o}^2}{4z_{\rm o}^3}-\frac{\mu x_{\rm r}}{z_{\rm r}}+\frac{2\mu x_{\rm r}^3}{4z_{\rm r}^3}+\frac{2\mu x_{\rm r}y_{\rm r}^2}{4z_{\rm r}^3}\right)\\
&-2y\left(\frac{y_{\rm c}}{z_{\rm c}}\frac{2y_{\rm c}^3}{4z_{\rm c}^3}-\frac{2y_{\rm c}x_{\rm c}^2}{4z_{\rm c}^3}+\frac{y_{\rm o}\mu}{z_{\rm o}}-\frac{2\mu y_{\rm o}^3}{4z_{\rm o}^3}-\frac{2\mu y_{\rm o}x_{\rm o}^2}{4z_{\rm o}^3}\right.\\
&\qquad\left.-\frac{\mu y_{\rm r}}{z_{\rm r}}+\frac{2\mu y_{\rm r}^3}{4z_{\rm r}^3}+\frac{2\mu y_{\rm r}x_{\rm r}^2}{4z_{\rm r}^3}\right)\\
&-x^4\left(\frac{1}{4z_{\rm c}^3}+\frac{\mu}{4z_{\rm o}^3}-\frac{\mu}{4z_{\rm r}^3}\right)-y^4\left(\frac{1}{4z_{\rm c}^3}+\frac{\mu}{4z_{\rm o}^3}-\frac{\mu}{4z_{\rm r}^3}\right)\\
&-2x^2y^2\left(\frac{1}{4z_{\rm c}^3}+\frac{\mu}{4z_{\rm o}^3}-\frac{\mu}{4z_{\rm r}^3}\right)+4x^3\left(\frac{x_{\rm c}}{4z_{\rm c}^3}+\frac{\mu x_{\rm o}}{4z_{\rm o}^3}-\frac{\mu x_{\rm r}}{4z_{\rm r}^3}\right)\\
&+4y^3\left(\frac{y_{\rm c}}{4z_{\rm c}^3}+\frac{\mu y_{\rm o}}{4z_{\rm o}^3}-\frac{\mu y_{\rm r}}{4z_{\rm r}^3}\right)+4xy^2\left(\frac{x_{\rm c}}{4z_{\rm c}^3}+\frac{\mu x_{\rm o}}{4z_{\rm o}^3}-\frac{\mu x_{\rm r}}{4z_{\rm r}^3}\right)\\
&+4x^2y\left(\frac{y_{\rm c}}{4z_{\rm c}^3}+\frac{\mu y_{\rm o}}{4z_{\rm o}^3}-\frac{\mu y_{\rm r}}{4z_{\rm r}^3}\right)-8xy\left(\frac{x_{\rm c}y_{\rm c}}{4z_{\rm c}^3}+\frac{\mu x_{\rm o}y_{\rm o}}{4z_{\rm o}^3}-\frac{\mu x_{\rm r}y_{\rm r}}{4z_{\rm r}^3}\right)\Bigg\}
\end{aligned}
\tag{8}
$$

where $\lambda_{\rm o}/\lambda_{\rm c}=\mu$.

We now consider eq. (8) as representing the Gaussian sphere with $z_{\rm v}$ its radius and $a_{\rm v}, b_{\rm v}$ the coordinates of its centre which determine the Gaussian image point.

## 3. Derivation of third-order aberration

The third-order term of the phase of the Gaussian reference sphere is given by

$$
\begin{aligned}
\phi_{\rm v}=&\left(\frac{\pi}{\lambda_{\rm c}}\right)\left(\frac{1}{2z_{\rm v}}\right)(x^2+y^2-2xa_{\rm v}-2yb_{\rm v})\\
&-\frac{2\pi}{\lambda_{\rm c}}\frac{1}{8z_{\rm v}^3}(x^4+y^4+2x^2y^2-4x^3a_{\rm v}-4y^3b_{\rm v}-4x^2yb_{\rm v}\\
&-4xy^2a_{\rm v}+6x^2a_{\rm v}^2+6y^2b_{\rm v}^2+2x^2b_{\rm v}^2+2y^2a_{\rm v}^2+8xya_{\rm v}b_{\rm v}-4xa_{\rm v}^3\\
&-4yb_{\rm v}^3-4xa_{\rm v}b_{\rm v}^2-4ya_{\rm v}^2b_{\rm v}^2)
\end{aligned}
\tag{9}
$$

where

$$
\frac{1}{z_{\rm v}}=\frac{1}{z_{\rm c}}+\frac{\mu}{z_{\rm o}}-\frac{\mu}{z_{\rm r}}-\frac{3x_{\rm c}^2}{2z_{\rm c}^3}+\frac{y_{\rm c}^2}{2z_{\rm c}^3}-\frac{3\mu x_{\rm o}^2}{2z_{\rm o}^3}-\frac{\mu y_{\rm o}^2}{2z_{\rm o}^3}+\frac{3\mu x_{\rm r}^2}{2z_{\rm r}^3}+\frac{y_{\rm r}^2}{2z_{\rm r}^3}. \tag{10}
$$

Changing to polar coordinates $\rho$ and $\theta$ defined by

$$
\rho^2=x^2+y^2,\quad x=\rho\cos\theta,\quad y=\rho\sin\theta, \tag{11}
$$

$$\phi_{\mathrm{v}} = \frac{2\pi}{\lambda_{\mathrm{c}}}\frac{1}{z_{\mathrm{v}}^3}\left\{\frac{-\rho^4}{8} + \frac{\rho^3}{2}(a_{\mathrm{v}}\cos\theta + b\sin\theta) \right.$$
$$-\rho^2/2(a_{\mathrm{v}}^2\cos^2\theta + b_{\mathrm{v}}^2\sin^2\theta + 2a_{\mathrm{v}}b_{\mathrm{v}}\cos\theta\sin\theta)$$
$$-\rho^2/4(a_{\mathrm{v}}^2 + b_{\mathrm{v}}^2) + \rho/2(a_{\mathrm{v}}^3 + a_{\mathrm{v}}b_{\mathrm{v}}^2)\cos\theta$$
$$\left. +(b_{\mathrm{v}}^3 + a_{\mathrm{v}}^2 b_{\mathrm{v}})\sin\theta \right\}. \qquad (12)$$

The third-order term of the phase of the actual wave front is given by the third-order term $\phi_{\mathrm{v}} =$ third-order term of $\phi_{\mathrm{c}}+$ third-order term of $\phi_{\mathrm{o}}-$third-order term of $\phi_{\mathrm{r}}$

$$= \frac{-2\pi}{\lambda_{\mathrm{c}}}\frac{1}{8z_{\mathrm{c}}^3}\{x^4 + y^4 + 2x^2y^2 - 4x^3x_{\mathrm{c}} - 4y^3y_{\mathrm{c}} - 4x^2yy_{\mathrm{c}}$$
$$-4xy^2x_{\mathrm{c}} + (4+2)x^2x_{\mathrm{c}}^2 + 6y^2y_{\mathrm{c}}^2 + 2x^2y_{\mathrm{c}}^2 + 2y^2x_{\mathrm{c}}^2$$
$$+8xyx_{\mathrm{c}}y_{\mathrm{c}} - 4xx_{\mathrm{c}}^3 - 4yy_{\mathrm{c}}^3 - 4xx_{\mathrm{c}}y_{\mathrm{c}}^2 - 4yx_{\mathrm{c}}^2y_{\mathrm{c}}\}$$
$$\frac{-2\pi}{\lambda_{\mathrm{o}}}\frac{1}{8z_{\mathrm{o}}^3}\{x^4 + y^4 + 2x^2y^2 - 4x^3x_{\mathrm{o}} - 4y^3y_{\mathrm{o}} - 4x^2yy_{\mathrm{o}}$$
$$-4xy^2x_{\mathrm{o}} + (4+2)x^2x_{\mathrm{o}}^2 + 6y^2y_{\mathrm{o}}^2 + 2x^2y_{\mathrm{o}}^2 + 2y^2x_{\mathrm{o}}^2$$
$$+8xyx_{\mathrm{o}}y_{\mathrm{o}} - 4xx_{\mathrm{o}}^3 - 4yy_{\mathrm{o}}^3 - 4xx_{\mathrm{o}}y_{\mathrm{o}}^2 - 4yx_{\mathrm{o}}^2y_{\mathrm{o}}\}$$
$$\frac{-2\pi}{\lambda_{\mathrm{r}}}\frac{1}{8z_{\mathrm{r}}^3}\{-x^4 - y^4 - 2x^2y^2 + 4x^3x_{\mathrm{r}} + 4y^3y_{\mathrm{r}} + 4x^2yy_{\mathrm{r}}$$
$$+4xy^2y_{\mathrm{r}} + 4xy^2x_{\mathrm{r}} - (4+2)x^2x_{\mathrm{r}}^2 - 6y^2y_{\mathrm{r}}^2 - 2x^2y_{\mathrm{r}}^2 - 2y^2x_{\mathrm{r}}^2$$
$$-8xyx_{\mathrm{r}}y_{\mathrm{r}} + 4xx_{\mathrm{r}}^3 + 4yy_{\mathrm{r}}^3 + 4xx_{\mathrm{r}}y_{\mathrm{r}}^2 + 4yx_{\mathrm{r}}^2y_{\mathrm{r}}\} \qquad (13)$$

Wave front aberrations are defined as the phase difference between Gaussian sphere and actual wave front in the hologram plane.

## 4. Evaluation of spherical aberration coefficient $S_{\mathrm{v}}$, coefficients for coma $C_x$, astigmatism $A_x$ and distortion $D_x$

The spherical aberration coefficient $S_{\mathrm{v}}$, is the difference between the coefficient $-(2\pi/\lambda_{\mathrm{c}})$ $(x^4/8)$ from eq. (13) and coefficient of $(2\pi/\lambda_{\mathrm{c}})(\rho^4/8)$ from eq. (12) and can be written as

$$S_{\mathrm{v}} = \mu\left\{(-\mu^2+1)\left(\frac{1}{z_{\mathrm{o}}^3} - \frac{1}{z_{\mathrm{r}}^3}\right) - \frac{3\mu}{z_{\mathrm{c}}}\left(\frac{1}{z_{\mathrm{o}}^2} - \frac{1}{z_{\mathrm{r}}^2}\right)\right.$$
$$\left. +3\left(-\frac{1}{z_{\mathrm{c}}^2} + \frac{\mu^2}{z_{\mathrm{o}}z_{\mathrm{r}}}\right)\left(\frac{1}{z_{\mathrm{o}}} - \frac{1}{z_{\mathrm{r}}}\right) + \frac{6\mu}{z_{\mathrm{o}}z_{\mathrm{c}}z_{\mathrm{r}}}\right\}. \qquad (14)$$

Similarly the aberration coefficients for coma, astigmatism and distortion can be obtained by taking the difference between the coefficient $-(2\pi/\lambda_{\mathrm{c}})(x^n/2)$ from eq. (13) and coefficient of $(2\pi/\lambda_{\mathrm{c}})(\rho^n/2)$ from eq. (12). When $n = 3$, one gets coma coefficient $C_x$, for $n = 2$, astigmatism coefficient $A_x$ and for $n=1$ distortion coefficient, $D_x$. These coefficients are found to be

$$C_x = \frac{x_c}{z_c}\left\{\frac{1}{z_c^2} - \left(\frac{1}{z_c} + \frac{\mu}{z_o} - \frac{\mu}{z_r}\right)^2\right\} + \frac{\mu x_o}{z_o}\left\{\frac{1}{z_o^2} - \left(\frac{1}{z_c} + \frac{\mu}{z_o} - \frac{\mu}{z_r}\right)^2\right\} - \frac{\mu x_r}{z_r}\left\{\frac{1}{z_r^2} - \left(\frac{1}{z_c} + \frac{\mu}{z_o} - \frac{\mu}{z_r}\right)^2\right\} \quad (15)$$

$$A_x = \frac{x_c^2}{z_c^3} + \frac{\mu x_o^2}{z_o^3} - \frac{\mu x_r^2}{z_r^3} - \left(\frac{x_c}{z_c} + \frac{\mu x_o}{z_o} - \frac{\mu x_r}{z_r}\right)^2 \left(\frac{1}{z_c} + \frac{\mu}{z_o} - \frac{\mu}{z_r}\right) \quad (16)$$

and

$$D_x = -\frac{\mu x_o^3(\mu^2-1)}{z_o^3} - \frac{3\mu^2 x_o^2}{z_o^2}\left(\frac{x_c}{z_c} - \frac{\mu x_r}{z_r}\right) + \frac{3\mu x_o}{z_o}\left(\frac{x_c}{z_c} - \frac{\mu x_r}{z_r}\right)^2 + \frac{3\mu x_c x_r}{z_r z_c}\left(\frac{x_c}{z_c} - \frac{\mu x_r}{z_r}\right) + \frac{\mu x_r^3(\mu^2-1)}{z_r^3} - \frac{\mu^2 y_o^2}{z_o^2}\left(\frac{x_c}{z_c} - \frac{\mu x_r}{z_r}\right) - \frac{\mu x_o y_o^2(\mu^2-1)}{z_o^3}. \quad (17)$$

**Table 1.** Comparison of aberration results with that of Meier's results.

| Aberration coefficient | Meier's expression for $\mu = m = 1$ and $z_c = \infty$ | Expression using eqs (14)–(17) for $\mu = 1$ and $z_c = \infty$ |
|---|---|---|
| Spherical aberrations coefficient $S_v$ | $\frac{3}{z_o z_r}\left(\frac{1}{z_o} - \frac{1}{z_r}\right)$ | $\frac{3}{z_o z_r}\left(\frac{1}{z_o} - \frac{1}{z_r}\right)$ |
| Coma coefficient $C_s$ | $\frac{x_o}{z_o}\left\{\frac{1}{z_o^2} - \left(\frac{1}{z_o} - \frac{1}{z_r}\right)^2\right\}$ | $\frac{x_o}{z_o}\left\{\frac{1}{z_o^2} - \left(\frac{1}{z_o} - \frac{1}{z_r}\right)^2\right\} - \frac{x_r}{z_r}\left\{\frac{1}{z_r^2} - \left(\frac{1}{z_r} - \frac{1}{z_o}\right)^2\right\}$ |
| Astigmatism coefficient $A_x$ | $\frac{x_o^2}{z_o^3} - \frac{x_r^2}{z_r^3} - \left(\frac{x_o}{z_o} - \frac{x_r}{z_r}\right)^2 \left(\frac{1}{z_o} - \frac{1}{z_r}\right)$ | $\frac{x_o^2}{z_o^3} - \frac{x_r^2}{z_r^3} - \left(\frac{x_o}{z_o} - \frac{x_r}{z_r}\right)^2 \left(\frac{1}{z_o} - \frac{1}{z_r}\right)$ |
| Distortion coefficient $D_x$ | $\left(\frac{3x_o^2 x_r}{z_o^2 z_r} - \frac{3x_r^2 x_o}{z_r^2 z_o}\right)$ | $\left(\frac{3x_o^2 x_r}{z_o^2 z_r} - \frac{3x_r^2 x_o}{z_r^2 z_o}\right) + \frac{y_o^2 x_r}{z_o^2 z_r}$ |

## 5. Results and conclusion

Meier's expressions for third-order aberration become simplified under the requirement $\mu = 1$ and $z_c = \infty$. Ananda Rao and Pappu [5] have used these results in magnification of holographic images. In table 1 we have compared our aberration expressions with that of Meier's simplified expressions.

It is found from the table that our expressions for $S_v$, $A_x$ tally with that of Meier's results but coefficients $C_x, D_x$ do not tally. Many authors have used Meier's aberration results. For example Ananda Rao *et al* have used these results for magnification of holographic image. As pointed out by these authors magnification of any desired value can be obtained by suitably choosing the values of $z_o$ and $d$ (the distance between the object and the hologram) so that the aberrations should be within the tolerance limit. They have taken $z_o = -49.9$ cm, $z_r = -50$ cm for a magnification of 100× but unfortunately experimentally they realized the magnification of 40×. This they have pointed out is due to the low power of the laser beam used. But it is found that for magnification $40{\times}z_o$ and $z_r$ may be taken as $-19.5$ cm and $-20$ cm respectively. Under these values of $z_o$ and $z_r$, spherical aberration found using the Meier's result in table 1 came within the tolerance limit but not the coma. We think this discrepancy is due to the omission of one extra term in the expression for aberration coefficient of coma. Thus we believe that the magnification of 40× can be achieved approximately for the above mentioned values of $z_o$ and $z_r$, if the extra term in coma coefficient as mentioned in table 1 is included. It may be noted here that the third-order expression for magnification should be used instead of first order to get the exact value of magnification 40×.

In conclusion, we want to point out that in the third-order aberration coefficients of Meier's, some extra terms should be included in case of coma and distortion as mentioned in table 1. We have justified the addition of extra term in coma but failed to do so for distortion due to lack of experimental data.

## References


[1] R W Meier, *J. Opt. Soc. Am.* **55**, 987 (1965)
[2] H M Smith, *Principles of holography* (John Wiley & Sons, New York, 1969)
[3] W Neil Mohon, *Aberrations in holography lenses*, Naval Training Equipment Centre, Orlando, Florida, Report No TN-40 p. 12, June (1974)
[4] C Sweatt William, *Describing holography optical elements as lenses* (Optical Science Centre, University of Arizona, Tucson, Arizona, Technical Report, 1977)
[5] S Ananda Rao and S V Papu, *Opt. Laser Tech.* **12**, 35 (1980)