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Relativistic longitudinal non-Abelian oscillations in quark–antiquark plasma

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Abstract. We study the relativistic version of the non-Abelian, longitudinal wave in quark–antiquark plasma reported earlier by Bhat *et al* [*Phys. Rev.* **D39**, 649 (1989)]. We have also relaxed various approximations they made in their analysis. Both the quark and antiquark dynamics are taken in our analysis. The non-linearity arising from non-Abelian field as well as from plasma are included. Hence it is an exact longitudinal mode in relativistic quark–antiquark plasma, relevant to the study of quark gluon plasma. We find that earlier results are reproduced for non-relativistic and low amplitude oscillations, but are modified for relativistic or large amplitude waves. Further more, the above results are based on just four first-order equations for gauge invariant quantities derived from gauge covariant twelve first-order equations.

Keywords. Quark–antiquark plasma; non-Abelian waves; relativistic; non-linear.

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In relativistic heavy ion collisions (RHICs), it is believed that at sufficiently high energy density quark gluon plasma (QGP) will be formed and expands and hadronizes. Many experiments were conducted and analyzed to see the QGP in RHICs. Various signatures were proposed as diagnostic for QGP. Plasma by definition is a matter which exhibits collective behavior. Since QGP is a plasma governed by non-Abelian theory, namely, quantum chromodynamics (QCD), it is interesting to study new features due to non-Abelian effects. One such attempt was done by Bhat *et al* [1] using non-relativistic fluid equations and SU(2) gauge theory. They found a new non-Abelian oscillation, in addition to the normal Abelian oscillations for the weak non-Abelian coupling. These two modes repeat alternatively with time as the system evolve. For the strong non-Abelian coupling, these modes lead to chaotic behavior. We extend their work using relativistic fluid theory and relaxing various approximations they made in their work. Important approximations they made are, firstly, one of the quark, they assumed, to be infinite mass. Secondly, plasma non-linearity is neglected, hence some limitations on the validity of the results whereas here we study exact, fully non-linear, relativistic, non-Abelian (SU(2)) quark–antiquark plasma. Even though we start with relativistic fluid set of equations, non-Abelian Maxwell's equations and color dynamics equations, by making use of *moving frame ansatz* and with some algebra, we end up with just two second-order equations for gauge invariant quantities.

We consider a quark–antiquark plasma and each species, quarks and antiquarks, obey the relativistic equations of motion [2], given by

$$m\frac{\mathrm{d}u^{\mu}}{\mathrm{d}\tau} = gI_a G_a^{\mu\nu} u_{\nu}, \tag{1}$$

where m, τ and g are mass, proper time and QCD coupling constant respectively. u^{μ} is the 4-velocity of the relativistic particle, which is also the fluid velocity in cold plasma limit. Since our main goal is to look for non-Abelian features in QGP, we neglect thermal effects for simplicity. Thermal effect introduces new features like pressure gradients and viscosity terms in the equation of motion. $G_a^{\mu\nu}$ is the field tensor, defined as, $G_a^{\mu\nu} = \partial^{\mu} A_a^{\nu} - \partial^{\nu} A_a^{\mu} + g\varepsilon_{abc} A_b^{\mu} A_c^{\nu}$. I_a is a color charge and is a dynamical quantity and it's evolution is governed by Wong's equation [3],

$$\frac{\mathrm{d}I_a}{\mathrm{d}\tau} = -g\varepsilon_{abc} u_{\mu} A_b^{\mu} I_c. \tag{2}$$

Fluid motion and color fluctuations give rise to the current, $j_a^{\nu} = g\sum nI_a u^{\nu}$, which in turn gives self-consistent vector potential A_a^{μ} obeying the equation

$$\partial_{\mu} G_a^{\mu\nu} + g\varepsilon_{abc} A_{\mu b} G_c^{\mu\nu} = j_a^{\nu}, \tag{3}$$

where n is the density to be determined by continuity equation

$$\partial_{\mu}(nu^{\mu}) = 0, \tag{4}$$

where summation $\sum$ is over different species; quark and antiquark. Originally, this picture of plasma described by fluid theory and self-consistently produced chromomagnetic fields along with the concept of dynamical color charge, was studied by Kajantie and Montonen [4]. They were studying some specific solutions like plane wave type for Yang–Mills fields in the plasma.

Since we are interested only in longitudinal oscillations, let us put A_{ya} and A_{za} equal to zero. We also consider a special case where all quantities depend only on t and x. Since our equations are all covariant, we can work in any frame. Let us consider a frame where $\partial^{\mu} \equiv (0, -\partial/\partial x)$, i.e., static solution. This is equivalent to a *moving frame ansatz* at non-relativistic limit, widely used in the non-linear study of plasma physics as well as the earlier study in QGP by Bhat *et al*. In this frame, after some algebra, we get, from eq. (3)

$$\ddot{A}_{xa} - g\gamma\varepsilon_{abc} A_{xb} \dot{A}_{xc} = -gn_0 \sum I_a \frac{\bar{u}_0}{\bar{u}_x}, \tag{5}$$

where quantities with bar are in new frame and dot refers to differentiation with respect to x. $\bar{u}_0$ and $\bar{u}_x$ are zeroth and x components of 4-vector $\bar{u}^{\mu}$. n_0 is the density of quarks or antiquarks at equilibrium. We have also chosen a gauge $A_{0a} = 0$. From equations of motion, eq. (1), we also get $(\bar{u}_0/\bar{u}_x) = -1/\sqrt{1-(\gamma+\gamma\beta(gI_a A_{xa}/m))^{-2}}$, where we assumed that at equilibrium, i.e., $A_{xa} = 0$, plasma is at rest, i.e., $(\bar{u}_0/\bar{u}_x) = -1/\beta$. Here we assumed that the frame is moving with velocity β and $\gamma = 1/\sqrt{1-\beta^2}$ so on. We also get a constraint equation from eq. (3)

$$\sum I_a = \frac{\beta\gamma}{n_0} \varepsilon_{abc} \dot{A}_b A_c, \tag{6}$$

where we have dropped subscript x for vector potential. Making use of the above relations, eq. (5) may be written as

$$\ddot{A}_a + g\gamma\varepsilon_{abc}\dot{A}_b A_c \left(1+\beta\frac{\bar{u}_{02}}{\bar{u}_2}\right) = -gn_0 I_a \left(\frac{\bar{u}_{01}}{\bar{u}_1} - \frac{\bar{u}_{02}}{\bar{u}_2}\right), \tag{7}$$

where $I_a \equiv I_{1a}$ and subscripts 1 and 2 refers to quark and antiquark. Similarly, color dynamics equation becomes

$$\dot{I}_a = g\gamma\varepsilon_{abc}A_b I_c \left(1+\beta\frac{\bar{u}_{01}}{\bar{u}_1}\right). \tag{8}$$

Let us rescale the variables as $(I_a/I_0) \to I_a, (gI_0 A_a/m\gamma^2\beta) \to A_a$ etc. and we get

$$\ddot{A}_a + \gamma^4\beta^2\varepsilon_0\varepsilon_{abc}\dot{A}_b A_c \left(1+\beta\frac{\bar{u}_{02}}{\bar{u}_2}\right) = -\frac{I_a}{2}\left(\beta\frac{\bar{u}_{01}}{\bar{u}_1} - \beta\frac{\bar{u}_{02}}{\bar{u}_2}\right), \tag{9}$$

and

$$\dot{I}_a = \gamma^4\beta^2\varepsilon_0\varepsilon_{abc}A_b I_c \left(1+\beta\frac{\bar{u}_{01}}{\bar{u}_1}\right), \tag{10}$$

where $\omega_p^2 \equiv 2g^2 I_0^2 n_0/m\gamma^2\beta^2$ and $\varepsilon_0 \equiv m\sqrt{m}/gI_0^2\sqrt{n_0}$. We have also assumed that quark and antiquark is of the same flavor so that $m_1 = m_2 = m$. Further, defining $f_i = (1+\beta\bar{u}_{0i}/\bar{u}_i)$, $X \equiv \sum_a I_a A_a$ and $\varepsilon = \gamma^4\beta^2\varepsilon_0$ etc., eqs (9), (10) reduce to

$$\ddot{A}_a + \varepsilon\varepsilon_{abc}\dot{A}_b A_c f_2(X) = -\frac{I_a}{2}(f_1(X) - f_2(X)), \tag{11}$$

and

$$\dot{I}_a = \varepsilon\varepsilon_{abc}A_b I_c f_1(X), \tag{12}$$

where $\bar{u}_{01}/\bar{u}_1$ and $\bar{u}_{02}/\bar{u}_2$ are given by

$$\frac{\bar{u}_{01}}{\bar{u}_1} = -1\Bigg/\sqrt{1-\frac{(1-\beta^2)}{(1+\gamma^2\beta^2 X)^2}} \quad \text{and} \quad \frac{\bar{u}_{02}}{\bar{u}_2} = -1\Bigg/\sqrt{1-\frac{(1-\beta^2)}{(1-\gamma^2\beta^2 X)^2}},$$

where we have also used the result $I_{2a}A_a = -I_{1a}A_a = -X$, which follows from eq. (6).

The above set of equations have obvious constants of motion $I^2 \equiv \sum_a I_a I_a$ and $K^2 \equiv \sum_a K_a K_a$, where K_a is defined as, $K_a \equiv \varepsilon\varepsilon_{abc}A_b\dot{A}_c + (I_a/2)$. One more constant of integration is

$$\dot{A}^2 - \frac{1}{\gamma^2}\left(\sqrt{1-2\gamma^2 X+\gamma^4\beta^2 X^2} + \sqrt{1+2\gamma^2 X+\gamma^4\beta^2 X^2}\right) = 2E, \tag{13}$$

where $\dot{A}^2 \equiv \sum_a \dot{A}_a\dot{A}_a$. Since f_1 and f_2 depend on X, let us derive a dynamical equation for X from eq. (11) as

$$\ddot{X} - (K^2 + I^2/4 - \varepsilon^2(2Y\dot{A}^2 - \dot{Y}^2))(f_2(X) - f_1(X)) = 0, \tag{14}$$

where $Y \equiv \sum_a A_a A_a/2$ and again, from eq. (11), it obeys a dynamical equation

$$\ddot{Y} - \dot{A}^2 - X(f_2(X) - f_1(X))/2 = 0, \tag{15}$$

where $\dot{A}^2$ is given by eq. (13). As $\varepsilon \to 0$, coefficient of $(f_2 - f_1) \to (K^2 + I^2/4)$ in eq. (14), is a constant. Further on linearizing, $(f_2 - f_1) \to -2X$ and we get Abelian oscillation.

We solve the above set of coupled non-linear equations, eqs (14) and (15), numerically using Runge–Kutta method for various values of β. Only the parameter in the problem is $\varepsilon = \gamma^4\beta^2\varepsilon_0$. Taking QGP at, say, $T = 200$ MeV etc. we can estimate

$$\varepsilon_0 = \frac{\sqrt{2n_0 m}\, g}{\omega_{\rm pl}^2} \approx \frac{\sqrt{7\pi^2 T^3 m/30}\, g}{g^2T^2/18} \approx \sqrt{\frac{m}{T}}\, 9\pi/g \approx 30,$$

for T of the order of m and g of the order of 1. Here we have taken plasma frequency, $\omega_{\rm pl} \approx gT/\sqrt{18}$ and $n_0 \approx (7\pi^2/60)T^3$, appropriate to quark–antiquark plasma. At higher temperature ε_0 will always be less than 30. Now ε depends on β. For non-relativistic case, $\beta \ll 1$ and hence non-Abelian effects are negligible, provided the amplitude of the wave is small enough. But it becomes important for relativistic case and new non-Abelian oscillations of X with varying amplitudes show up after a few Abelian oscillations as shown in figure 1 for $\beta = 0.3$. These non-Abelian oscillations are at higher frequency than that of Abelian oscillations. Then after a few non-Abelian oscillations, Abelian oscillations develop again and this phenomena of alternate repetition of Abelian and non-Abelian oscillations continue. Over all plot looks like a *modulated wave* of Abelian and non-Abelian waves, repeating alternatively. As β increases, the number of Abelian oscillations as well as non-Abelian oscillations decreases and hence the repetition of Abelian and non-Abelian oscillations become quicker and quicker, as shown in figures 2 and 3. In other words, the period of the envelope of modulated wave (say, envelope wave) decreases with the increase in β and as a result, Abelian and non-Abelian waves can make only fewer oscillations. For

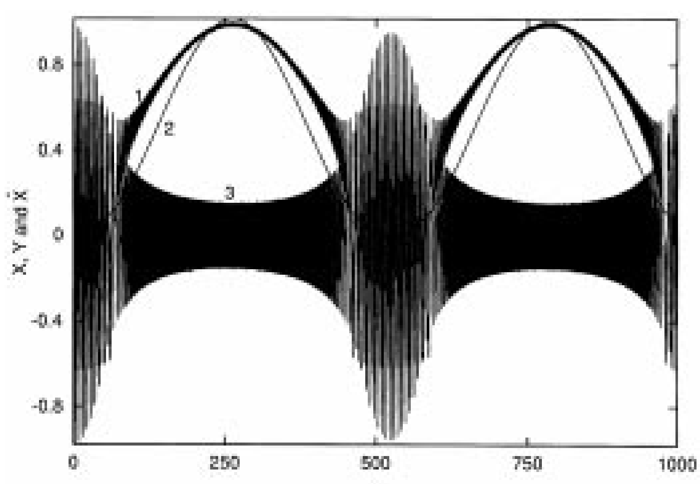

Figure 1. Plots of X (curve 1), Y (curve 2) and $\dot{X}$ (curve 3), with their maximum normalized to 1, as a function of x for $\beta = 0.3$.

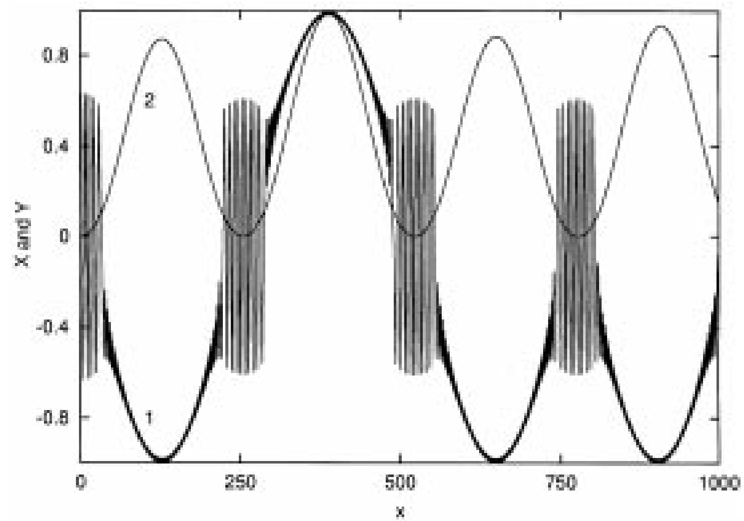

Figure 2. Plots of X (curve 1) and Y (curve 2), with their maximum normalized to 1, as a function of x for $\beta = 0.4$.

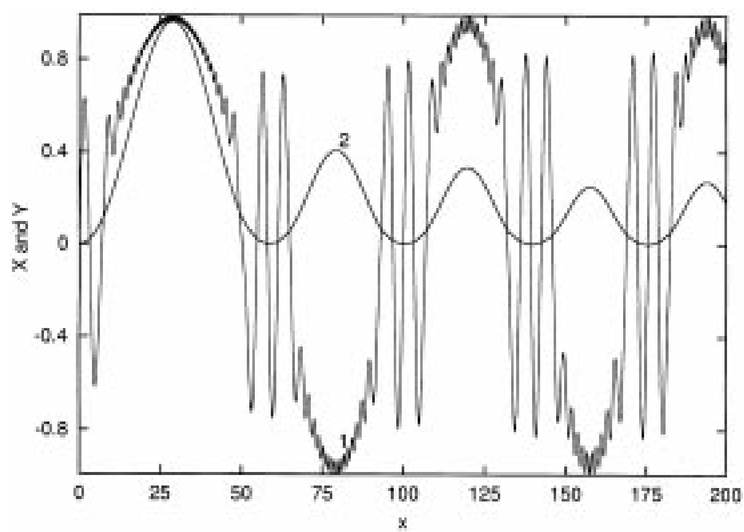

Figure 3. Plots of X (curve 1) and Y (curve 2), with their maximum normalized to 1, as a function of x for $\beta = 0.6$.

further increase in β, envelope wave oscillations become so fast that Abelian and non-Abelian oscillations are smeared out and leads to irregular oscillations as shown in figure 4, for $\beta = 0.75$. Finally for $\beta = 0.9$, Abelian and non-Abelian oscillations disappear as shown in figure 5, which looks like regular oscillations with varying frequency without any separate relations to Abelian or non-Abelian oscillations. For large amplitudes, even

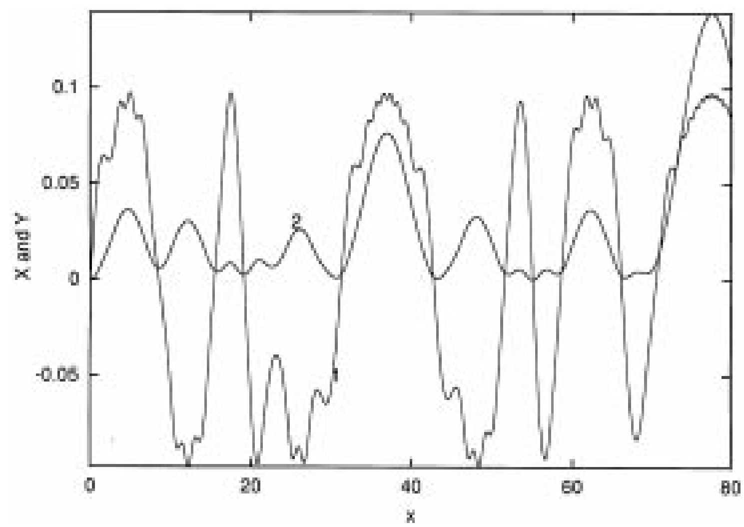

Figure 4. Plots of X (curve 1) and Y (curve 2), as a function of x for $\beta = 0.75$.

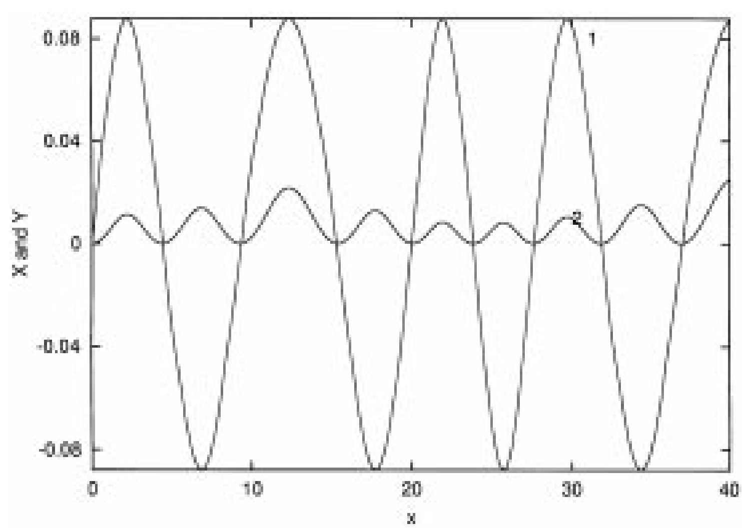

Figure 5. Plots of X (curve 1) and Y (curve 2), as a function of x for $\beta = 0.9$.

if β is small, non-Abelian effects shown up, but only after a very large number of Abelian oscillations, as if the envelope wave here has long period. In figures 1–5, we also plotted Y as a function of x. We see that as β increases a pulse-like structure with constant amplitude changes to smaller pulses with varying amplitudes. In figure 1, we also plotted $\dot{X}$ as a function of x and it also exhibits Abelian and non-Abelian oscillations with the same frequencies as that of X, but oscillations are symmetric about x-axis. It is interesting to see that the non-Abelian oscillations in X and $\dot{X}$ occurs around the peak of Y as if the amplitude of Y is related to the frequency of non-Abelian oscillations. This can also be seen from the approximate equations given below.

Let us apply the approximation made by Bhat *et al* to our problem. As one of the quark, say, $m_1 \to \infty$ and hence $f_1 \to 0$, there is not much change in the structure of the equation. Further, neglecting plasma non-linearity leads to, from eqs (14), (15),

$$\ddot{X} + (K^2 + I^2/2)X - 2\varepsilon^2 \left(\left(2E + \frac{1}{\gamma^2} \right) XY - X^3 Y - X\dot{Y}^2 \right) = 0 \tag{16}$$

and

$$\ddot{Y} + X^2/2 - \left(2E + \frac{1}{\gamma^2} \right) = 0. \tag{17}$$

Equation (16) shows Abelian oscillation term plus non-linear term, due to non-Abelian effects, with coefficient ε^2. Non-Abelian term, in fact, depends on Y which is seen in figure 1. So even with these two approximations non-Abelian effects still exist, consistent with the results of Bhat *et al*. Note that the amplitudes of X, Y and $\dot{X}$ are in general different, but normalized to 1 in figures 1–3. Amplitudes of X and $\dot{X}$ are independent of β for given initial conditions, but Y decreases as β increases.

In summary, we have studied the fully non-linear, relativistic, non-Abelian quark–antiquark plasma, relevant to the study of QGP at RHICs. We reproduced the earlier results of existence of new non-Abelian oscillations of X, as shown in figure 1, in addition and alternating with Abelian oscillations, using our equations for a small amplitude waves with the small value of β. As we discussed earlier, this is because the basic structure of the equation does not change by the approximation made by Bhat *et al*. However, we notice in our exact calculation that (1) frequency square is now doubled due to dynamics of both the quarks. (2) Frequency is also modified by a relativistic factor γ. (3) For $\beta \leq 0.4$, non-Abelian frequency is approximately 10 times that of Abelian oscillations, but for $\beta > 0.4$ there is no relationship between Abelian and non-Abelian frequencies as shown figure 3. (4) We can see from figures 1–3 that the plot for X looks like a wave (envelope wave) modulated by Abelian and non-Abelian oscillations. As β increases the period of envelope wave decreases and hence the repetition of Abelian and non-Abelian oscillations occur quicker and quicker. Finally, for large β Abelian and non-Abelian oscillations are smeared out and we see an irregular oscillation as shown in figure 4, for $\beta = 0.75$ and then a regular oscillation with varying frequency as shown in figure 5.

As a future work, it may be interesting to study other non-linear properties, such as chaos, fractals etc. of our final equations (eqs (14) and (15)) which may give more features of QGP due to strong interaction.

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