PRAMANA
— journal of
physics

© Indian Academy of Sciences

Vol. 59, No. 3
September 2002
pp. 465–477

Bounds on neutrino mixing with exotic singlet neutrinos

J K SINGHAL[1,*], SARDAR SINGH[2], ASHOK K NAGAWAT[2] and N K SHARMA[2]
[1]Department of Physics, Government College, Sawai Madhopur 322 001, India
[2]Department of Physics, University of Rajasthan, Jaipur 302 004, India
[*]Email: jksinghal@hotmail.com

MS received 29 January 2002; revised 10 April 2002

Abstract. We examine the effects of mixing induced non-diagonal light–heavy neutrino weak neutral currents on the amplitude for the process $\nu_a \bar{\nu}_a \to ZZ$ (with $a = e, \mu$ or τ). By imposing constraint that the amplitude should not exceed the perturbative unitarity limit at high energy $(\sqrt{s} = \Lambda)$, we obtain bounds on light–heavy neutrino mixing parameter $\sin^2\theta_{\mathrm{L}}^{\nu_a}$ where $\theta_{\mathrm{L}}^{\nu_a}$ is the mixing angle. In the case of one heavy neutrino (mass m_ξ) or mass degenerate heavy neutrinos, for $\Lambda = 1$ TeV, no bound is obtained for $m_\xi < 0.50$ TeV. However, $\sin^2\theta_{\mathrm{L}}^{\nu_a} \leq 3.8 \times 10^{-6}$ for $m_\xi = 5$ TeV and $\sin^2\theta_{\mathrm{L}}^{\nu_a} \leq 6.0 \times 10^{-8}$ for $m_\xi = 10$ TeV. For $\Lambda = \infty$, no constraint is obtained for $m_\xi < 0.99$ TeV and $\sin^2\theta_{\mathrm{L}}^{\nu_a} \leq 3.8 \times 10^{-2}$ (for $m_\xi = 5$ TeV) and $\sin^2\theta_{\mathrm{L}}^{\nu_a} \leq 9.6 \times 10^{-3}$ (for $m_\xi = 10$ TeV).

Keywords. Neutrino; neutrino mixing; exotic neutrino; singlet neutrino.

PACS Nos 14.60.St; 13.15.+g

1. Introduction

Neutral heavy leptons or heavy neutrinos (HNs) are predicted by various extensions of the standard model (SM) [1–4]. Searches of HNs are listed in the Review of Particle Physics [4], which reveal that the heavy neutrino masses are experimentally bounded to be greater than 73.5–100 GeV [4,5]. If HNs exist then even in the case in which these new particles are too heavy to be produced, their presence could still manifest through their mixing with the SM neutrinos.

A comprehensive analysis of the mixing between ordinary fermions with canonical SU(2) × U(1) assignments (i.e. left handed (L) fermions as SU(2) doublets, while right handed (R) fermions as SU(2) singlets) and possible heavy fermions with exotic (non-canonical) SU(2) × U(1) assignments (i.e. L-fermions as SU(2) singlets, while R-fermions as SU(2) doublets) has been performed by Langacker and London [1]. In this analysis new neutrinos $N_{1\mathrm{L}}, N_{2\mathrm{L}}$ and $N_{3\mathrm{L}}$, in the representation

$$\begin{bmatrix} N_1 \\ E^- \end{bmatrix}_{\mathrm{L}}, \quad \begin{bmatrix} E^+ \\ N_2 \end{bmatrix}_{\mathrm{L}}, \quad N_{3\mathrm{L}}, \tag{1}$$

are considered. In the general Majorana neutrinos case, all N_{1L}, N_{2L} and N_{3L} can mix with the known neutrinos. In the Dirac neutrinos case, N_{1L} and N_{3L} can mix with the known neutrinos. The neutrino mixing in general modifies the weak charged and weak neutral currents of light (known) neutrinos and in special cases (exotic and/or singlet neutrino mixing) may induce light–heavy neutrino non-diagonal weak neutral currents [1]. The analysis of these modifications leads to constraints on neutrino mixing parameters [1,6].

In the neutrino mixing formalism [1,6], the weak eigenstates (ψ_L^0, ψ_R^0) and mass eigenstates (ψ_L, ψ_R) are related by the transformations

$$\psi_L^0 = U_L \psi_L \quad \text{and} \quad \psi_R^0 = U_R \psi_R, \tag{2}$$

where $U_{L,R}$ are unitary matrices which diagonalize the fermion mass matrix. The elements of the unitary matrices U_L and U_R are assumed to be independent of the energy scale at which weak interaction processes involving the ψ fields are considered. Such constraints obtained on elements of the mixing matrix $U_{L,R}$ at one energy scale are assumed to be valid at any other energy scale. With this in view we consider, in this paper the process $\nu_a \bar{\nu}_a \to ZZ$ (where $a = e, \mu$ or τ) with the inclusion of neutrino mixing. The threshold of the process $\sqrt{s} = 2m_Z$ is high, and masses of light neutrinos are too small to be kinematically relevant. This allows us to neglect the masses of light (known) neutrinos. In the SM the process occurs at tree level via light neutrino exchange in t- and u-channels. The neutrino mixing in special cases (e.g. mixing of exotic and/or singlets), induces non-diagonal light–heavy neutrino neutral currents and additional contribution to the process arises via heavy neutrino exchange in t- and u-channels. Any such additional contribution, in principle, modifies the partial wave amplitude in comparison to the no mixing case. By using the constraint that the amplitude should not exceed the perturbative unitarity limit at high energies [7–10], we obtain theoretical bounds on the heavy neutrino mixing parameter.

The paper is organized as follows: In §2, following the neutrino mixing formalism of Langacker and London [1], we examine the non-diagonal light–heavy neutrino weak neutral currents and evaluate the amplitude for the process $\nu_a \bar{\nu}_a \to ZZ$ with the inclusion of neutrino mixing. In §3, the unitarity bounds are obtained, and we summarize our results in §4.

2. Neutrino mixing formalism and amplitude for the process $\nu_a \bar{\nu}_a \to ZZ$

2.1 *Mixing formalism*

We follow the neutrino mixing formalism and notation of Langacker and London [1]. We consider the three SU(2) × U(1) assignments,

$$\begin{bmatrix} n_{OL}^0 \\ e_L^{0-} \end{bmatrix}, \quad \begin{bmatrix} e_L^{0+} \\ n_{EL}^0 \end{bmatrix}, \quad n_{SL}^0, \tag{3}$$

where e_L^{0-} and e_L^{0+} are weak eigenstates of charged leptons $(e_L^-, \mu_L^-, \tau_L^-, \ldots)$ and antileptons $(e_L^+, \mu_L^+, \tau_L^+, \ldots)$. the n_{OL}^0 are ordinary SU(2) doublet neutrinos, n_{EL}^0 are exotic neutrinos, occurring in doublets with left handed anti-leptons and n_{SL}^0 are SU(2) singlets [11]. These are themselves column vectors consisting of individual neutrinos. Let their dimensions are n-, m-, and p- respectively, i.e.,

$$n^0_{\mathrm{OL}} = (\nu^0_{e\mathrm{L}}\ \nu^0_{\mu\mathrm{L}}\ \nu^0_{\tau\mathrm{L}}\ \nu^0_{4\mathrm{L}} \dots \nu^0_{n\mathrm{L}})^T,$$
$$n^0_{\mathrm{EL}} = (N^0_{1\mathrm{L}}\ N^0_{2\mathrm{L}} \dots N^0_{m\mathrm{L}})^T,$$
$$n^0_{\mathrm{SL}} = (N^0_{(m+1)\mathrm{L}} N^0_{(m+2)\mathrm{L}} \cdots N^0_{(m+p)\mathrm{L}})^T. \tag{4}$$

For convenience, all of the weak eigenstate neutrinos are arranged into a column vector

$$n^0_{\mathrm{L}} = (n^0_{\mathrm{OL}}\ n^0_{\mathrm{EL}}\ n^0_{\mathrm{SL}})^T. \tag{5}$$

Following Langacker and London [1] we assume that the mass eigenstate neutrinos are all either light (massless, i.e., with masses too small to be kinematically relevant and whose masses are neglected in our calculation) or heavy allowing the arrangement of the mass eigenstate into a column vector

$$n_{\mathrm{L}} = (n_{\mathrm{lL}}\ n_{\mathrm{hL}})^T. \tag{6}$$

The n_{lL} and n_{hL} are themselves column vectors consisting of q light and r heavy neutrinos respectively, i.e.,

$$n_{\mathrm{lL}} = (\nu_{1\mathrm{L}}\ \nu_{2\mathrm{L}} \dots \nu_{q\mathrm{L}})^T,$$
$$n_{\mathrm{hL}} = (N_{1\mathrm{L}}\ N_{2\mathrm{L}} \dots N_{r\mathrm{L}})^T. \tag{7}$$

The weak and mass eigenstates are related by unitary transformation

$$n^0_{\mathrm{L}} = U_{\mathrm{L}} n_{\mathrm{L}}. \tag{8}$$

The matrix U_{L} can be written in the following block form [11]

$$U_{\mathrm{L}} = \begin{bmatrix} A_{\mathrm{L}} & E_{\mathrm{L}} \\ F_{\mathrm{L}} & G_{\mathrm{L}} \\ H_{\mathrm{L}} & J_{\mathrm{L}} \end{bmatrix}, \tag{9}$$

where $A_{\mathrm{L}}, F_{\mathrm{L}}$ and H_{L} are $(n \times q)$-, $(m \times q)$- and $(p \times q)$-dimension matrices describing the overlap of light neutrinos with ordinary doublets (n^0_{OL}), exotic doublets (n^0_{EL}) and singlets (n^0_{SL}) respectively. Similarly $E_{\mathrm{L}}, G_{\mathrm{L}}$ and J_{L} are $(n \times r)$-, $(m \times r)$- and $(p \times r)$-dimension matrices describing the overlap of heavy neutrinos.

The weak neutral current involving ordinary (n^0_{OL}) and exotic (n^0_{EL}) weak eigenstate neutrinos (the n^0_{SL} do not participate in the weak interactions) is [1]

$$\frac{1}{2} J^\mu_Z = \bar{n}^0_{\mathrm{OL}} \gamma^\mu t_3 n^0_{\mathrm{OL}} + n^0_{\mathrm{EL}} \gamma^\mu t_3 n^0_{\mathrm{EL}}. \tag{10}$$

In this mass eigenstate basis $n_{\mathrm{L}} = (n_{\mathrm{lL}}\ n_{\mathrm{hL}})^T$, the weak neutral current is obtained by using eq. (10), eqs (4)–(9) and respective t_3 assignment from eq. (3). It involves light and heavy neutrino column vectors n_{lL} and n_{hL}. In the mass eigenstate basis the weak neutral current is given by (here, repeated indices are summed with ranges: $b = 1$ to $n, c = 1$ to $m, \alpha = 1$ to $q, \xi = 1$ to $r, \delta = 1$ to q and $\phi = 1$ to r),

$$\frac{1}{2}J_Z^\mu = \frac{1}{2}\begin{bmatrix} \bar{\nu}_{\alpha\mathrm{L}}\gamma^\mu((A_\mathrm{L}^\dagger)_{\alpha b}(A_\mathrm{L})_{b\delta} - (F_\mathrm{L}^\dagger)_{\alpha c}(F_\mathrm{L})_{c\delta})\nu_{\delta\mathrm{L}} \\ +\bar{N}_{\xi\mathrm{L}}\gamma^\mu((E_\mathrm{L}^\dagger)_{\xi b}(A_\mathrm{L})_{b\delta} - (G_\mathrm{L}^\dagger)_{\xi c}(F_\mathrm{L})_{c\delta})\nu_{\delta\mathrm{L}} \\ +\bar{\nu}_{\alpha\mathrm{L}}\gamma^\mu((A_\mathrm{L}^\dagger)_{\alpha b}(E_\mathrm{L})_{b\phi} - (F_\mathrm{L}^\dagger)_{\alpha c}(G_\mathrm{L})_{c\phi})N_{\phi\mathrm{L}} \\ +\bar{N}_{\xi\mathrm{L}}\gamma^\mu((E_\mathrm{L}^\dagger)_{\xi b}(E_\mathrm{L})_{b\phi} - (G_\mathrm{L}^\dagger)_{\xi c}(G_\mathrm{L})_{c\phi})N_{\phi\mathrm{L}} \end{bmatrix}$$

$$= \frac{1}{2}\begin{bmatrix} \bar{n}_{\mathrm{lL}}\gamma^\mu(A_\mathrm{L}^\dagger A_\mathrm{L} - F_\mathrm{L}^\dagger F_\mathrm{L})n_{\mathrm{lL}} \\ +\bar{n}_{\mathrm{hL}}\gamma^\mu(E_\mathrm{L}^\dagger A_\mathrm{L} - G_\mathrm{L}^\dagger F_\mathrm{L})n_{\mathrm{lL}} \\ +\bar{n}_{\mathrm{lL}}\gamma^\mu(A_\mathrm{L}^\dagger E_\mathrm{L} - F_\mathrm{L}^\dagger G_\mathrm{L})n_{\mathrm{hL}} \\ +\bar{n}_{\mathrm{hL}}\gamma^\mu(E_\mathrm{L}^\dagger E_\mathrm{L} - G_\mathrm{L}^\dagger G_\mathrm{L})n_{\mathrm{hL}} \end{bmatrix}. \tag{11}$$

The first term involves light neutrinos only and represents reduction of the strength of the normal $Z\nu\nu$ neutral current due to neutrino mixing. The second and third terms represent neutrino mixing induced $Z\nu N$ light–heavy neutrino neutral currents. The second and third terms survive if $Y = E_\mathrm{L}^\dagger A_\mathrm{L} - G_\mathrm{L}^\dagger F_\mathrm{L} \neq 0$.

The process $\nu_a + \bar{\nu}_a \to ZZ$ (with $a = e, \mu$, or τ) with the inclusion of neutrino mixing, in general (with masses of light neutrinos neglected), occurs through t- and u-channels light neutrino $\nu_{\alpha\mathrm{L}}$ (figure 1a) and heavy neutrino $N_{\xi\mathrm{L}}$ (figure 1b) exchange. The contributions of figure 1b will survive if $(Y^\dagger Y)_{aa} \neq 0$. To the second order in light–heavy neutrino mixing, we get (see Appendix for proof)

$$(Y^\dagger Y)_{aa} = (4\lambda_F^a + \lambda_H^a)(\sin^2\theta_\mathrm{L}^{\nu_a}), \tag{12}$$

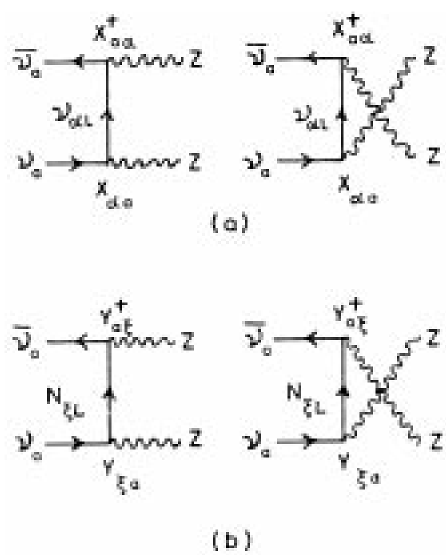

Figure 1. (**a**) The light neutrinos exchange in t- and u-channels, $X_{\alpha a} = (A_\mathrm{L}^\dagger A_\mathrm{L} - F_\mathrm{L}^\dagger F_\mathrm{L})_{\alpha a}$. (**b**) The heavy neutrino exchange in t- and u-channels induced by neutrino mixing, $Y_{\xi a} = (E_\mathrm{L}^\dagger A_\mathrm{L} - G_\mathrm{L}^\dagger F_\mathrm{L})_{\xi a}$.

where the parameters λ and $\theta_{\rm L}^{\nu_a}$ (interpreted as light–heavy mixing angle [1]) are defined through the relations (for details of the following see Appendix of [1])

$$(F^{\dagger}F)_{aa} = \lambda_F^a(\sin^2\theta_{\rm L}^{\nu_a}),$$
$$(H^{\dagger}H)_{aa} = \lambda_H^a(\sin^2\theta_{\rm L}^{\nu_a}), \tag{13}$$

with $0 \le \lambda \le 1$. Thus if extra neutrinos occur only in $n_{\rm OL}^0$, i.e., $U_{\rm L} = [A_{\rm L}\ \ E_{\rm L}]$, then $(Y^{\dagger}Y)_{aa} = 0$ and non-diagonal light–heavy neutrino neutral current vanishes. However, if extra neutrinos occur in $n_{\rm EL}^0$ or $n_{\rm SL}^0$ non-diagonal light–heavy neutrino neutral current survives. It may be noted that the mixing of extra $n_{\rm EL}^0$ occurs only for Majorana case. For Dirac case, only extra neutrinos in $n_{\rm SL}^0$ give rise to $Z\nu N$ couplings, with

$$(Y^{\dagger}Y)_{aa} = \lambda_H^a(\sin^2\theta_{\rm L}^{\nu_a}). \tag{14}$$

2.2 *Amplitude for the process* $\nu_a\overline{\nu_a} \to ZZ$

With the inclusion of neutrino mixing, the process (with $a = e, \mu$ or τ)

$$\nu_a(k_1,\sigma) + \overline{\nu_a}(\overline{k}_1,\overline{\sigma}) \to Z(k_2,\lambda) + Z(\overline{k}_2,\overline{\lambda}), \tag{15}$$

(where the arguments indicate the four momenta and helicities of the respective particles) occur via q light neutrinos $\nu_{\alpha{\rm L}}(\alpha = 1$ to $q)$ and r heavy neutrinos $N_{\xi{\rm L}}(\xi = 1$ to $r)$ exchange in t- and u-channels, as shown in figure 1. In our calculations we are confining to the possibility of extra neutrinos to $n_{\rm SL}^0$ only. The recent search for heavy singlet neutrinos in e^+e^- annihilation at LEP reports no evidence in a mass range 80 GeV–205 GeV [12]. However, heavier neutrinos and their mixings are not ruled out. For singlet neutrinos, $\lambda_F^a = 0$ and $\lambda_H^a = 1$ [11], so that

$$(Y^{\dagger}Y)_{aa} = \sin^2\theta_{\rm L}^{\nu_a}. \tag{16}$$

As the threshold ($\sqrt{s} = 2m_Z$) of the process is sufficiently high the masses of light neutrinos are neglected. Therefore $\sigma = -1$ and $\overline{\sigma} = +1$ giving total angular momentum $J = 1$ for the initial state in the center of mass frame. We evaluate the amplitude for the process in the helicity basis following the technique discussed in detail by Renard [13] and Hagiwara and Zeppenfeld [14]. In the c.m. frame, the momenta of the particles in eq. (15) are

$$k_1 = \frac{\sqrt{s}}{2}[1,\ 0,\ 0,\ 1]; \qquad \overline{k}_1 = \frac{\sqrt{s}}{2}[1,0,0,-1];$$
$$k_2 = \frac{\sqrt{s}}{2}[1,\beta\sin\theta,0,\beta\cos\theta]; \qquad \overline{k}_2 = \frac{\sqrt{s}}{2}[1,-\beta\sin\theta,0,-\beta\cos\theta],$$

where $\sqrt{s}$ is the total c.m. energy, $\beta = \sqrt{1-4m_Z^2/s}$ and θ is the scattering angle of $Z(q,\lambda)$ with the incident neutrino direction. The polarization vectors of the final states gauge bosons in Jacob–Wick phase convention are [13]

$$\varepsilon_1^* = \frac{1}{\sqrt{2}}[0, -\lambda\cos\theta, i, \lambda\sin\theta], \qquad \text{for } \lambda = \pm 1,$$
$$\varepsilon_1^* = \frac{\sqrt{s}}{2m_Z}[\beta, \sin\theta, 0, \cos\theta], \qquad \text{for } \lambda = 0,$$
$$\varepsilon_2^* = \frac{1}{\sqrt{2}}[0, \overline{\lambda}\cos\theta, i, -\overline{\lambda}\sin\theta], \qquad \text{for } \overline{\lambda} = \pm 1,$$
$$\varepsilon_2^* = -\frac{\sqrt{s}}{2m_Z}[\beta, -\sin\theta, 0, -\cos\theta], \qquad \text{for } \overline{\lambda} = 0.$$

The $\lambda, \overline{\lambda} = \pm 1$ correspond to transverse gauge bosons and $\lambda, \overline{\lambda} = 0$ correspond to longitudinal gauge bosons in the final state. The polarization vector of the longitudinal gauge boson contains the factor $\gamma = \sqrt{s}/2m_Z$, which with its extra power of energy, is expected to give a possible breakdown of unitarity at high energies. Therefore, we consider the production of longitudinal gauge bosons to examine the unitarity constraints.

For the massless light neutrinos, $\Delta\sigma = (\sigma - \overline{\sigma})/2 = -1$, and for the final state longitudinal Z-bosons $\Delta\lambda = \lambda - \overline{\lambda} = 0$ giving the minimum angular momentum, $\underline{J} = \max(|\Delta\lambda|, |\Delta\sigma|) = 1$. Following [13] and confining ourselves to the specific case $\lambda = \overline{\lambda} = 0$, we separate the contributions to the amplitude from t- and u-channels $\nu_{\alpha\mathrm{L}}$-exchange and $N_{\xi\mathrm{L}}$-exchange graphs in figure 1 as

$$M = M^\nu + M^N, \tag{17}$$

where the $\nu_{\alpha\mathrm{L}}$-exchange contribution is (with $X_{\alpha a} = (A_\mathrm{L}^\dagger A_\mathrm{L} - F_\mathrm{L}^\dagger F_\mathrm{L})_{\alpha a}$)

$$M^\nu = \frac{-e^2}{\sin^2\theta_W\cos^2\theta_W}\sum_{\alpha=1}^{q} X_{a\alpha}^\dagger X_{\alpha a}\frac{1}{\gamma^2}\left[\frac{2\sin\theta\cos\theta}{4\beta^2\sin^2\theta + 1/\gamma^4}\right]\delta_{J,1}, \tag{18}$$

and the $N_{\xi\mathrm{L}}$-exchange contribution is (with $Y_{\xi a} = (E_\mathrm{L}^\dagger A_\mathrm{L} - G_\mathrm{L}^\dagger F_\mathrm{L})_{\xi a}$)

$$M^N = \frac{-e^2}{\sin^2\theta_W\cos^2\theta_W}\sum_{\xi=1}^{r} Y_{a\xi}^\dagger Y_{\xi a} \times \left[\frac{\left(\frac{1}{\gamma^2} + \frac{m_\xi^2}{m_Z^2}\right)2\sin\theta\cos\theta}{4\beta^2\sin^2\theta + \frac{1}{\gamma^4} + \frac{8m_\xi^2}{S}\left(1 + \beta^2 + \frac{4m_\xi^2}{S}\right)}\right]\delta_{J,1} \tag{19}$$

where m_ξ is the mass of the heavy neutrino $N_{\xi\mathrm{L}}$. In the high energy domain $\sqrt{s} \gg 2m_Z$, we use the approximation $\beta \to 1$ and $\gamma \to \infty$. Then eqs (18) and (19) reduce to

$$M^\nu \to 0, \tag{20}$$

$$M^N \approx \frac{-e^2}{2\sin^2\theta_W\cos^2\theta_W}\sum_{\xi=1}^{r} Y_{a\xi}^\dagger Y_{\xi a}\left[\frac{\frac{m_\xi^2}{m_Z^2}\sin\theta\cos\theta}{\sin^2\theta + \frac{4m_\xi^2}{s}\left(1 + \frac{2m_\xi^2}{s}\right)}\right]\delta_{J,1}. \tag{21}$$

Thus, if heavy neutrinos are absent, we do not expect unitarity violation as $M^{\nu} \to 0$. However, if heavy neutrinos exist, then through neutrino mixing, they induce non-zero amplitude M^N which depends on parameter $Y^{\dagger}_{a\xi} Y_{\xi a}$ and heavy neutrino mass m_{ξ}. In the high energy limit ($\sqrt{s} \gg 2m_Z$), the total amplitude for the process with the inclusion of neutrino mixing, using eqs (20) and (21) in eq. (17) (with $e^2/(m_Z^2 \sin^2\theta_W \cos^2\theta_W) = 4\sqrt{2}G_F$) becomes

$$M \approx -2\sqrt{2}G_F \sum_{\xi=1}^{r} Y^{\dagger}_{a\xi} Y_{\xi a} m_{\xi}^2 \frac{\sin\theta\cos\theta}{\left[\sin^2\theta + \frac{4m_{\xi}^2}{s}\left(1+\frac{2m_{\xi}^2}{s}\right)\right]} \delta_{J,1}. \tag{22}$$

3. Bounds on neutrino mixing

Unitarity of scattering matrix together with the optical theorem imposes certain constraints for the partial waves [7–10]. For the process AB $\to$ CD, in terms of partial wave decomposition the amplitude is written as [8,10]

$$M(s,t,u) = 16\pi \sum_{J=0}^{\infty} (2J+1) P_J(\cos\theta)\ a_J(s), \tag{23}$$

where s,t,u are the Mandelstam variables, $a_J(s)$ is the Jth partial wave and $P_J(\cos\theta)$ are the Legendre polynomials. The differential cross-section is given by [8]

$$\frac{\mathrm{d}\sigma}{\mathrm{d}\cos\theta} = \frac{1}{32\pi}|M(s,t,u)|^2. \tag{24}$$

Using the fact that Legendre polynomials are orthogonal, we write cross-section as [8,10]

$$\sigma = \frac{16\pi}{s} \sum_{J=0}^{\infty} (2J+1) P_J(\cos\theta) |a_J(s)|^2. \tag{25}$$

The optical theorem together with the above expression gives the following unitarity constraint [8,10]

$$|\mathrm{Re}\ a_J(s)| \le 1/2. \tag{26}$$

The partial wave $a_J(s)$ is obtained from eq. (23) and is given by

$$a_J(s) = \frac{1}{32\pi} \int_{-1}^{1} M(s,t,u) P_J(\cos\theta)\ \mathrm{d}(\cos\theta). \tag{27}$$

Substituting the value of M from eq. (22) and remembering that $J = 1$, we obtain, after solving the integral, the following expression:

$$a_1(s) = \frac{-G_F}{16\sqrt{2}} \sum_{\xi=1}^{r} Y^{\dagger}_{a\xi} Y_{\xi a} m_{\xi}^2 \left[1 + \frac{8m_{\xi}^2}{s}\left(1+\frac{2m_{\xi}^2}{s}\right)\right]. \tag{28}$$

Then the unitarity constraint (eq. (26)) gives

$$\sum_{\xi=1}^{r} Y^{\dagger}_{a\xi} Y_{\xi a} m_{\xi}^{2} \left[1+\frac{8m_{\xi}^{2}}{s}\left(1+\frac{2m_{\xi}^{2}}{s}\right)\right] \leq \frac{8\sqrt{2}}{G_F}. \qquad (29)$$

The one important question in beyond the standard model physics is what are the scales of new physics [15]. *A priori*, the recent BNL E821 measurement of the anomalous magnetic moment of muon favors new physics at the TeV scale [16]. Further, it has been suggested that new physics should be expected at some scale ≤ 1 TeV, in order to stabilize the gauge hierarchy [17]. We assume that $\sqrt{s} = \Lambda$ is the energy scale at which new physics emerges, i.e., it is the energy scale at which perturbative unitarity is violated. Then bound of eq. (29) reads

$$\sum_{\xi=1}^{r} Y^{\dagger}_{a\xi} Y_{\xi a} m_{\xi}^{2} \left[1+\frac{8m_{\xi}^{2}}{\Lambda^2}\left(1+\frac{2m_{\xi}^{2}}{\Lambda^2}\right)\right] \leq \frac{8\sqrt{2}}{G_F}. \qquad (30)$$

A large number of unknown masses m_{ξ} and mixing parameters in $Y_{\xi a}$ make a thorough analysis impractical. However, if we consider the case of mass degenerate heavy neutrinos [18], or only one heavy neutrino then the discussion becomes tractable. For these cases eq. (30) becomes

$$(Y^{\dagger}Y)_{aa} \leq \frac{8\sqrt{2}}{G_F m_{\xi}^{2}\left[1+\frac{8m_{\xi}^{2}}{\Lambda^2}\left(1+\frac{2m_{\xi}^{2}}{\Lambda^2}\right)\right]}. \qquad (31)$$

Use of eq. (16) then, gives the following bound on the light–heavy neutrino mixing angle $\theta_{\mathrm{L}}^{\nu_a}$ as a function of heavy neutrino mass m_{ξ}

$$\sin^2\theta_{\mathrm{L}}^{\nu_a} \leq \frac{8\sqrt{2}}{G_F m_{\xi}^{2}\left[1+\frac{8m_{\xi}^{2}}{\Lambda^2}\left(1+\frac{2m_{\xi}^{2}}{\Lambda^2}\right)\right]}. \qquad (32)$$

The upper limits on $\sin^2\theta_{\mathrm{L}}^{\nu_a}$ as a function of mass m_{ξ} obtained from eq. (32) with $G_F = 1.16639\times 10^{-5}$ GeV^{-2} [4] are shown in figure 2, for $\Lambda = 1$ TeV, 2 TeV and ∞. For $\Lambda = 1$ TeV, no constraint on $\sin^2\theta_{\mathrm{L}}^{\nu_a}$ is obtained for $m_{\xi} < 0.50$ TeV. However, for heavy neutrino mass $m_{\xi} = 5$ TeV, we find $\sin^2\theta_{\mathrm{L}}^{\nu_a} \leq 3.8\times 10^{-6}$ and for $m_{\xi} = 10$ TeV, $\sin^2\theta_{\mathrm{L}}^{\nu_a} \leq 6.0\times 10^{-8}$. Similarly, for $\Lambda = \infty$, no unitarity constraint is obtained for $m_{\xi} < 0.99$ TeV. However, $\sin^2\theta_{\mathrm{L}}^{\nu_a} \leq 3.8\times 10^{-2}$ for $m_{\xi} = 5$ TeV and $\sin^2\theta_{\mathrm{L}}^{\nu_a} \leq 9.6\times 10^{-3}$ for $m_{\xi} = 10$ TeV.

One immediate phenomenological consequence in a model with heavy neutrinos is that the light–heavy neutrino admixture in the weak currents leads to the lepton flavor violating (LFV) processes such as $\mu \to e\gamma$, $\mu \to eee$ etc. [19–21]. There have been numerous theoretical and experimental studies [20–23] on LFV process $\mu \to e\gamma$. The neutrino mixing aspect and its effect on the branching ratio (BR) for the process $\mu \to e\gamma$ with the inclusion of neutrino mixing has been discussed by Langacker and London [20]. The BR in the notation of [18], is (see eq. (37) of [20])

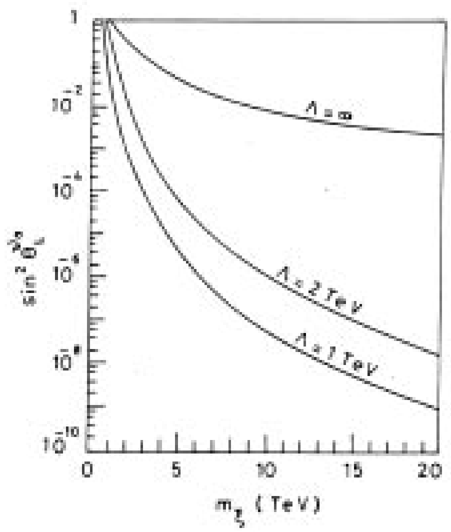

Figure 2. Upper bounds on light–heavy neutrino mixing parameter $\sin^2\theta_{\rm L}^{\nu_a}$ as a function of heavy neutrino mass m_ξ, at different Λ (the scale assumed for validity of perturbative unitarity).

$$\mathrm{BR}(\mu\to e\gamma) = \frac{3\alpha}{2^7\pi}(c_{\rm L}^e)^2(c_{\rm L}^\mu)^2\left|2\sum(E_{\rm L})_{e\xi}F\left(\frac{m_\xi^2}{m_W^2}\right)(E_{\rm L}^\dagger)_{\xi\mu}\right|^2, \tag{33}$$

where $c_{\rm L}^e$ and $c_{\rm L}^\mu$ are the cosines of the light–heavy mixing angles in the charged lepton sector [1], and with $x = m_\xi^2/m_W^2$, the $F(x)$ is [18]

$$F(x) = \frac{x(1-6x+3x^2+2x^3-6x^2\ln x)}{(1-x)^4}. \tag{34}$$

The behavior of $F(x)$ as a function of m_ξ is shown in figure 3. For the specific case considered in this paper, eq. (33) reduces to

$$\mathrm{BR}(\mu\to e\gamma) = \frac{3\alpha}{32\pi}[F(x)]^2|\lambda_{e\mu}^{\rm L}|^2, \tag{35}$$

where

$$|\lambda_{e\mu}^{\rm L}|^2 = (c_{\rm L}^e)^2(c_{\rm L}^\mu)^2|(E_{\rm L}E_{\rm L}^\dagger)_{e\mu}|^2. \tag{36}$$

The present experimental limit $\mathrm{BR}(\mu\to e\gamma)\le 1.2\times 10^{-11}$ [24], constraints the $|\lambda_{e\mu}^{\rm L}|^2$, as a function of heavy neutrino mass m_ξ. These constraints are shown in figure 4. It may be noted that the involvement of charged leptons light–heavy mixing angle parameters $c_{\rm L}^e, c_{\rm L}^\mu$ limits the usefulness of figure 4 in arriving at any bound on ordinary $(n_{\rm OL}^0)$–heavy $(n_{\rm hL})$

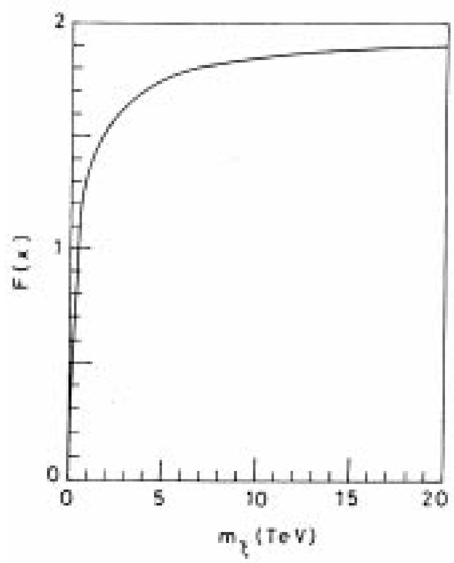

Figure 3. Behavior of $F(x)$ with m_ξ.

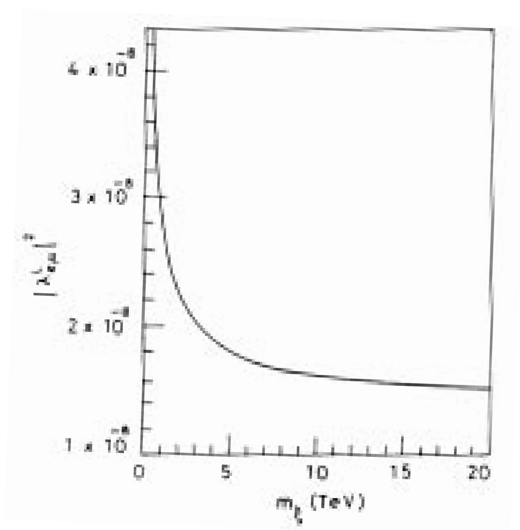

Figure 4. Constraints on $|\lambda^{\mathrm{L}}_{e\mu}|^2$ as a function of m_ξ.

neutrinos overlap parameters in E_{L}. Furthermore, the analysis of BR$(\mu \to e\gamma)$ [20–23] do not conclusively constrain the exotic (n^0_{EL})–light (n_{lL}) neutrino overlap parameters occurring in sub-matrix F_{L} or singlet (n^0_{SL})–light (n_{lL}) neutrino overlap parameters occurring in sub-matrix H_{L}. As such our analysis of the process $\nu_a\bar{\nu}_a \to ZZ$ with inclusion of neutrino mixing is still valid and the unitarity bounds in light–heavy neutrino mixing angle parameter $\sin^2\theta^{\nu_a}_{\mathrm{L}}$ are of interest.

4. Conclusions

We have examined the process $\nu_a \overline{\nu}_a \to ZZ$ with the inclusion of non-diagonal light–heavy neutrino weak neutral currents induced by mixing of exotic (singlet) neutrino with ordinary neutrinos. We applied the unitarity constraint on the $J = 1$ amplitude and obtained theoretical bounds on light–heavy neutrino mixing parameter $\sin^2 \theta_{\mathrm{L}}^{\nu_a}$ where $\theta_{\mathrm{L}}^{\nu_a}$ is the mixing angle. If $\sqrt{s} = \Lambda$ is the energy scale at which unitarity is assumed to be violated, then for one heavy neutrino (mass m_ξ) or mass degenerate heavy neutrinos we find the following:

(i) For $\Lambda = 1$ TeV, no unitarity constraint is obtained for heavy neutrino mass $m_\xi < 0.50$ TeV. However, for heavy neutrino mass $m_\xi = 5$ TeV, $\sin^2 \theta_{\mathrm{L}}^{\nu_a} \le 3.8 \times 10^{-6}$ and for $m_\xi = 10$ TeV, $\sin^2 \theta_{\mathrm{L}}^{\nu_a} \le 6.0 \times 10^{-8}$.

(ii) For $\Lambda = 2$ TeV, no unitarity constraint is obtained for heavy neutrino mass $m_\xi < 0.68$ TeV. For $m_\xi = 5$ TeV, $\sin^2 \theta_{\mathrm{L}}^{\nu_a} \le 5.7 \times 10^{-5}$ and for $m_\xi = 10$ TeV, $\sin^2 \theta_{\mathrm{L}}^{\nu_a} \le 9.5 \times 10^{-7}$.

(iii) For $\Lambda = \infty$, not unitarity constraint is obtained for $m_\xi < 0.99$ TeV. For $m_\xi = 5$ TeV, $\sin^2 \theta_{\mathrm{L}}^{\nu_a} \le 3.8 \times 10^{-2}$ and for $m_\xi = 10$ TeV $\sin^2 \theta_{\mathrm{L}}^{\nu_a} \le 9.6 \times 10^{-3}$.

Appendix

Ignoring, for the moment, the subscript ‘L’ in the matrices A, E, G, and F, we write

$$(Y^\dagger Y)_{aa} = (A^\dagger E E^\dagger A - A^\dagger E G^\dagger F - F^\dagger G E^\dagger A + F^\dagger G G^\dagger F)_{aa}. \tag{A1}$$

We use in the above the following relations obtained from unitarity condition $UU^\dagger = I$,

$$EE^\dagger = I - AA^\dagger, \qquad GG^\dagger = I - FF^\dagger,$$
$$EG^\dagger = -AF^\dagger, \qquad GE^\dagger = -FA^\dagger. \tag{A2}$$

Then

$$(Y^\dagger Y)_{aa} = ((A^\dagger A + FF^\dagger) - (A^\dagger A - F^\dagger F)^2)_{aa}. \tag{A3}$$

The unitarity condition $U^\dagger U = I$ requires

$$A^\dagger A + F^\dagger F = I - H^\dagger H, \tag{A4}$$

and

$$A^\dagger A - F^\dagger F = I + \Delta, \tag{A5}$$

where $\Delta = -2F^\dagger F - H^\dagger H$. It has been shown in the Appendix of [1] that each component of Δ is of $\mathrm{O}(s^2)$ where $s^2 = \sin^2 \theta_{\mathrm{L}}^{\nu_a}$. Therefore, using eqs (A4) and (A5) in (A3) gives

$$(Y^\dagger Y)_{aa} = (I - H^\dagger H - (I + \Delta)^2)_{aa},$$

which up to $\mathrm{O}(s^2)$ is

$$(Y^\dagger Y)_{aa} = (-H^\dagger H - 2\Delta)_{aa} = (4F^\dagger F + H^\dagger H)_{aa}. \tag{A6}$$

Using (for justification see Appendix of [1]),

$$(F^\dagger F)_{aa} = \lambda_F^a (\sin\theta_{\mathrm{L}}^{\nu_a})^2,$$

and

$$(H^\dagger H)_{aa} = \lambda_H^a (\sin\theta_{\mathrm{L}}^{\nu_a})^2,$$

where $0 \le \lambda \le 1$, we get

$$(Y^\dagger Y)_{aa} = (4\lambda_F^a + \lambda_H^a)(\sin\theta_{\mathrm{L}}^{\nu_a})^2. \tag{A7}$$

Acknowledgements

JKS thanks University Grants Commission and NKS thanks DST for financial assistance.

References

[1] P Langacker and D London, *Phys. Rev.* **D38**, 886 (1988)
[2] A Djouadi, J Ng and T G Rizzo, in *Electroweak symmetry breaking and beyond the standard model* edited by T Barklow, S Dawson, H E Haber and S Siegrist (World Scientific, 1997) p. 416 (hep-ph/9504210)
[3] A large number of references to the models containing possible types of new fermions are given in [1] and [2]. Table 1 of ref. [1] and §1 (overview) of [2] contains possible types of new fermions.
[4] D E Groom *et al*, *Europhys. J.* **C15**, 1 (2000)
[5] M Acciarri *et al*, *Phys. Lett.* **B462**, 354 (1999)
W Rodejohann and K Zuber, *Phys. Rev.* **D62**, 094017 (2000)
P Achard *et al*, *Phys. Lett.* **B517**, 75 (2001)
[6] J L Hewett and T G Rizzo, *Phys. Rep.* **183**, 193 (1989)
E Nardi, E Roulet and D Tommasini, *Nucl. Phys.* **B386**, 239 (1992); *Phys. Rev.* **D46**, 3040 (1992); *Phys. Lett.* **B327**, 319 (1994); **B344**, 225 (1994)
F M L Almeida Jr., Y A Coutinho, J A Martins Simoes and M A B do Vale, *Phys. Rev.* **D62**, 075004 (2000)
[7] See for example, S Weinberg, *The quantum theory of fields* (Cambridge University Press, Cambridge, 1995) vol. I, p. 147
[8] S Dawson, Lectures at the 1994 TASI Summer School (hep-ph/9411325)
[9] B W Lee, C Quigg and H B Thacker, *Phys. Rev.* **D16**, 1519 (1977)
W Marciano, G Valencia and S Willenbrock, *Phys. Rev.* **D40**, 1725 (1989)
[10] C Kolda and H Murayama, *J. High Energy Phys.* **07**, 35 (2000)
[11] For more details see ref. [1]
[12] P Achard *et al*, *Phys. Lett.* **B517**, 67 (2001)
[13] F M Renard, *Basics of electron and positron collisions* (Editions Frontiers, Gifsur Yvette, France, 1981)

[14] K Hagiwara and D Zeppenfeld, *Nucl. Phys.* **B274**, 1 (1986)
[15] M Dine, *Possible scales of new physics*, Plenary talks on *Beyond the SM Physics*, DPF 99 (UCLA) hep-ph/9905219 (1999)
[16] J Eillis, D V Nanopoulos and K A Olive, *Phys. Lett.* **B508**, 65 (2001)
[17] J Ellis, G Ganis, D V Nanopoulos and K A Olive, *Phys. Lett.* **B502**, 171 (2001)
[18] New almost-degenerate neutral heavy leptons are a feature of a number of theories of physics beyond the Standard Model, see for example, B C Allanach, C M Harris, M A Parker, P Richardson and B R Webber, *J. High Energy Phys.* **0108**, 51 (2001)
[19] D Wyler and L Wolfenstein, *Nucl. Phys.* **B218**, 205 (1983)
R N Mohapatra and J W F Valle, *Phys. Rev.* **D34**, 1642 (1986)
J W F Valle, *Nucl. Phys. (Proc. Suppl.)* **B11**, 118 (1989)
[20] P Langacker and D London, *Phys. Rev.* **D38**, 907 (1988)
[21] T P Cheng and L F Li, *Phys. Rev. Lett.* **38**, 381 (1977); **45**, 1908 (1980); see also T P Cheng and L F Li, *Gauge theory of elementary particle physics* (Oxford University Press, reprint 1988) p. 420
[22] J Brenabeu, A Santamaria, J Vidal, A Mendez and J W F Valle, *Phys. Lett.* **B187**, 303 (1987)
S L Glashow, *Phys. Lett.* **B187**, 367 (1987)
M C Ganzalez-Garcia and J W Valle, *Phys. Lett.* **B216**, 360 (1989)
[23] For a most recent review of current theoretical and experimental status, see Y Kuno and Y Okada, *Rev. Mod. Phys.* **73**, 151 (2001)
[24] MEGA Collaboration: M L Brooks, *Phys. Rev. Lett.* **83**, 1521 (1999), see also refs [4,21]