PRAMANA
— journal of
physics

© Indian Academy of Sciences

Vol. 59, No. 3
September 2002
pp. 433–443

Radial oscillations of neutron stars in strong magnetic fields

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MS received 8 October 2001; revised 21 February 2002

Abstract. The eigen frequencies of radial pulsations of neutron stars are calculated in a strong magnetic field. At low densities we use the magnetic BPS equation of state (EOS) similar to that obtained by Lai and Shapiro while at high densities the EOS obtained from the relativistic nuclear mean field theory is taken and extended to include strong magnetic field. It is found that magnetized neutron stars support higher maximum mass whereas the effect of magnetic field on radial stability for observed neutron star masses is minimal.

Keywords. Neutron stars; radial oscillations; magnetic field.

PACS No. **97.60.Jd**

1. Introduction

It is well-known that intense magnetic fields ($B \sim 10^{12-13}$ G) exist on the surface of many neutron stars. Objects with even higher magnetic fields have been surmised and detected recently. Recent observational studies and several independent arguments link the class of soft γ-ray repeaters and perhaps certain anomalous X-ray pulsars with neutron stars having ultra strong magnetic fields, the so called magnetars. Kuoveliotou *et al* [1] found a soft γ-ray repeater SGR 1806-20 with a period of 7.47 s and a spin-down rate of 2.6×10^{-3} s yr^{-1} from which they estimated the pulsar age to be about 1500 years and field strength of $\sim 8\times10^{14}$ G. Since the magnetic field in the core, according to some models, could be 10^3–10^5 times higher than its value on the surface, it is possible that ultra strong magnetic fields of order 10^{18} G or greater can exist in the core of certain neutron stars. According to Kuoveliotou *et al* [1], a statistical analysis of the population of soft γ-ray repeaters indicates that instead of being just isolated examples, as many as 10% of neutron stars could be magnetars. It is therefore of interest to study the equation of state of nuclear matter and various properties of neutron stars under such magnetic fields. Equations of state at low densities in the presence of magnetic field have been extensively studied in the literature [2,3]. In recent years the effect of strong magnetic field on the EOS of cold, charge neutral, super dense, interacting nuclear matter in β-equilibrium has been studied in a relativistic mean field theoretical framework [4–6]. In some of these studies [5,6] not only the effect of Landau quantization but also the contribution of anomalous magnetic moment of nucleons was incorporated in a relativistic description and it was found that for magnetic fields $\geq 10^{18}$ G, this effect cannot be ignored. The effect of magnetic field has been basically to increase the proton fraction and thereby lowering the threshold for direct URCA process to proceed and to modify the EOS in comparison to the field-free case. However, a super strong magnetic field capable of substantially modifying the EOS would also modify the structure of the star because of the electromagnetic stresses leading to anisotropy. Boucquet *et al* [7] studied the structure of rotating magnetized stars in a general relativistic framework but neglected the change induced by the magnetic field in the pressure and energy of the nuclear matter whereas the authors of ref. [4] assumed spherical symmetry and investigated the mass–radius relationship. In this paper we study the radial oscillations of neutron stars in the presence of super strong magnetic fields. Studies of radial oscillations is of interest since Cameron [8] suggested more than three decades ago that vibrations of neutron stars could excite motions that can have interesting astrophysical applications. X-ray and γ-ray burst phenomena are clearly explosive in nature. These explosive events probably perturb the associated neutron star and the resulting dynamical behavior may eventually be deduced from such observations. Observations of quasi-periodic pulses of pulsars have also been associated with oscillations of underlying neutron stars [9]. As a first step towards studying the effect of magnetic field on radial oscillations, we assume spherical symmetry and incorporate its effect only on the nuclear matter EOS. A consistent calculation of rotating, magnetized neutron star structure and radial oscillations incorporating both the magnetic EOS and general relativistic framework will be the subject matter of future work. The EOS is central to the calculation of most neutron star properties as it determines the mass range, the red shift as well as mass–radius relationship for these stars. Since neutron stars span a very wide range of densities, no one EOS is adequate to describe the properties of neutron stars. In the low density regions from the neutron drip density ($\sim 4\times 10^{11}$) and up to $\rho_{\rm n} \simeq 3.0\times 10^{14}$ g/cc the density at which the nuclei just begin to dissolve and merge together, the nuclear matter EOS is adequately described by the BPS model [10] which is based on the semi-empirical nuclear mass formula. We adopt this BPS EOS and its magnetized version as given by Lai and Shapiro [3] in this density range. In the high density range above the neutron drip density $\rho_{\rm n}$, the physical properties of matter are still uncertain. Many models for the description of nuclear matter at such high densities have been proposed over the years. One of the most studied models is the relativistic nuclear mean field theory, in which the strong interactions among various particles involved are mediated by a scalar field σ, an isoscalar–vector field ω and an isovector–vector field ρ. Along with scalar self-interaction terms it can reproduce the values of experimentally known quantities relevant to nuclear matter, viz., the binding energy per neucleon, the nuclear density at saturation, the asymmetry energy, the effective mass and the bulk modulus and provide a good description of nuclear matter for densities up to a few times the saturation density $\rho_{\rm c}$. In our study, following ref. [5] we use this nuclear mean field theory and its modification in the presence of a magnetic field.

In §2 a brief discussion of the EOS is given at zero temperature. In §3 we present the formalism for radial pulsations of the neutron star models computed here as a result of integration of the relativistic equations. Section 4 deals with Results and Discussion.

2. The equation of state (EOS) for nuclear matter

We shall describe nuclear matter at high densities by the relativistic nuclear mean field model, in the presence of constant magnetic field with baryons interacting through the exchange of σ–ω–ρ mesons. For densities less than the neutron drip density we adopt the BPS model in the presence of magnetic field as developed by Lai and Shapiro.

2.1 *The nuclear mean field EOS at high densities*

We consider the charge neutral nuclear matter consisting of neutrons, protons and electrons in β-equilibrium in the presence of a magnetic field and at zero temperature ($T = 0$). Following ref. [5], the thermodynamic potential of the system is given by

$$\Omega = -\frac{1}{2}m_\omega^2\langle\omega_0\rangle^2 - \frac{1}{2}m_\rho^2\langle\rho_0\rangle^2 + \frac{1}{2}m_\sigma^2\langle\sigma\rangle^2 + U(\sigma) + \frac{B^2}{8\pi} + \sum_{i=\mathrm{n,p,e}}\Omega_i \tag{1}$$

where

$$U(\sigma) = \frac{1}{3}bm_\mathrm{n}(g_{\sigma\mathrm{N}}\sigma)^3 + \frac{1}{4}c(g_{\sigma\mathrm{N}}\sigma)^4 \tag{2}$$

$$\Omega_\mathrm{e} = -\frac{eB}{4\pi^2}\sum_\nu(2-\delta_{\nu,0})\left[\mu_\mathrm{e}p_\mathrm{f}^\mathrm{e}(\nu) - \bar{m}_\mathrm{e}^2\ln\frac{\mu_\mathrm{e}+p_\mathrm{f}^\mathrm{e}(\nu)}{\bar{m}_\mathrm{e}}\right] \tag{3}$$

$$\Omega_\mathrm{p} = -\frac{eB}{4\pi^2}\sum_s\sum_\nu(2-\delta_{\nu,0})\left[\mu_\mathrm{p}^*p_\mathrm{f}^\mathrm{p}(s,\nu) - \bar{m}_\mathrm{p}^2\ln\frac{\mu_\mathrm{p}^*+p_\mathrm{f}^\mathrm{p}(s,\nu)}{\bar{m}_\mathrm{p}}\right] \tag{4}$$

$$\Omega_\mathrm{n} = -\frac{1}{8\pi^2}\sum_s\left[\frac{1}{3}\mu_\mathrm{n}^*p_\mathrm{f}^{\mathrm{n}^3}(s) - \frac{1}{2}\bar{m}_\mathrm{n}\mu_\mathrm{n}^*p_\mathrm{f}^\mathrm{n}(s) + \frac{1}{2}\bar{m}_\mathrm{n}^4\ln\frac{\mu_\mathrm{n}^*+p_\mathrm{f}^\mathrm{n}(s)}{\bar{m}_\mathrm{n}}\right] \tag{5}$$

and

$$\begin{aligned}
\mu_\mathrm{N}^* &= \mu_\mathrm{N} - g_\omega\langle\omega_0\rangle + g_\rho\tau_{3\mathrm{N}}\langle\rho_0\rangle\\
m_\mathrm{N}^* &= m_\mathrm{N} - g_\sigma\langle\sigma_0\rangle\\
\bar{m}_\mathrm{e} &= \sqrt{m_\mathrm{e}^2 + 2\nu eB}\\
\bar{m}_\mathrm{p} &= \sqrt{m_\mathrm{p}^{*2} + 2\left(\nu + \frac{1}{2} + \frac{s}{2}\right)eB} + sK_\mathrm{p}B\\
\bar{m}_\mathrm{n} &= m_\mathrm{n}^* + sK_\mathrm{n}B
\end{aligned} \tag{6}$$

where $K_{\rm n}$, $K_{\rm p}$ are the anomalous magnetic moments of the neutron and proton given by $K_{\rm n} = \frac{e}{2m_{\rm n}}\frac{g_{\rm n}}{2}$, $K_{\rm p} = \frac{e}{2m_{\rm p}}\left(\frac{g_{\rm p}}{2}-1\right)$ with Lande g factors $g_{\rm n} = -3.02$ and $g_{\rm p} = 5.58$ respectively.

The Fermi momenta $p_{\rm f}^{\rm e}(\nu)$, $p_{\rm f}^{\rm p}(s,\nu)$ and $p_{\rm f}^{\rm n}(s)$ are given by

$$p_{\rm f}^{\rm e}(\nu) = \sqrt{\mu_{\rm e}^2 - \bar{m}_{\rm e}^2}$$
$$p_{\rm f}^{\rm p}(s,\nu) = \sqrt{\mu_{\rm p}^{*2} - \bar{m}_{\rm p}^2}$$
$$p_{\rm f}^{\rm n}(s) = \sqrt{\mu_{\rm n}^{*2} - \bar{m}_{\rm n}^2}. \tag{7}$$

In the mean field approximation, the thermodynamic quantities are expressed in terms of thermodynamic averages of meson field which are assumed to be constant and are related to the baryonic and scalar number densities through the field equations viz.

$$m_\sigma^2\langle\sigma\rangle + \frac{\partial U}{\partial\sigma} = g_{\sigma{\rm N}}(n_{\rm n}^s + n_{\rm p}^s) \tag{8}$$

$$m_\omega^2\langle\omega_0\rangle = g_{\omega{\rm N}}(n_{\rm n} + n_{\rm p}) \tag{9}$$

$$m_\rho^2\langle\rho_0\rangle = g_{\rho{\rm N}}(n_{\rm p} - n_{\rm n}). \tag{10}$$

The number densities are given by

$$n_{\rm n} = \frac{1}{2\pi^2}\sum_s\left[\frac{1}{3}p_{\rm f}^{{\rm n}3}(s) - \frac{1}{2}sK_{\rm n}B\left(\mu_{\rm n}^{*2}\left(\sin^{-1}\frac{\bar{m}_{\rm n}}{\mu_{\rm n}^*} - \frac{\pi}{2}\right) + \bar{m}_{\rm n}p_{\rm f}^{\rm n}(s)\right)\right]$$
$$n_{\rm p} = \frac{eB}{2\pi^2}\sum_s\sum_\nu p_{\rm f}^{\rm p}(s,\nu)$$
$$n_{\rm e} = \frac{eB}{2\pi^2}\sum(2-\delta_{\nu,0})p_{\rm f}^{\rm e}(\nu) \tag{11}$$

and the scalar densities are given by

$$n_{\rm p}^s = \frac{eB}{2\pi^2}\sum_s\sum_\nu \bar{m}_{\rm p}\ln\frac{\mu_{\rm p}^* + p_{\rm f}^{\rm p}(s,\nu)}{\bar{m}_p}$$
$$n_{\rm n}^s = \frac{\bar{m}_{\rm n}}{4\pi^2}\sum_s\left[\mu_{\rm n}^* p_{\rm f}^{\rm n}(s) - \bar{m}_{\rm n}^2\ln\frac{\mu_{\rm n}^* + p_{\rm f}^{\rm n}(s)}{\bar{m}_{\rm n}}\right]. \tag{12}$$

From the thermodynamic potential, all the thermodynamic quantities can be determined by the usual relation $P = -\Omega$ and $\varepsilon = \Omega + \Sigma\mu_i n_i (i = {\rm n, p, e})$ by solving the field equations [eqs (8), (9) and (10)] along with the charge neutrality condition

$$n_{\rm p} = n_{\rm e} \tag{13}$$

and the condition of β-equilibrium

$$\mu_{\rm n} = \mu_{\rm p} + \mu_{\rm e} \tag{14}$$

self consistently for a given baryon density

$$n_{\rm B} = n_{\rm p} + n_{\rm n} \tag{15}$$

and for a given set of nuclear-meson and scalar self-interaction coupling constants. We thus compute the high density equation of state.

Table 1. BPS equilibrium nuclei below neutron drip.

Element	BPS Mass-energy (in units of 10^4 MeV)
Fe^{56}_{26}	5.2103
Ni^{62}_{28}	5.7686
Ni^{64}_{28}	5.9549
Ni^{66}_{28}	6.1413
Kr^{86}_{36}	8.0025
Se^{84}_{34}	7.8170
Ge^{82}_{32}	7.6316
Zn^{80}_{30}	7.4466
Ni^{78}_{28}	7.2621
Ru^{126}_{44}	11.7337
Mo^{124}_{42}	11.5495
Zr^{122}_{40}	11.3655
Sr^{120}_{38}	11.1818
Sr^{122}_{38}	11.3655
Kr^{118}_{36}	10.9985

2.2 *The magnetic BPS model*

The BPS model describes the EOS for cold, β-equilibrated catalyzed matter below neutron drip, i.e., below $\rho \sim 4.4 \times 10^{11}$ g/cc. Following ref. [3], the total pressure of the hadron matter condensing into a perfect crystal lattice with a nuclear species (A, Z) at the lattice sites is given by

$$P = P_{\mathrm{e}}(n_{\mathrm{e}}) + P_{\mathrm{L}} = P_{\mathrm{e}}(n_{\mathrm{e}}) + \frac{1}{3}\varepsilon_{\mathrm{L}}(Z, n_{\mathrm{e}}) \tag{16}$$

where ε_{L} is the bcc Coulomb lattice energy given by

$$\varepsilon_{\mathrm{L}} = -1.444 Z^{2/3} e^2 e^2 n_{\mathrm{e}}^{4/3} \tag{17}$$

and P_{e} the pressure, and n_{e} the density of the electrons in the presence of the magnetic field are given by equations in [3] and [11]. The energy density is given by

$$\varepsilon = \frac{n_{\mathrm{e}}}{Z} W_{\mathrm{N}}(A, Z) + \varepsilon'_{\mathrm{e}}(n_{\mathrm{e}}) + \varepsilon_{\mathrm{L}}(Z, n_{\mathrm{e}}) \tag{18}$$

where W_{N} is the mass-energy of the nucleus (including the rest mass of nucleons and Z electrons) and $\varepsilon'_{\mathrm{e}}$ is the free electron energy excluding the rest mass of electrons. For W_{N} we use the experimental values for laboratory nuclei as tabulated by Wapastra and Bos [11]. The elements taken in this paper are given in table 1 along with their mass energy $W_{\mathrm{N}}(A, Z)$. At a given pressure P, the equilibrium values of A and Z are determined by minimizing the Gibbs free energy per nucleon

$$g = \frac{\varepsilon + P}{n} = \frac{W_{\rm N}(A,Z)}{A} + \frac{Z}{A}(\mu_{\rm e} - m_{\rm e}c^2) + \frac{4Z\varepsilon_{\rm L}}{3An_{\rm e}}. \tag{19}$$

The neutron drip point is determined by the condition

$$g_{\rm min} = m_{\rm n}c^2. \tag{20}$$

Thus knowing A and Z the energy can be determined from eq. (18).

3. Radial pulsations of a non-rotating neutron star

The equations governing infinitesimal radial pulsations of a non-rotating star in general relativity were given by Chandrasekhar [9]. The structure of the star in hydrostatic equilibrium is described by the Tolman–Openheimer–Volkoff equations

$$\frac{{\rm d}p}{{\rm d}r} = \frac{-G(p+pc^2)\left(m + \frac{4\pi r^3 p}{c^2}\right)}{c^2 r^2\left(1 - \frac{2Gm}{c^2 r}\right)} \tag{21}$$

$$\frac{{\rm d}m}{{\rm d}r} = 4\pi r^2 \rho \tag{22}$$

$$\frac{{\rm d}\nu}{{\rm d}r} = \frac{Gm\left(1 + \frac{4\pi r^3 p}{mc^2}\right)}{c^2 r\left(1 - \frac{2Gm}{c^2 r}\right)}. \tag{23}$$

Given an equation of state $p(\rho)$, eqs (21)–(23) can be integrated numerically for a given central density to obtain the radius R and gravitational mass $M = M(R)$ of the star. The metric used is given by

$${\rm d}s^2 = -{\rm e}^{2\nu}c^2\,{\rm d}t^2 + e^{2\lambda}\,{\rm d}r^2 + r^2({\rm d}\theta^2 + \sin^2\theta\,{\rm d}\phi^2). \tag{24}$$

If Δr is the radial displacement

$$\xi = \frac{\Delta r}{r} \tag{25}$$

$$\zeta = r^2 e^{-\nu}\xi \tag{26}$$

and the time dependence of the harmonic oscillations is written as ${\rm e}^{{\rm i}\sigma t}$, one gets the equation governing radial adiabatic oscillations [9,12,13]

$$F\frac{{\rm d}\zeta}{{\rm d}r^2} + G\frac{{\rm d}\zeta}{{\rm d}r} + H\zeta = \sigma^2\zeta \tag{27}$$

where

$$F = -\frac{{\rm e}^{2\nu - 2\lambda}(\Gamma p)}{p + \rho c^2} \tag{28}$$

$$G = -\frac{e^{2\nu-2\lambda}}{p+\rho c^2}\left[(\Gamma p)\left(\frac{d\lambda}{dr}+3\frac{d\nu}{dr}\right)+\frac{d}{dr}(\Gamma p)-\frac{2}{r}(\Gamma p)\right] \tag{29}$$

$$H = \frac{e^{2\nu-2\lambda}}{p+\rho c^2}\left[\frac{4}{r}\frac{dp}{dr}+\frac{8\pi G}{c^4}e^{2\lambda}p(p+\rho c^2)-\frac{1}{p+\rho c^2}\left(\frac{dp}{dr}\right)^2\right] \tag{30}$$

$$\lambda = -\ln\left[1-\frac{2Gm}{rc^2}\right]^{1/2}. \tag{31}$$

In the above equations Γ the adiabatic index is given by

$$\Gamma = \frac{p+\rho c^2}{c^2 p}\frac{dp}{d\rho}. \tag{32}$$

The boundary conditions to solve eq. (27) are

$$\zeta(r=0)=0$$
$$\delta p(r=R)=0. \tag{33}$$

The expression for δp as given by Chandrasekhar [11] is

$$\delta p(r) = -\frac{dp}{dr}\frac{e^{\nu}\zeta}{r^2}-\frac{\Gamma p e^{\nu}}{r^2}\frac{d\zeta}{dr}. \tag{34}$$

All these equations are totally model independent and are in fact the same whether we are considering neutron stars, quark stars or any other dense stellar object. The nature of the object being considered and the particular model affects the structure of the star and the frequency of radial pulsations only through the EOS. Note that in Chandrasekhar [9] and Datta *et al* [12] the pulsation equations were written in terms of ξ instead of ζ. Equation (27) along with the boundary conditions represent a Sturm–Liouville eigenvalue problem for σ^2. From the theory of such equations we have the well-known results: (i) Eigenvalues σ^2 are all real and (ii) they form an infinite discrete sequence

$$\sigma_0^2 < \sigma_1^2 < \sigma_2^2 \ldots .$$

An important consequence of (ii) is that if the fundamental radial mode of a star is stable ($\sigma_0^2 > 0$), then all the radial modes are stable.

4. Results and discussions

To study the structure and radial oscillations of neutron stars in the presence of a strong magnetic field we have employed the BPS model with its generalization in a magnetic field given by Lai and Shapiro [3] below the neutron drip and the RMF theory above it [5]. We have used the values of various couplings [6,12] which provide the known values of various nuclear matter parameters namely

$$\left(\frac{g_\sigma}{m_\sigma}\right) = 0.01525\,\text{MeV}^{-1}, \quad \left(\frac{g_\omega}{m_\omega}\right) = 0.011\,\text{MeV}^{-1}$$

$$\left(\frac{g_\rho}{m_\rho}\right) = 0.011\,\text{MeV}^{-1}$$

$$b = 0.003748, \quad c = 0.01328. \tag{35}$$

As explained in §2.1 and 2.2, for the RMF theory the equations are solved in a self-consistent manner for the effective masses and chemical potentials, and the EOS at high density obtained. Below the neutron drip, the EOS is obtained by the minimization of

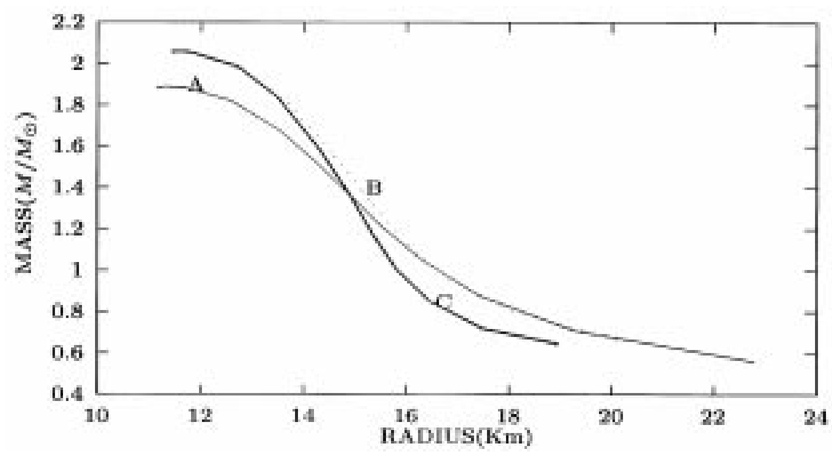

Figure 1. Plot of mass in solar mass unit vs. radius in km for magnetic fields $0, 1\times10^4$, 2×10^4 MeV^2 represented by the curves A, B and C respectively.

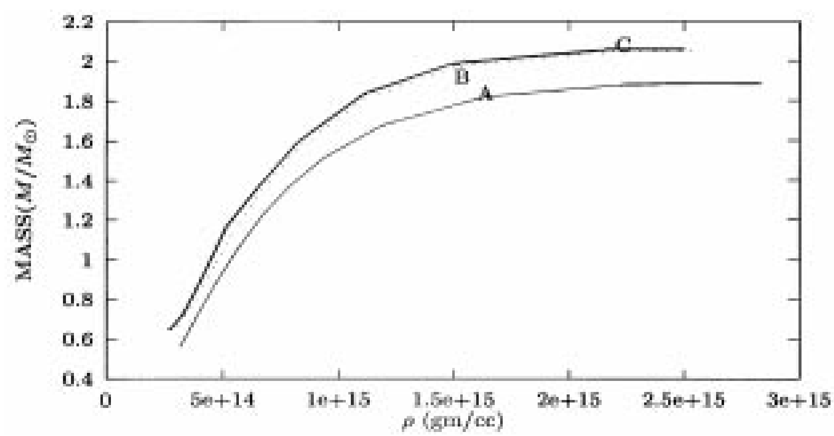

Figure 2. Plot of mass in solar mass unit vs. energy density for different magnetic fields. Curves A, B and C as in figure 1.

Gibb's free energy as functions of A and Z. For this purpose we have employed 14 nuclei listed in the table. The problem is solved separately for $B = 0$ and $B \neq 0$. For each B this gives the EOS in the form $P(n_B)$ and $\rho(n_B)$. The structure of the neutron star is then obtained from the integration of the Oppenheimer–Volkoff equations using Runge–Kutta integration procedure. This also gives the profile of m, p and ν as a function of r for each star. One more quantity that is required is Γ which is calculated directly from the EOS for all densities by using a quadratic difference formula for the derivative $\mathrm{d}p/\mathrm{d}(\rho c^2)$.

Along with the M–R relationship, one also obtains the gravitational red shift

$$Z = \left[1 - \frac{2Gm}{c^2 r}\right]^{-1/2} - 1 \tag{36}$$

which can in principle be observed experimentally.

For radial oscillation frequency equations [eqs (27)–(32)] are also solved using Runge–Kutta of order 4 integration procedure for the boundary conditions (33). We use a trial value of σ for a given set of values of $\zeta(r = 0)$ and $\zeta'(r = 0)$ and integrate the equation outword from the centre up to the surface. The trial value of σ is varied till the boundary condition

$$\delta p(r) = 0 \qquad \text{at } r = R$$

is satisfied. The discrete values of σ^2 for which the boundary condition is satisfied are the eigen frequencies of the radial pulsations. We check that we get zero frequency modes at the maximum as well as at the minimum of the mass curves. By changing the number of mesh points, it was estimated that σ^2 is accurate to one part in 10^3. In figure 1 we plot mass in solar mass unit vs. radius in km for magnetic fields 0, 1×10^4, 2×10^4 MeV^2 ($1\ \mathrm{MeV}^2 = 1.69 \times 10^{14}$ G) represented by the curves A, B and C respectively. It is worthwhile to note that the magnetized neutron stars support higher masses. For very high magnetic fields the stars become relatively more compact. In figure 2 we present mass vs. central energy density and in figure 3 we have plotted gravitational red shift vs. mass. In figure 4 a plot of time period of fundamental mode vs. gravitational red shift is given for the same magnetic fields as in figure 1. It is interesting to note that for the observed neutron

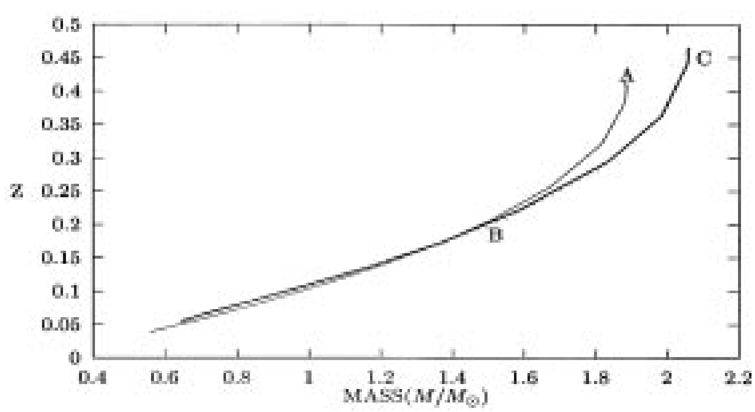

Figure 3. Plot of gravitational red shift (Z) vs. mass in solar mass unit for different magnetic fields. Curves A, B and C as in figure 1.

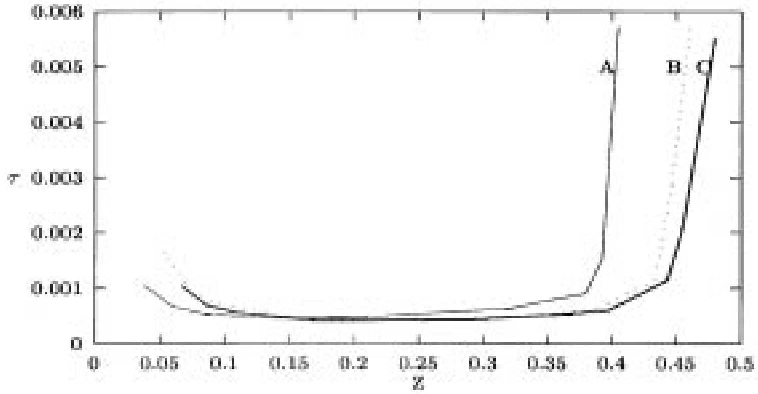

Figure 4. Plot of time period τ for fundamental mode vs. gravitational red shift (Z) for different magnetic fields. Curves A, B and C as in figure 1.

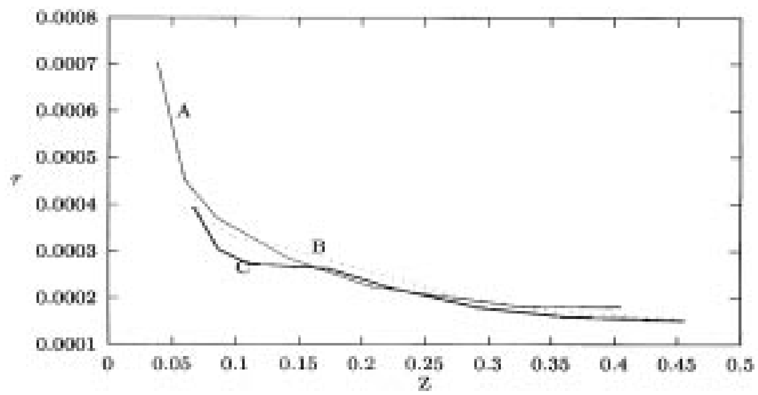

Figure 5. Plot of time period τ for $n = 1$ mode vs. gravitational red shift (Z) for different magnetic fields. Curves A, B and C as in figure 1.

star mass (1.4 $M_{\odot}$), the magnetic field has practically no influence on radial stability. Similar trend is seen for the first excited mode as shown in figure 5.

References

[1] Kuoveliotou *et al*, *Nature* **393**, 235 (1998)
[2] I Fushiki, E H Gumundsson and C J Pethic, *Astrophys. J.* **342**, 958 (1989)
A M Abrahams and S L Shapiro, *Astrophys. J.* **374**, 652 (1991)
O E Rognvaldsson, I Fushiki, E H Gudmundsson and C J Pethic, *Astrophys. J.* **416**, 276 (1993)
A Thorlofsson, O E Rognvaldsson, J Yngvasan and E H Gudmundsson, *Astrophys. J.* **502**, 847 (1998)
[3] D Lai and S Shapiro, *Astrophys. J.* **383**, 745 (1991)
[4] S Chakrabarty, *Phys. Rev.* **D54**, 1306 (1996)
S Chakrabarty, D Bandyopadhyay and S Pal, *Phys. Rev. Lett.* **78**, 2898 (1997)

[5] Ashok Goyal, V K Gupta, Kanupriya Goswami and Vinita Tuli, *Int. J. Mod. Phys.* **A16**, 347 (2001)
[6] A Broderick, M Prakash and J M Lattimer, *Astrophys. J.* **537**, 351 (2000)
[7] M Boucquet, S Bonozzola, E Gourgoulhon and J Novah, *Astron. Astrophys.* **301**, 757 (1995)
[8] A G W Cameron, *Nature* (London) **205**, 787 (1965)
[9] S Chandrasekhar, *Phys. Rev. Lett.* **12**, 114 (1964)
S Chandrasekhar, *Astrophys. J.* **140**, 417 (1964)
C Cutler, L Lindblow and R J Splinter, *Astrophys. J.* **363**, 603 (1990)
[10] G Bayam, C Pethick and Sutherland, *Astrophys. J.* **170**, 299 (1971)
[11] A H Wapastra and K Bos, *Atomic Data Nucl. Data Tables* **17**, 474 (1976)
A H Wapastra and K Bos, *Atomic Data Nucl. Data Tables* **19**, 175 (1977)
[12] B Datta, S S Hasan, P K Sahu and A R Prassana, *Int. J. Mod. Phys.* **D7**, 49 (1998)
J Ellis, J I Kapusta and K A Olive, *Nucl. Phys.* **B348**, 345 (1991)
P L Hong, *Phys. Lett.* **B445**, 36 (1998)
[13] J D Anand, N Chandrika Devi, V K Gupta and S Singh, *Astrophys. J.* **538**, 870 (2000)