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Tunneling time distribution by means of Nelson's quantum mechanics and wave-particle duality

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Abstract. We construct a tunneling time distribution by means of Nelson's quantum mechanics and investigate statistical properties of the tunneling time distribution. As a result, we find that the relationship between the average and the variance of the tunneling time shows 'wave-particle duality'.

Keywords. Tunneling time distribution; wave-particle duality.

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1. Introduction

It was suggested that there is a time associated with the passage of a particle under a tunneling barrier, the so-called tunneling time [1]. The existence of such a time is now accepted. In fact, the time has been measured experimentally. However, there is no clear consensus about the existence of a simple expression for this time, and the exact nature of that expression. See ref. [2] and references therein for reviews of this problem.

In this work, our objective is to construct the tunneling time distribution by means of Nelson's quantum mechanics [3] and investigate the statistical properties of the distribution. The tunneling phenomena are purely quantum mechanical. If it is possible to measure the tunneling time, it is expected that the tunneling time does not take a unique value but is distributed over a rather wide region. In spite of this prediction, in many approaches discussions are centered only on the average value because it is difficult to consider the tunneling time distribution using the conventional framework of quantum mechanics. However, Nelson's quantum mechanics can afford to predict such a distribution. This method gives us the same results as conventional quantum mechanics from the ensemble of sample paths generated by the following Langevin equation

$$\mathrm{d}x(t) = [u(x(t),t) + v(x(t),t)]\mathrm{d}t + \mathrm{d}w(t), \tag{1}$$

where $u(x(t),t)$ and $v(x(t),t)$ are the osmotic and current velocities defined by such relations

$$u = \mathrm{Re}\frac{\hbar}{m}\frac{\partial}{\partial x}\ln\psi(x,t), \text{ and } v = \mathrm{Im}\frac{\hbar}{m}\frac{\partial}{\partial x}\ln\psi(x,t), \tag{2}$$

with ψ as the solution of the Schrödinger equation, and $\mathrm{d}w(t)$ a Gaussian white noise with the statistical properties of

$$\langle \mathrm{d}w(t)\rangle = 0, \text{ and } \langle \mathrm{d}w(t)\mathrm{d}w(t)\rangle = \frac{\hbar}{m}\mathrm{d}t, \tag{3}$$

$\langle\cdots\rangle$ indicating the ensemble average with respect to the noise. Thus, this method enables us to obtain all information on the time spent by a certain sample path in a tunneling barrier [4,5]. In this work, for simplicity, we analyze the tunneling phenomenon with a 1-dimensional static rectangular potential barrier which has a potential height V_0, and a width d. The initial Gaussian wave packet with a rather wide variance Δx^2 is injected into the potential barrier. We use the natural unit $m = \hbar = 1$, and perform numerical simulations by using 100,000 sample paths.

2. Tunneling time distribution and wave-particle duality

From the ensemble of sample paths, we define a distribution of tunneling times as follows:

$$P(\tau)\Delta\tau \equiv \Delta n(\tau)/N, \tag{4}$$

where $\Delta n(\tau)$ is the number of sample paths with tunneling times between τ and $\tau + \Delta\tau$, and N is the total number of sample paths. Figure 1 shows the numerical result of the distribution as a function of the tunneling time.

In a previous paper [6], from the discussion of the tunneling time average $\langle\tau\rangle$, we found that there are two characteristic regions classified by comparing the action κd with $\hbar$. Here κ is the imaginary wave number inside the potential defined as $\kappa \equiv \sqrt{2m(V_0 - E_0)/\hbar^2}$. In one region $\kappa d/\hbar$ is small and $\langle\tau\rangle$ is not in agreement with the WKB time $\tau_{\mathrm{WKB}} = md/\hbar\kappa$, and in the other region $\kappa d/\hbar$ is large and $\langle\tau\rangle$ agrees with the WKB time. Then, we call the former region 'translucent' and the latter 'opaque'.

By changing the width of barrier with fixed potential heights, figure 1 shows the relationship between the average $\langle\tau\rangle$ and the deviation $\Delta\tau$ of tunneling times.

The arrows in this figure point out the data in the case of $\kappa d = 2$, and the dashed and dotted lines are straight lines with grade of 1 and $1/2$, respectively. The former shows the relation that the deviation is proportional to the average, and the latter shows the relation that the deviation is proportional to the square root of the average. Consequently, we find some characteristics related with the tunneling time distribution. The 'transition' from linearity to the square root behavior occurs around $\kappa d = 2$. It seems that the shape of the distribution is dominantly determined by only κd. The deviation is proportional to the average in the translucent case, and the square root of the average in the opaque case. It seems that the 'wave-particle duality' exists in such a relationship. In the translucent case, it has the property $\Delta\tau \propto \langle\tau\rangle$ which is often found for an 'oscillation' or a 'wave'. That is, this relation is associated with the 'wave' property. On the other hand, the property $\Delta\tau \propto \sqrt{\langle\tau\rangle}$ is found in statistics of, for example, the Poisson distribution, and the Brownian motion, and so on. These examples are associated with the 'point-like' particle, or the

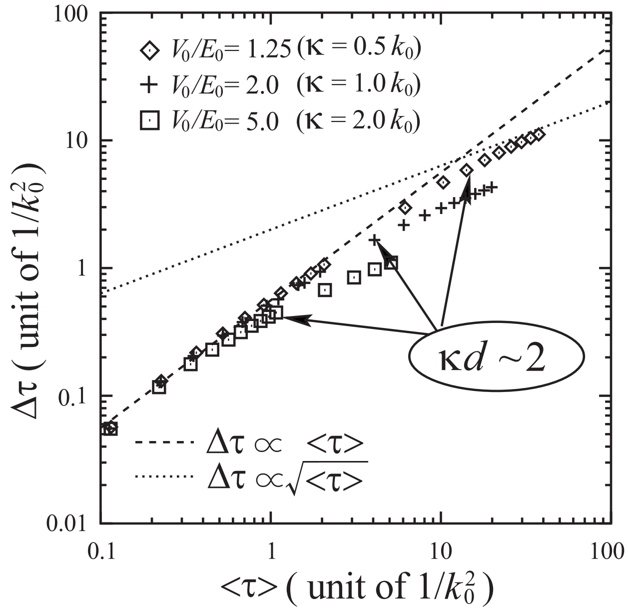

Figure 1. Relationship between the average $\langle\tau\rangle$ and the deviation $\Delta\tau$ of tunneling times. Dashed line shows the relation $\Delta\tau \propto \langle\tau\rangle$. Dotted line shows the relation $\Delta\tau \propto \sqrt{\langle\tau\rangle}$.

ensemble of that. Such a relation is associated with the 'particle-like' property. That is, the two relations obtained from tunneling time distributions show 'wave-like' property in the translucent case, and 'particle-like' property in the opaque case, respectively. So, we think that the relation between the deviation and the average of tunneling times reveals 'wave-particle duality'.

3. Summary

We have constructed the tunneling time distribution by means of Nelson's quantum mechanics and investigated statistical properties of the tunneling time distribution. By considering two characteristic regions, viz., 'translucent' and 'opaque' classified by comparing the action κd with $\hbar$, we found that the relationship between the average and the deviation of tunneling times shows 'wave-particle duality'.

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References

[1] L A MacColl, *Phys. Rev.* **40**, 621 (1932)
[2] R Landauer and Th Martin, *Rev. Mod. Phys.* **66**, 217 (1994)
[3] E Nelson, *Phys. Rev.* **150**, 1079 (1966)
[4] K Imafuku, I Ohba and Y Yamanaka, *Phys. Lett.* **A204**, 329 (1995)
[5] K Imafuku, I Ohba and Y Yamanaka, *Phys. Rev.* **A56**, 1142 (1997)
[6] K Hara and I Ohba, *Phys. Rev.* **A62**, 032104 (2000)