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Violation of Bell's inequality in neutral kaons system

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Abstract. We show by general considerations that it is not possible to test violation of the existing versions of Bell's inequality in entangled neutral kaons system using experimentally accessible thin regenerators. We point out the loophole in the recent argument (A Bramon and M Nowakowski, *Phys. Rev. Lett.* **83**, 1 (1999)) that claimed such a test to be possible.

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Bell's inequality (BI) represents the fact that if one assumes local realism (LR) then the correlation between spatially separated constituents of a bipartite system is bound by 2, whereas quantum theory (QT) predicts such correlations exceeding 2 for certain types of entangled states. Hence LR can be experimentally confronted with QT by means of BI. Such experiments should be performed in various complementary branches of physics so that not only one can avoid the loopholes encountered in photon experiments [1], but also can explore different ramifications of quantum nonlocality. EPR-type entangled pairs consisting of massive, neutral K-mesons (kaons) is a system that has advantage over the massless photon system in two ways: (i) in defining the spatial separability in terms of localized entities, and (ii) having highly efficient particle physics detectors.

Taking into account the charge conjugation ($=-1$) of the Φ-meson the EPR–Bohm type entangled state of the neutral kaon pair coming from Φ decay, as at the DAΦNE Φ-factory [2], can be written as

$$|\Phi\rangle = \frac{1}{\sqrt{2}}[|K^0\rangle \otimes |\bar{K^0}\rangle - |\bar{K^0}\rangle \otimes |K^0\rangle]. \quad (1)$$

Their time evolution under weak interactions is given by [3]

$$|\Phi\rangle = \frac{N(t)}{\sqrt{2}}\left[|K_S\rangle \otimes |K_L\rangle - |K_L\rangle \otimes |K_S\rangle\right], \quad (2)$$

where $|N(t)| = (1+|\varepsilon|^2)/(|1-\varepsilon^2|) \times e^{-1/2}(\Gamma_S + \Gamma_L)t$ reflecting the extinction of the beams via weak interaction induced kaon decays characterized by decay constants Γ_S and Γ_L but without modifying the perfect antisymmetry of the initial state. The mass eigenstates $|K_S\rangle$ and $|K_L\rangle$ (with mass m_S and m_L respectively) are written in terms of CP eigenstates $|K_1\rangle$ and $|K_2\rangle$ and the CP violation parameter ε as

$$|K_S\rangle = \frac{1}{\sqrt{1+|\varepsilon|^2}}[|K_1\rangle + \varepsilon|K_2\rangle],$$
$$|K_L\rangle = \frac{1}{\sqrt{1+|\varepsilon|^2}}[|K_2\rangle + \varepsilon|K_1\rangle], \tag{3}$$

where

$$|K_{1,2}\rangle = \frac{1}{\sqrt{2}}[|K^0\rangle \pm |\bar{K^0}\rangle]. \tag{4}$$

The state (1) denotes a bipartite system formed by two subsystems which fly apart getting spatially separated thus defining after collimation a left- and a right-hand kaon beam. Its structure in the strangeness space is identical to that of the spin-singlet state decaying into two spin-1/2 particles in the EPR–Bohm configuration except for an important difference that reduces considerably the possibilities of Bell-tests with kaons. In the two spin-1/2 case spin projections (with individual measurement outcomes given by a dichotomic variable) on each one of the two distant subsystems can be measured along any *arbitrary* space direction chosen at will. One does not have such a privilege when one tries to measure S through decay channels over which one has no real control. Various interesting suggestions [4,5] to perform joint strangeness measurements at two different times on the right beam and at two other times on the left beam, in analogy with four different choices of measurement directions in spin case, unfortunately had yielded inequalities that are never violated by QT because of the dominance of rapid weak decays of kaons over their slow strangeness oscillations.

Another possible measurement with dichotomic outcomes is based [6] on identification of the K_S vs. K_L components of a beam via kaon weak decays and strong interactions. However, since the K_S and K_L states are not strictly orthogonal, $\langle K_S|K_L\rangle = 2\Re\varepsilon/(1+|\varepsilon|^2) \simeq 3.2\times 10^{-3}$, their identification cannot be exact, even in principle, unless one neglects the small CP-violation parameter $|\varepsilon| \simeq 2.3\times 10^{-3}$, and designs suitable experiments based on the fact that the decay probabilities of the two components are so different ($\Gamma_S \simeq 579\Gamma_L$).

In a recent important paper [7], it was pointed out that a situation equivalent to the spin-$1/2$ system can be achieved by measuring the strangeness S of a neutral kaon beam through its interaction with a regenerator (homogeneous slab of dense nucleonic matter). Then for thin regenerators with regeneration parameter ρ, the regeneration effect can be seen as pseudo-rotations (where only first order terms in the small *complex* parameter $r \equiv (im_S - im_L + \frac{1}{2}\Gamma_S - \frac{1}{2}\Gamma_L) \times \rho\Delta t$ have been kept):

$$|K_S\rangle \to |K_S\rangle + r|K_L\rangle$$
$$|K_L\rangle \to |K_L\rangle + r|K_S\rangle, \tag{5}$$

in the kaon quasi-spin space analogous to the strangeness oscillations but without requiring additional time intervals.

After establishing this complete analogy between the kaons case and the spin-1/2 EPRB configuration through the use of thin regenerators they proceeded to show the violation of the Wigner version of the Bell inequality

$$P(\mathbf{a},+;\mathbf{b},+) \le P(\mathbf{a},+;\mathbf{c},\sigma_c) + P(\mathbf{c},\sigma_c;\mathbf{b},+), \tag{6}$$

where σ_c can be either + or − and **c** stands for a given direction common to both left ($\mathbf{c} = \mathbf{a}'$) and right ($\mathbf{c} = \mathbf{b}'$) hand sides. Introducing the regenerator on the right beam and then reverting to the $K^0\bar{K}^0$-basis, eq. (2) becomes

$$|\Phi(0)\rangle \to |\Phi(t)\rangle \simeq \frac{N(t)}{\sqrt{2}}[(1-r)|K^0\rangle \otimes |\bar{K}^0\rangle - (1+r)|\bar{K}^0\rangle \otimes |K^0\rangle]. \tag{7}$$

As in the spin case, the antisymmetry of the initial state has been lost although not in the same way, as expected from the differences between rotation of spin by magnetic field and the pseudo-rotation introduced by regenerators. Then, one can unambiguously compute the detection probabilities in the context of quantum mechanics by simply projecting eq. (7) over the appropriate states

$$\begin{aligned}
&P(0,K^0;r,\bar{K}^0) = P(r,\bar{K}^0;0,K^0) \simeq N(t)/2 - N(t)\mathrm{Re}(r),\\
&P(0,\bar{K}^0;r,K^0) = P(r,K^0;0,\bar{K}^0) \simeq N(t)/2 + N(t)\mathrm{Re}(r),\\
&P(0,\bar{K}^0;r,\bar{K}^0) = P(r,\bar{K}^0;0,\bar{K}^0) \simeq 0,\\
&P(0,K^0;r,K^0) = P(r,K^0;0,K^0) \simeq 0,
\end{aligned} \tag{8}$$

where the left equalities are obvious consequences of rotation invariance and the approximated ones in the right are valid up to first order in r. In this notation, the K^0 or $\bar{K}^0$ indicates the outcome of the measurement and r or 0 denotes the presence or absence of the regenerator. Under the same conditions as before, one can now invoke local realistic theories to establish Wigner-inequalities such as

$$\begin{aligned}
&P(0,K^0;0,\bar{K}^0) \le P(0,K^0;r,K^0) + P(r,K^0;0,\bar{K}^0),\\
&P(0,K^0;0,\bar{K}^0) \le P(0,K^0;r,\bar{K}^0) + P(r,\bar{K}^0;0,\bar{K}^0).
\end{aligned} \tag{9}$$

The incompatibility between quantum mechanics and local realism appears when one introduces the probabilities (8) in (9): the first inequality leads to $\mathrm{Re}\,(r) \ge 0$, while the second one leads to $\mathrm{Re}(r) \le 0$. Hence, in any case (i.e., independently of the specific properties of the regenerator) one of the Wigner-inequalities (9) is violated by quantum mechanics.

The drawback of these analyses is that, up to now, they only refer to thin regenerators and the predicted violations of Bell's inequalities (below a few per cent) are hardly observable. Secondly and more importantly, the Wigner's version of BI that was used is derived with a crucial assumption [8] that for any pair of identical settings B_1 and B_2 in Bell type experiments, one has a perfect anticorrelation, i.e.,

$$P(\mathbf{b},+;\mathbf{b},+) = 0, \tag{10}$$

which translates into

$$P(K^0,0;K^0,0) = P(\bar{K}^0,0;\bar{K}^0,0) \tag{11}$$

for the entangled neutral kaons system. Although this is certainly what is predicted by QT for a singlet state, this cannot be established directly from the experimental data – these always show some 'error' due to misalignment of the axis of measuring devices relative to one another, and even the most minute departure from zero on the right hand side of the above equation spoils the condition of perfect correlation. Thus one cannot use Wigner's version of BI to test the experimental result directly against the LR hypothesis.

Because of the objections given above, we intend to study the violation of BI in entangled neutral kaons systems through the use of Bell's original inequalities and the CHSH version of BI. Introducing the regenerators r' and r on the left and right beam, respectively, and then reverting to the K^0, $\bar{K^0}$ basis, one obtains

$$|\Phi(0)\rangle \to |\Phi(t)\rangle \simeq \frac{N(t)}{\sqrt{2}}[(r-r'-1)|K^0\rangle \otimes |\bar{K^0}\rangle - (r-r'+1)|\bar{K^0}\rangle \otimes |K^0\rangle]. \tag{12}$$

Then using quantum mechanics one gets the detection probabilities as follows:

$$\begin{aligned}
&P(r',K^0;r,\bar{K^0}) \simeq N(t)[1/2 - \Re(r) + \Re(r')],\\
&P(r',\bar{K^0};r,K^0) \simeq N(t)[1/2 + \Re(r) + \Re(r')],\\
&P(r',K^0;r,K^0) = P(r',\bar{K^0};r,\bar{K^0}) \simeq 0,\\
&P(0,\bar{K^0};0,K^0) = P(0,K^0;0,\bar{K^0}) = N(t),\\
&P(0,K^0;r,\bar{K^0}) = 1/2 - \Re(r),\\
&P(r',K^0;0,\bar{K^0}) = 1/2 + \Re(r').
\end{aligned} \tag{13}$$

Bell's inequalities of the CHSH form

$$|P(\mathbf{a},\mathbf{b}) + P(\mathbf{a},\mathbf{b}') + P(\mathbf{a}',\mathbf{b}) - P(\mathbf{a}',\mathbf{b}')| \leq 2 \tag{14}$$

in this case yield the following conditions:

$$\begin{aligned}
\mathrm{e}^{-(\Gamma_\mathrm{S}+\Gamma_\mathrm{L})t} &\leq 2,\\
\mathrm{e}^{-(\Gamma_\mathrm{S}+\Gamma_\mathrm{L})t}|1-2\Re(r)+2\Re(r')| &\leq 2,\\
\mathrm{e}^{-(\Gamma_\mathrm{S}+\Gamma_\mathrm{L})t}|1-2\Re(r)| &\leq 2,\\
\mathrm{e}^{-(\Gamma_\mathrm{S}+\Gamma_\mathrm{L})t}|1+2\Re(r')| &\leq 2.
\end{aligned} \tag{15}$$

It is easy to see that consistency among the above four conditions requires

$$|1-2\Re(r)+2\Re(r')| \leq 1, \tag{16}$$

which is always satisfied. Thus CHSH version of BI is not violated for this case of thin regenerators.

Similarly, BI of the type [9]

$$|\langle \mathbf{ab}\rangle - \langle \mathbf{ac}\rangle| \leq 1 - \langle \mathbf{bc}\rangle \tag{17}$$

yields

$$|[1+2\Re(r')][-2\Re(r)]| \leq 4\mathrm{e}^{(\Gamma_S+\Gamma_L)t} - [1-2\Re(r)],$$
$$|[1+2\Re(r')][-2\Re(r')]| \leq 4\mathrm{e}^{(\Gamma_S+\Gamma_L)t} - [1-2\Re(r)][1-2\Re(r)+2\Re(r')], \tag{18}$$

which can be simultaneously satisfied up to first order in $\Re(r)$ and $\Re(r')$ provided

$$\mathrm{Re}(r) - \Re(r') \leq \mathrm{e}^{(\Gamma_S+\Gamma_L)t} - \frac{1}{4}. \tag{19}$$

Since $\Re(r)$ and $\Re(r')$ are of the order 10^{-2} and the exponential function on the right-hand side of the above equation grows with time starting from 1 at $t = 0$, the above condition will always be satisfied. Thus this type of BI is also not violated for thin regenerators.

To summarize, we have shown that contrary to the recent claim [7] the Wigner version of Bell's inequality cannot be used to experimentally verify the violation of BI in entangled neutral kaons system. We have also shown that other versions of BI like CHSH etc. are not violated for the case of thin regenerators. Hence we conclude that there is a need for checking the applicability of generalized Bell's inequalities [10] as well as formulating new types of Bell's inequality to confront LR with QT in entangled neutral kaons system.

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