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Subquantum information and computation

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Abstract. It is argued that immense physical resources – for nonlocal communication, espionage, and exponentially-fast computation – are hidden from us by quantum noise, and that this noise is not fundamental but merely a property of an equilibrium state in which the universe happens to be at the present time. It is suggested that 'non-quantum' or nonequilibrium matter might exist today in the form of relic particles from the early universe. We describe how such matter could be detected and put to practical use. Nonequilibrium matter could be used to send instantaneous signals, to violate the uncertainty principle, to distinguish non-orthogonal quantum states without disturbing them, to eavesdrop on quantum key distribution, and to outpace quantum computation (solving NP-complete problems in polynomial time).

Keywords. Hidden variables; de Broglie–Bohm; quantum information; computation; nonlocality.

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1. Introduction and motivation

In quantum theory the Born probability rule is regarded as a fundamental law of Nature: a system with wave function ψ has an associated probability distribution $\rho = |\psi|^2$. However, there are reasons to believe that this distribution is not fundamental, but merely corresponds to a special 'equilibrium' state, analogous to thermal equilibrium [1–7]. For there seems to be a 'conspiracy' in the known laws of physics: long-distance quantum correlations suggest that our universe is fundamentally nonlocal, and yet the nonlocality cannot be used for practical instantaneous signaling. This apparent conspiracy may be explained if one supposes that signal-locality is merely a property of the special state $\rho = |\psi|^2$, in which nonlocality happens to be hidden by quantum noise; while for a general distribution $\rho \neq |\psi|^2$, nonlocality would be directly visible. While $\rho = |\psi|^2$ to high accuracy now (for all systems probed so far), perhaps $\rho \neq |\psi|^2$ in the early universe, the relaxation $\rho \to |\psi|^2$ having taken place soon after the big bang. Thus our experience happens to be restricted to an equilibrium state $\rho = |\psi|^2$ in which locality and uncertainty *appear* to be fundamental.

A heuristic analogy may be drawn with physics in a universe that has reached a state of thermal 'heat death', in which all systems have the same temperature [2]. In such a universe there is a universal probability distribution given by the Boltzmann rule $\rho = \mathrm{e}^{-E/kT}/Z$, analogous to our universal Born rule $\rho = |\psi|^2$; all systems are subject to a universal thermal noise, analogous to our universal uncertainty noise; and it is impossible to convert thermal energy into useful work, just as it is impossible in our universe to convert quantum nonlocality into a useful instantaneous signal.

A precise model of this scenario is obtained in deterministic hidden-variables theories such as the pilot-wave theory of de Broglie and Bohm [1–14]. These nonlocal theories allow one to discuss the properties of hypothetical nonequilibrium distributions $\rho \neq |\psi|^2$, for which it may be shown that there are instantaneous signals at the statistical level [2,15,16]. Thus in these theories it may be asserted that quantum theory is just the theory of a special state $\rho = |\psi|^2$ in which nonlocality happens to be hidden by statistical noise. And in pilot-wave theory at least, the relaxation $\rho \to |\psi|^2$ may be accounted for by an H-theorem [1,5], much as in classical statistical mechanics, so that $\rho = |\psi|^2$ is indeed merely an equilibrium state [17].

Here we shall work with the pilot-wave model. The details of that model may or may not be correct: but it has qualitative features, such as nonlocality, that are known to be properties of all hidden-variables theories; and it is helpful to work with a specific, well-defined theory. In this model, a system with wave function $\psi(x,t)$ has a configuration $x(t)$ whose velocity is determined by $\dot{x}(t) = j(x,t)/|\psi(x,t)|^2$, where j is the quantum probability current. Quantum theory is recovered if one assumes that an ensemble of systems with wave function $\psi_0(x)$ begins with a 'quantum equilibrium' distribution of configurations $\rho_0(x) = |\psi_0(x)|^2$ at $t = 0$ (guaranteeing $\rho(x,t) = |\psi(x,t)|^2$ for all t). In effect, the Born rule is assumed as an initial condition. But the theory also allows one to consider arbitrary 'nonequilibrium' initial distributions $\rho_0(x) \neq |\psi_0(x)|^2$, which violate quantum theory [1–7], and whose evolution is given by the continuity equation

$$\frac{\partial \rho(x,t)}{\partial t} + \nabla \cdot (\dot{x}(t)\rho(x,t)) = 0$$

(the same equation that is satisfied by $|\psi(x,t)|^2$).

Our working hypothesis, then, is that $\rho = |\psi|^2$ is an equilibrium state, analogous to thermal equilibrium in classical mechanics. This state has special properties – in particular locality and uncertainty – which are not fundamental. It then becomes clear that a lot of new physics must be hidden behind quantum noise, physics that is unavailable to us only because we happen to be trapped in an equilibrium state.

This new physics might be accessible if the universe began in nonequilibrium $\rho \neq |\psi|^2$. First, in theories of cosmological inflation, early corrections to quantum fluctuations would change the spectrum of primordial density perturbations imprinted on the cosmic microwave background [6,7]. Second, relic cosmological particles that decoupled at sufficiently early times might still be in quantum nonequilibrium today, violating quantum mechanics [3–7].

The second possibility is particularly relevant here. If relic nonequilibrium matter from the early universe was discovered, what could we do with it? Thermal and chemical nonequilibrium have myriad technological applications; we expect that quantum nonequilibrium would be just as useful.

2. Detection and use of quantum nonequilibrium

First we need to consider how a nonequilibrium distribution $\rho \neq |\psi|^2$ might be deduced by statistical analysis of a random sample of relic matter [7].

Consider the unrealistic but simple example of a large number N of hydrogen atoms in the ground state $\psi_{100}(r)$. Assume they make up a cloud of gas somewhere in space. Because the phase of ψ_{100} has zero gradient, the de Broglie–Bohm velocity field vanishes, and pilot-wave theory predicts that each electron is at rest relative to its nucleus. We then have a static distribution $\rho(r)$, which may or may not equal the quantum equilibrium distribution $\rho_{\text{eq}}(r) = |\psi_{100}(r)|^2 \propto \mathrm{e}^{-2r/a_0}$. To test this, one could draw a random sample of N' atoms ($N' << N$) and measure the electron positions. The sample $r_1, r_2, r_3, \ldots, r_{N'}$ may be used to make statistical inferences about the parent distribution $\rho(r)$. In particular, one may estimate the likelihood that $\rho(r) = \rho_{\text{eq}}(r)$. Should one deduce that, almost certainly, the cloud as a whole has a nonequilibrium distribution $\rho(r) \neq \rho_{\text{eq}}(r)$, the rest of the cloud may then be used as a resource for new physics.

For example, one could test $\rho(r)$ via the sample mean $\bar{r}$. If $\rho(r)$ has mean μ and variance σ^2, the central limit theorem tells us that for large N' the random variable $\bar{r}$ has an approximately normal distribution with mean μ and variance σ^2/N'. We can then calculate the probability that $\bar{r}$ differs from μ, and we can test the hypothesis that $\rho(r) = \rho_{\text{eq}}(r)$ with $\mu = \mu_{\text{eq}} = (3/2)a_0$. A standard technique is to compare the probability $P(\bar{r}|\rho_{\text{eq}})$ of obtaining $\bar{r}$ from a distribution ρ_{eq} with the probability $P(\bar{r}|\rho_{\text{noneq}})$ of obtaining $\bar{r}$ from some nonequilibrium distribution ρ_{noneq}. One usually refers to $P(\bar{r}|\rho_{\text{eq}})$ and $P(\bar{r}|\rho_{\text{noneq}})$ as the 'likelihoods' of ρ_{eq} and ρ_{noneq} respectively, given the sample mean $\bar{r}$. If $P(\bar{r}|\rho_{\text{eq}}) << P(\bar{r}|\rho_{\text{noneq}})$, one concludes that nonequilibrium is much more likely. Similarly, using standard techniques such as the chi-square test, one may deduce the most likely form of the parent distribution $\rho(r)$, which almost certainly applies to the rest of the cloud [18].

In what follows, then, we assume that at $t = 0$ we have a large number of particles with the same known wave function $\psi_0(x)$, and with positions x that have a *known* nonequilibrium distribution $\rho_0(x) \neq |\psi_0(x)|^2$.

3. Instantaneous signalling

The most obvious application of such 'non-quantum' matter would be for instantaneous signalling across space [7].

Suppose we take pairs of nonequilibrium particles and prepare each pair in an entangled state $\psi(x_A, x_B, t_0)$ at time t_0 (by briefly switching on an interaction). Given the details of the preparation, we may use the Schrödinger equation to calculate the evolution of the wave function of each pair, from $\psi(x_A, x_B, 0) = \psi_0(x_A)\psi_0(x_B)$ at $t = 0$ to $\psi(x_A, x_B, t_0)$ at $t = t_0$. We then know the de Broglie–Bohm velocity field throughout $(0, t_0)$, and so we may use the continuity equation to calculate the evolution of the joint distribution for the pairs from $\rho(x_A, x_B, 0) = \rho_0(x_A)\rho_0(x_B)$ at $t = 0$ to $\rho(x_A, x_B, t_0) \neq |\psi(x_A, x_B, t_0)|^2$ at $t = t_0$ [19]. We then have the situation discussed in detail elsewhere [2]. The marginal distribution $\rho_A(x_A, t_0) \equiv \int \mathrm{d}x_B\, \rho(x_A, x_B, t_0)$ at A is known, and its subsequent evolution will depend instantaneously on perturbations applied at B, however remote B may be from A. Thus instantaneous signals may be sent from B to A.

It might be thought that superluminal signals would necessarily lead to causal paradoxes. However, it could well be that at the nonlocal hidden-variable level there is a preferred slicing of spacetime, labelled by a time parameter that defines a fundamental causal sequence [3,7,20–22].

4. Subquantum measurement

Let us now consider how our nonequilibrium particles could be used to perform novel measurements on ordinary, equilibrium systems [7].

Assume once again that we have an ensemble of what we shall now call 'apparatus' particles with known wave function $g_0(y)$ and known *nonequilibrium* distribution $\pi_0(y) \neq |g_0(y)|^2$. (The position y may be regarded as a 'pointer' position.) And let us now use them to measure the positions of ordinary 'system' particles with known wave function $\psi_0(x)$ and known *equilibrium* distribution $\rho_0(x) = |\psi_0(x)|^2$. We shall see that, if the apparatus distribution $\pi_0(y)$ were arbitrarily narrow, we could measure the system position x_0 without disturbing the system wavefunction $\psi_0(x)$, to arbitrary accuracy, in complete violation of the uncertainty principle.

We shall illustrate the idea with an exactly-solvable model. At $t = 0$, we take a system particle and an apparatus particle and switch on an interaction between them described by the Hamiltonian $\hat{H} = a\hat{x}\hat{p}_y$, where a is a coupling constant and p_y is the momentum canonically conjugate to y. (This is the standard interaction Hamiltonian for an ideal quantum measurement of x using the pointer y.) For simplicity, we neglect the Hamiltonians of x and y themselves [23]. For $t > 0$, the joint wave function $\Psi(x,y,t)$ satisfies the Schrödinger equation

$$\frac{\partial \Psi(x,y,t)}{\partial t} = -ax\frac{\partial \Psi(x,y,t)}{\partial y},$$

while $|\Psi(x,y,t)|^2$ obeys the continuity equation

$$\frac{\partial |\Psi(x,y,t)|^2}{\partial t} + ax\frac{\partial |\Psi(x,y,t)|^2}{\partial y} = 0.$$

The hidden-variable velocity fields $\dot{x}$ and $\dot{y}$ must satisfy

$$\frac{\partial |\Psi(x,y,t)|^2}{\partial t} + \frac{\partial \left(|\Psi(x,y,t)|^2 \dot{x}\right)}{\partial x} + \frac{\partial (|\Psi(x,y,t)|^2 \dot{y})}{\partial y} = 0,$$

from which we deduce the (non-standard) guidance equations [24] $\dot{x} = 0$, $\dot{y} = ax$ and the de Broglie–Bohm trajectories $x(t) = x_0$, $y(t) = y_0 + ax_0 t$.

Now the initial product wave function $\Psi_0(x,y) = \psi_0(x)g_0(y)$ evolves into the entangled wave function $\Psi(x,y,t) = \psi_0(x)g_0(y - axt)$. In the limit $at \to 0$, we have $\Psi(x,y,t) \approx \psi_0(x)g_0(y)$ and the system wave function $\psi_0(x)$ is undisturbed. Yet, no matter how small at may be, at the hidden-variable level the 'pointer' position $y(t) = y_0 + ax_0 t$ contains information about the value of x_0 (and of $x(t) = x_0$). And this 'subquantum' information about x will be visible to us if the initial pointer distribution $\pi_0(y)$ is sufficiently narrow.

For consider an ensemble of similar experiments, where x and y have the initial joint distribution $P_0(x,y) = |\psi_0(x)|^2 \pi_0(y)$ (equilibrium for x and nonequilibrium for y). The continuity equation

$$\frac{\partial P(x,y,t)}{\partial t} + ax\frac{\partial P(x,y,t)}{\partial y} = 0$$

implies that at later times $P(x,y,t) = |\psi_0(x)|^2 \pi_0(y - axt)$. If $\pi_0(y)$ is localized – say $\pi_0(y) \approx 0$ for $|y| > w/2$ – then from a standard measurement of y we may deduce that x lies in the interval $(y/at - w/2at,\, y/at + w/2at)$ (so that $P(x,y,t) \neq 0$), where the error margin $w/2at \to 0$ as the width $w \to 0$.

Thus, if the nonequilibrium 'apparatus' distribution $\pi_0(y)$ has an arbitrarily small width w, then to arbitrary accuracy we may measure the position x of each equilibrium particle without disturbing the wave function $\psi_0(x)$ [25].

We have for simplicity considered an exactly-solvable system with an unusual total Hamiltonian. Similar conclusions hold for more standard systems: if the interaction between x and y is sufficiently weak, then while $\psi_0(x)$ is hardly disturbed, at the hidden-variable level y generally contains information about x – information that is visible if y has a sufficiently narrow distribution.

Generalizing, if w is arbitrarily small, then by a sequence of such measurements, it is clear that for a system particle with arbitrary wave function $\psi(x,t)$ we can determine the hidden trajectory $x(t)$ without disturbing $\psi(x,t)$, to arbitrary accuracy.

5. Distinguishing non-orthogonal quantum states

In quantum mechanics, non-orthogonal states $|\psi_1\rangle$, $|\psi_2\rangle$ (with $\langle\psi_1|\psi_2\rangle \neq 0$) cannot be distinguished without disturbing them [26]. This theorem breaks down in the presence of nonequilibrium matter [7].

For example, if $|\psi_1\rangle$, $|\psi_2\rangle$ are distinct initial states of a single spinless particle, then in de Broglie–Bohm theory the velocity fields $j_1(x,t)/|\psi_1(x,t)|^2$, $j_2(x,t)/|\psi_2(x,t)|^2$ generated by the wave functions $\psi_1(x,t)$, $\psi_2(x,t)$ will in general be different, even if $\langle\psi_1|\psi_2\rangle = \int dx\, \psi_1^*(x,0)\psi_2(x,0) \neq 0$. The hidden-variable trajectories $x_1(t)$ and $x_2(t)$ – associated with $\psi_1(x,t)$ and $\psi_2(x,t)$ respectively – will generally differ if $\psi_1(x,0) \neq \psi_2(x,0)$ (even if $x_1(0) = x_2(0)$). Thus, a subquantum measurement of the particle trajectory (even over a short time) would generally enable us to distinguish the quantum states $|\psi_1\rangle$ and $|\psi_2\rangle$ without disturbing them, to arbitrary accuracy.

6. Eavesdropping on quantum key distribution

Alice and Bob want to share a secret sequence of bits that will be used as a key for cryptography. During distribution of the key between them, they must be able to detect any eavesdropping by Eve. Three protocols for quantum key distribution – BB84 [27], B92 [28], and E91 (or EPR) [29] – are known to be secure against classical or quantum attacks (that is, against eavesdropping based on classical or quantum physics) [30]. But these protocols are *not* secure against a 'subquantum' attack [7].

Both BB84 and B92 rely on the impossibility of distinguishing non-orthogonal quantum states without disturbing them. In BB84, Alice sends Bob a random sequence of spin-1/2 states $|+z\rangle$, $|-z\rangle$, $|+x\rangle$, $|-x\rangle$, while in B92 she sends a random sequence of arbitrary non-orthogonal states $|u_0\rangle$, $|u_1\rangle$ (the states being subjected to appropriate random measurements by Bob). In each case the sequence is chosen by Alice. But if Eve possesses non-quantum matter with an arbitrarily narrow nonequilibrium distribution, she may identify the states sent by Alice without disturbing them, to arbitrary accuracy, and so read the supposedly secret key. (For B92, $|u_0\rangle$, $|u_1\rangle$ could be states of a spinless particle with wave-functions $\psi_0(x,t)$, $\psi_1(x,t)$, which Eve may distinguish by monitoring the hidden-variable trajectories. Similarly for BB84 – though for spin-1/2 states one must consider pilot-wave theory for two-component wave functions [7,10].)

E91 is particularly interesting for it relies on the completeness of quantum theory – that is, on the assumption that there are no hidden 'elements of reality'. Pairs of spin-1/2 particles in the singlet state are shared by Alice and Bob, who perform spin measurements along random axes. For coincident axes the same bit sequence is generated at each wing, by apparently random quantum outcomes. 'The eavesdropper cannot elicit any information from the particles while in transit... because there is no information encoded there' [29]. But our Eve has access to information outside the domain of quantum theory. She can measure the particle positions while in transit, without disturbing the wave function, and so *predict* the outcomes of spin measurements at the two wings (for the publicly-announced axes) [31]. Thus Eve is able to predict the key shared by Alice and Bob.

7. Outpacing quantum computation

Quantum theory allows parallel Turing-type computations to occur in different branches of the state vector for a single computer [32]. However, owing to the effective collapse that occurs under measurement, an experimenter is able to access only one result; the outputs of the other computations are lost. Of course, by clever use of entanglement and interference, one can make quantum computation remarkably efficient for certain special problems. But in general, what at first sight seems to be a massive increase in computational power is not, in fact, realized in practice.

All the results of a parallel quantum computation could be read, however, if we had access to nonequilibrium matter with a very narrow distribution [3,7].

Each result could be encoded in an integer n, and stored as an energy eigenvalue E_n for a single spinless particle (a component of the computer). At the end of the computation the particle wave function will be a superposition

$$\psi(x,t) = \sum_{n\in S} \phi_n(x)\mathrm{e}^{-iE_nt}$$

of N energy eigenfunctions $\phi_n(x)$, where S is an *unknown* set of N quantum numbers. (We assume a Hamiltonian $\hat{H} = \hat{p}^2/2 + V(\hat{x})$, where the mass $m = 1$ and $\hbar = 1$.) In standard quantum theory an energy measurement for the particle yields just one value E_n. To find out what other eigenvalues are present, one would have to run the whole computation many times – to produce an ensemble of copies of the same wave function – and repeat the energy measurement for each. And so one may as well just run many computations on a single classical computer, one after the other.

But the hidden-variable particle trajectory $x(t)$ – determined by $\dot{x}(t) = j/|\psi|^2$ or $\dot{x} = \mathrm{Im}(\nabla\psi/\psi)$ – contains information about all the modes in the superposition (provided the $\phi_n(x)$ overlap in space). If we had a sample of nonequilibrium matter with a very narrow distribution, we could use it to measure $x(t)$ without disturbing $\psi(x,t)$. We could then read the set S of quantum numbers: having measured the values of $x(t)$, $\dot{x}(t)$ at N times

$t = t_1, t_2, \ldots, t_N$, the equation $\dot{x} = \mathrm{Im}(\nabla\psi/\psi)$ may be solved for the N quantum numbers n [33]. Thus we could read the results of all N arbitrarily long parallel computations (at the price of solving N simultaneous equations), even though the computer has been run only once.

By combining subquantum measurements with quantum algorithms, we could solve NP-complete problems in polynomial time, and so outpace all known quantum (or classical) algorithms [7].

To see this, consider the computational enhancement noted by Abrams and Lloyd in nonlinear quantum mechanics [34]. Let a quantum (equilibrium) computer begin with $n+1$ qubits in the state $|00\ \ldots\ 0\rangle$ and apply the Hadamard gate H (which maps $|0\rangle \to (|0\rangle + |1\rangle)/\sqrt{2}$, $|1\rangle \to (|0\rangle - |1\rangle)/\sqrt{2}$) to each of the first n qubits to produce $(1/\sqrt{2^n})\sum_x |x,\ 0\rangle$, where the n-bit 'input' x ranges from $00\ \ldots\ 0$ to $11\ \ldots\ 1$ (or from 0 to $2^n - 1$). Then use an 'oracle' or 'black box' to calculate – in parallel – a function $f(x) = 0$ or 1, whose value is stored in the last qubit, producing $(1/\sqrt{2^n})\sum_x |x,\ f(x)\rangle$. Applying H again to each of the first n qubits produces a state containing the term $(1/2^n)\sum_x |00\ \ldots\ 0,\ f(x)\rangle$. If upon quantum measurement of the first n qubits we obtain $|00\ \ldots\ 0\rangle$, the total effective state becomes $|00\ \ldots\ 0\rangle \otimes |\psi\rangle$ where $|\psi\rangle \propto |0\rangle(2^n - s)/2^n + |1\rangle s/2^n$ and s is the number of inputs x such that $f(x) = 1$ (the total number of inputs being 2^n) [35]. As Abrams and Lloyd point out, we could solve NP-complete problems if we could distinguish between $s = 0$ and $s > 0$ for the state $|\psi\rangle$ of the last qubit. This could be accomplished by nonlinear evolution, in which non-orthogonal states evolve into (distinguishable) orthogonal ones [34]. But equally, non-orthogonal qubits could be distinguished using our nonequilibrium matter. Here, the de Broglie–Bohm trajectory $x(t)$ of an equilibrium particle guided by $\psi(x,t) = \langle x|\psi(t)\rangle$ will in general be sensitive to the value of s, which may therefore be read by a subquantum measurement of $x(t)$ [7].

8. Conclusion

We have argued that immense physical resources are hidden from us by quantum noise, and that we will be unable to access those resources only for as long as we are trapped in the 'quantum heat death' – a state in which all systems are subject to the noise associated with the Born probability distribution $\rho = |\psi|^2$.

It is clear that hidden-variables theories offer a radically different perspective on quantum information theory. In such theories, a huge amount of 'subquantum information' is hidden from us simply because we happen to live in a time and place where the hidden variables have a certain 'equilibrium' distribution. As we have mentioned, nonequilibrium instantaneous signals occur not only in pilot-wave theory but in *any* deterministic hidden-variables theory [15,16]. And in pilot-wave theory at least, we have shown that the security of quantum cryptography depends on our being trapped in quantum equilibrium; and, that nonequilibrium would unleash computational resources far more powerful than those of quantum computers.

Some might prefer to regard this work as showing how the principles of quantum information theory depend on a particular axiom of quantum theory – the Born rule $\rho = |\psi|^2$. (One might also consider the role of the axiom of linear evolution [34,36].)

But if one takes hidden-variables theories seriously as physical theories of Nature, one can hardly escape the conclusion that we just happen to be confined to a particular state in which our powers are limited by an all-pervading statistical noise. It then seems important to search for violations $\rho \neq |\psi|^2$ of the Born rule [3–7].

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