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Information flow in quantum teleportation

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Abstract. The flow of information is discussed in the context of quantum teleportation. Situations are described which use a sequence of systems of particles in which, though there is no claim of faster-than-light signaling, it is plausible to suggest that information about measurement procedures in one wing of the apparatus does reach the other end in a non-local manner. The definition of the term 'parameter dependence' is discussed.

Keywords. Quantum information; quantum teleportation; parameter independence.

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1. Introduction

It is well-known that quantum teleportation requires both the quantum interaction between Alice and Bob's systems, and passage of a classical signal [1,2]. From one point of view, the necessity of a classical signal may be regarded as an obvious requirement to avoid the occurrence of faster-than-light (FTL) signaling. However the question of what elements of the overall situation may be translated from one part of the system to another in a non-local way, and which must be moved by classical means is intriguing.

In the EPR–Bohm–Bell context, the situation is usually described as follows. Any non-locality involved in such experiments relates to the results obtained in the two wings of the experiment. This may be referred to as a breakdown of *outcome dependence*, in the terms of Shimony [3,4], or of *completeness* in those of Jarrett [5]. Thus observation of measurement results in one wing of the experiment does give non-local information (usually probabilistic in nature) on those in the other wing. Since the results in either wing are not controlled by the experimenter, they may not, of course, be used to send signals.

In contrast, in these experiments it is usually argued that *parameter independence*, to use Shimony's term [3,4], locality to use Jarrett's [5], is maintained in these experiments. This means that a decision on which quantity to measure in one wing of the experiment cannot influence the results obtained in the other wing. If one interprets the latter phrase as 'the expectation value of the results', then certainly any breakdown of parameter independence would lead to FTL signaling. However there are cases [6] where FTL signaling is definitely prohibited, yet it may still be maintained that *some* information about experimental procedures in one wing of the experiment does reach the other wing. The purpose of the present paper is to examine this type of situation in the context of quantum teleportation.

2. Sketch of quantum teleportation

In order that we may refer to details of the quantum teleportation procedure, the process is sketched here. The quantum interaction between Alice and Bob is achieved by their sharing a pair of particles in an entangled state. The state-vector of this pair of particles may be written as

$$|\Psi_{23}\rangle = \frac{1}{\sqrt{2}}\{|\uparrow\rangle_2|\downarrow\rangle_3 - |\downarrow\rangle_2|\uparrow\rangle_3\}. \tag{1}$$

Here subscripts 2 and 3 signify particles that are in the possession of Alice and Bob respectively.

Particle 1 is in the possession of Alice, and it is the state of this particle that is to be teleported. This state may be written as

$$|\Phi\rangle_1 = a|\uparrow\rangle_1 + b|\downarrow\rangle_1. \tag{2}$$

Thus the initial state of the entire system of particles 1, 2 and 3 may be written as

$$|\Psi_{123}\rangle = \frac{a}{\sqrt{2}}\left\{|\uparrow\rangle_1|\uparrow\rangle_2|\downarrow\rangle_3 - |\uparrow\rangle_1|\downarrow\rangle_2|\uparrow\rangle_3\right\} + \frac{b}{\sqrt{2}}\left\{|\downarrow\rangle_1|\uparrow\rangle_2|\downarrow\rangle_3 - |\downarrow\rangle_1|\downarrow\rangle_2|\uparrow\rangle_3\right\}. \tag{3}$$

We now write the direct product state of particles 1 and 2, those particles in the possession of Alice, in the Bell basis as

$$|\Psi_{12}^{+}\rangle = \frac{1}{\sqrt{2}}\left\{|\uparrow\rangle_1|\downarrow\rangle_2 + |\downarrow\rangle_1|\uparrow\rangle_2\right\},$$

$$|\Psi_{12}^{-}\rangle = \frac{1}{\sqrt{2}}\left\{|\uparrow\rangle_1|\downarrow\rangle_2 - |\downarrow\rangle_1|\uparrow\rangle_2\right\},$$

$$|\Phi_{12}^{+}\rangle = \frac{1}{\sqrt{2}}\left\{|\uparrow\rangle_1|\uparrow\rangle_2 + |\downarrow\rangle_1|\downarrow\rangle_2\right\},$$

$$|\Phi_{12}^{-}\rangle = \frac{1}{\sqrt{2}}\left\{|\uparrow\rangle_1|\uparrow\rangle_2 - |\downarrow\rangle_1|\downarrow\rangle_2\right\}. \tag{4}$$

With the use of eq. (4), $|\Psi_{123}\rangle$ from eq. (3) may be expressed as

$$|\Psi_{123}\rangle = \frac{1}{2}\left\{(-a|\uparrow\rangle_3 + b|\downarrow\rangle_3)|\Psi_{12}^{+}\rangle + (-a|\uparrow\rangle_3 - b|\downarrow\rangle_3)|\Psi_{12}^{-}\rangle + (a|\downarrow\rangle_3 - b|\uparrow\rangle_3)|\Phi_{12}^{+}\rangle + (a|\downarrow\rangle_3 + b|\uparrow\rangle_3)|\Phi_{12}^{-}\rangle\right\}. \tag{5}$$

Alice now performs measurements on the joint system of particles 1 and 2 to distinguish between the Bell states given in eq. (4). Any measurement result she obtains is of course directly correlated to a particular Bell state, and thus also to a particular state of Bob's particle 3, which is thus left in one of four states, the probability of obtaining each being equally likely. The four states may be written as

$$\begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} a \\ b \end{pmatrix}, \quad -\begin{pmatrix} a \\ b \end{pmatrix}, \quad \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix} \begin{pmatrix} a \\ b \end{pmatrix} \text{ and } \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \begin{pmatrix} a \\ b \end{pmatrix}. \tag{6}$$

The second of these states differs from the original state of particle 1 only by an irrelevant phase factor. The other three are related to this original state by rotations of 180° about the x-, y- or z-axes. To recover the original state, Bob then has to apply the appropriate rotation in the case of the first, third or fourth states, or leave the system undisturbed in the case of the second. But, of course, he needs to know which action to take, and this is the message that Alice has to send him over the classical channel, on the basis of her measurement of the Bell state.

Once this is achieved, Bob has a replica of the original state, but Alice has no trace of this state remaining. Rather her system is left in the appropriate Bell state.

3. Measurements on groups of particles

We now consider a series of trials in which Alice takes a succession of particles as particle 1, each with the same state-vector, $\begin{pmatrix} a \\ b \end{pmatrix}$. This situation may be obtained by measuring a particular component of spin with a Stern–Gerlach experiment, and extracting those spins that fall into one of the two emerging beams. We imagine that Alice does a Bell state measurement on four pairs of spins 1 and 2 with spin 1 in this category, and we consider only trials where she gets a different result in each measurement. Obviously this will occur only in a fraction of these cases, and it is these that we consider.

For these four measurements of Alice, we consider that in each case Bob measures S_{z3}, and we look at the sum of all these measurements, which we call $S_{z(\text{total})}$. From eq. (5) it is clear that the probability of Bob obtaining S_{z3} equal to $+(\hbar/2)$ in each measurement is equal to a^4b^4. So

$$P(S_{z(\text{total})} = 2\hbar) = a^4b^4. \tag{7a}$$

Similarly the probabilities of obtaining other values of $S_{z(\text{total})}$ are

$$P(S_{z(\text{total})} = \hbar) = 2a^2b^2(a^4+b^4), \tag{7b}$$

$$P(S_{z(\text{total})} = 0) = a^8 + 4a^4b^4 + b^8, \tag{7c}$$

$$P(S_{z(\text{total})} = -\hbar) = 2a^2b^2(a^4+b^4), \tag{7d}$$

$$P(S_{z(\text{total})} = -2\hbar) = a^4b^4. \tag{7e}$$

When we look at expectation values, we find that, fairly naturally,

$$\langle S_{z(\text{total})} \rangle = 0, \tag{8}$$

but

$$\langle S^2_{z(\text{total})}\rangle = 4\hbar^2 a^2 b^2, \tag{9}$$

which is zero only if either *a* or *b* is zero.

Consider, though, the situation if Alice, instead of performing a Bell state measurement, measures S_{z1} and S_{z2}. Bob though does the same measurement. Again consider cases where Alice obtains four different pairs of results. Here we may use eq. (3), and we find in this case that

$$P(S_{z(\text{total})} = 2\hbar) = P(S_{z(\text{total})} = \hbar) = P(S_{z(\text{total})} = -\hbar)$$
$$= P(S_{z(\text{total})} = -2\hbar) = 0, \tag{10a}$$

$$P(S_{z(\text{total})} = 0) = 1. \tag{10b}$$

Naturally

$$\langle S_{z(\text{total})}\rangle = 0; \tag{11a}$$

$$\langle S^2_{z(\text{total})}\rangle = 0. \tag{11b}$$

The implication is that, although at the usual level to which attention is conventionally restricted, that of expectation value of the measured quantity, the results of Bob's measurements give no information about which measurements have been performed by Alice, at the level of the variance or standard deviation of Bob's measurement, such information is available.

It is convenient to define

$$\sigma = \frac{S_{z(\text{total})}}{2\hbar} \tag{12}$$

so, for all results,

$$-1 \leq \sigma \leq 1. \tag{13}$$

Then

$$\langle \sigma^2\rangle = a^2 b^2 \tag{14}$$

for Bob's measurement and the case where Alice has performed Bell state measurements. With the variance for Alice's measurements of σ given by V, and the standard deviation by s.d.,

$$V = a^2 b^2; \quad \text{s.d.} = |ab|. \tag{15}$$

If Alice performs measurements of S_{z1} and S_{z2}, then of course all these results are zero.

The practical application and theoretical implications of these results will be discussed briefly later, but we first widen them a little.

4. The dichotomic case

Since the probability of obtaining each measurement result once in a series of four measurements is quite low, one may refer to an analogous dichotomic case. Alice's measurement may only be to distinguish between obtaining, in one measurement, $|\Psi^{+}_{12}\rangle$ or $|\Psi^{-}_{12}\rangle$, and, in the other, $|\Phi^{+}_{12}\rangle$ or $|\Phi^{-}_{12}\rangle$. There are four possible measurement results in this category, for example $|\Psi^{+}_{12}\rangle$ and $|\Phi^{+}_{12}\rangle$, each occurring with probability 1/8. For each the probabilities of Bob obtaining particular results are as follows:

$$P(S_{z(\text{total})} = \hbar) = P(S_{z(\text{total})} = -\hbar) = a^2 b^2, \tag{16a}$$

$$P(S_{z(\text{total})} = 0) = a^4 + b^4. \tag{16b}$$

We then obtain

$$\langle S_{z(\text{total})} \rangle = 0, \tag{17}$$

and

$$\langle S^2_{z(\text{total})} \rangle = 2a^2 b^2 \hbar^2. \tag{18}$$

With

$$\sigma = \frac{S_{z(\text{total})}}{\hbar}, \tag{19}$$

so that, again,

$$-1 \leq \sigma \leq 1, \tag{20}$$

we have

$$\langle \sigma^2 \rangle = 2a^2 b^2 \tag{21}$$

and $\langle \sigma^2 \rangle$ has a maximum value of $1/2$ when $a = b = 1/\sqrt{2}$.

For this dichotomic case, the second type of measurement for Alice may be, for example, of S_{z2}, and it may be required that one result gives $+\hbar/2$, and the other $-\hbar/2$. Just as in §3, for the two particles Bob measures, the total z-component of spin is bound to be 0, and so $\langle S_{z(\text{total})} \rangle$ and $\langle S^2_{z(\text{total})} \rangle$ are also, of course, zero. The general argument is exactly as in §3.

5. The case of *N* groups of 4 particles

The original case of §3 is now extended to the case where $4N$ measurements are carried out by Alice, and equal numbers are obtained of each result.

In the calculation of $\langle S^2_{z(\text{total})} \rangle$ for this case, two types of term emerge. There are 25 terms involving two particles of which an example is

$$T_{2,2} = \sum_{i=1}^{N} {\sum_{j=1}^{N}}' (2\hbar)(2\hbar) P(S_{z(\text{total})}^{(i)} = 2\hbar) P(S_{z(\text{total})}^{(j)} = 2\hbar), \tag{22}$$

the dash on the second summation indicating $i \neq j$. Each of these terms is either identically zero or is equal and opposite to another of the terms. The term in eq. (22), for example, is equal and opposite to the term $T_{2,-2}$ in which the second $(2\hbar)$ is replaced by $(-2\hbar)$. These 25 terms then sum to zero.

The term $\langle S_{z(\text{total})}^2 \rangle$ is then given by the sum of the other type of terms, of which there are five, each involving a single particle. So

$$\begin{aligned}\langle S_{z(\text{total})}^2 \rangle = \sum_{i=1}^{N} [&(2\hbar)^2 P(S_{z(\text{total})} = 2) + (\hbar)^2 P(S_{z(\text{total})} = 1) \\ &+ (0) P(S_{z(\text{total})} = 0) + (-\hbar)^2 P(S_{z(\text{total})} = -1) \\ &+ (-2\hbar)^2 P(S_{z(\text{total})} = -2)].\end{aligned} \tag{23}$$

So

$$\langle S_{z(\text{total})}^2 \rangle = 4N\hbar^2 a^2 b^2, \tag{24}$$

and the appropriate standard deviation here is $2\sqrt{N}\hbar ab$. If we define

$$\sigma = \frac{S_{z(\text{total})}}{2N\hbar}, \tag{25}$$

then

$$\langle \sigma^2 \rangle = \frac{a^2 b^2}{N}, \tag{26}$$

and the appropriate standard deviation is given by $ab/\sqrt{N}$. We note that the standard deviation obtained from eq. (24) varies as $\sqrt{N}$, while that from eq. (26) varies as $1/\sqrt{N}$. The second would usually be spoken of as corresponding to the central limit theorem, but the first is the actual experimental quantity. (See the related discussion between Ghirardi, Rimini and Weber [7,8] and Newton [9].)

6. Experimental application

Experimental application is straightforward. A series of pairs of particles, entangled as in eq. (1) are sent to Alice and Bob. For each of these pairs, Alice also receives a particle in the state given by eq. (2), obtained, as mentioned above, from one beam of a Stern–Gerlach apparatus with the magnetic field at an appropriate angle.

It may well be preferable to aim at a situation with a substantial run, that is with N large. This is because, for N equal to 4, for example, there is a substantial probability that, when Alice performs a Bell state measurement, Bob's result for $S_{z(\text{total})}$ with a single group of such particles will duplicate that when she measures S_{z1} and S_{z2}, which is, of course, zero. This is demonstrated in eq. (7c). For Bob to be (virtually) sure which measurement Alice has made, one has to work with the distribution of a number of such groups of 4, as demonstrated by the difference between eqs (9) and (11b).

If N is large, however, then when Alice performs a Bell measurement for a single run, the probability that Bob will obtain the answer zero for $S_{z(\mathrm{total})}$ will be extremely small. Thus a single run will establish (with virtual certainty) which measurement Alice has made.

7. Theoretical implications

As already stated, there is, of course, no suggestion that FTL signaling is involved in the above. The necessity for the classical signal to confirm that Alice has had equal numbers of measurement results in each category assures that point. Neither may it be said that the type of measurement made by Alice affects the expectation value of Bob's measurement.

Yet it also difficult to maintain that the type of measurement made by Alice does not affect Bob's measurements at all. If and when Bob receives the message from Alice that her set of experimental results satisfied the appropriate criterion for a particular value of N, that message is neutral between the two possibilities for her measurement procedure. The same message may cause Bob to understand that she has carried out Bell-type measurements *or* that she has been measuring S_{z1} and S_{z2}. It is clear that the information which allows Bob to decide between these possibilities is with him before the classical signal. It is with him, in fact, in an FTL or non-local way. Clearly there are some surprising aspects of non-locality which are still only being understood.

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