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Generic entangling through quantum indistinguishability

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Abstract. We present a general scheme for entangling any degree of freedom of two uncorrelated identical particles from independent sources by a combination of two-particle interferometry and which-way detection. We show that this entanglement generation procedure works for completely random initial states of the variable to be entangled. We also demonstrate a curious complementarity exhibited by our scheme and its applications in estimating the generated entanglement as a function of wave packet overlap at the beamsplitter.

Keywords. Entanglement; quantum statistics; indistinguishability; identical particles; complementarity.

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1. Introduction

Consider the task of entangling *any* variable of two independent identical particles of *any* type. The particles in question may not interact with each other or with any other quantum system which can mediate entanglement between them. This seems to rule out all the *known* mechanisms for entanglement generation [1–4] which involve either a direct interaction between the systems involved (such as photon–photon interaction during down-conversion [2]) or indirect transfer of entanglement due to their interaction with other quantum systems (methods such as entanglement swapping [5] fall in this category). In this paper, we will present a very general method of entangling two identical particles from independent sources which does not fall into either of the above categories. In fact, it exploits *quantum indistinguishability* as an important entangling mechanism, rather than using explicit interactions.

The basic idea is as follows: Two identical particles in orthogonal states of the degree of freedom to be entangled (for example, opposite orientations in the case of spin) are mixed at a beamsplitter. Then the subensemble of cases when the two particles exit the beamsplitter through separate outputs is selected. If the degree of freedom to be entangled is kept unmeasured, then because of indistinguishability, the output channels automatically become the new identity labels of the particles. The relevant degree of freedom of the two particles is then found to be entangled. The procedure can be repeated if the particles do not exit the beamsplitter through separate channels. In this way, as we shall describe in greater detail in the next section, the particles can be entangled with nearly unit efficiency.

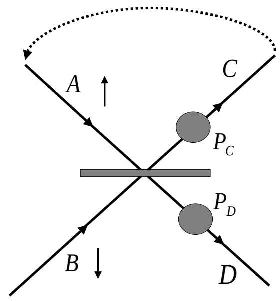

Figure 1. The set-up for generating entangled states using quantum indistinguishability. The feedback loop is shown in the figure as a dotted line. If the particles bunch in the same output channel, they are fed back into the beamsplitter. This is recursively continued till the particles anti-bunch, in which case they are entangled.

2. A detailed description of the entangling procedure

Our setup is depicted in figure 1. It consists of a beamsplitter with input channels A and B, output channels C and D and which-channel detectors P_C in C and P_D in D. These detectors are assumed to be *nonabsorbing* and are able to *determine the path without disturbing the spin* (this is possible since position and spin commute; feasibility is discussed later). Now consider two identical particles in different spin states (say $|\uparrow\rangle$ and $|\downarrow\rangle$) incident simultaneously on the beamsplitter from arms A and B as shown in figure 1. This state, in second quantized notation, is described as $a^\dagger_{A\uparrow}a^\dagger_{B\downarrow}|0\rangle$, where $|0\rangle$ is the vacuum state and $a^\dagger_{A\uparrow}$ and $a^\dagger_{B\downarrow}$ are creation operators for spin $\uparrow$ in path A and spin $\downarrow$ in path B, respectively. We will label the state concisely as $|A\uparrow;B\downarrow\rangle$. For fermions, $|A\uparrow;B\downarrow\rangle = -|B\downarrow;A\uparrow\rangle$ and for bosons $|A\uparrow;B\downarrow\rangle = |B\downarrow;A\uparrow\rangle$. The transformation done by the beamsplitter is [6,7]

$$|A\uparrow;B\downarrow\rangle \to \frac{1}{\sqrt{2}}\left\{\frac{1}{\sqrt{2}}(|D\uparrow;C\downarrow\rangle \pm |D\downarrow;C\uparrow\rangle)\right\} + \frac{i}{2}(|C\uparrow;C\downarrow\rangle + |D\uparrow;D\downarrow\rangle), \tag{1}$$

where the $+$ sign stands for fermions and the $-$ sign stands for bosons. After the detectors click, the combined state of the particles and the detectors is

$$\frac{1}{\sqrt{2}}\left\{\frac{1}{\sqrt{2}}(|D\uparrow;C\downarrow\rangle \pm |D\downarrow;C\uparrow\rangle)\right\}|P_C^*\rangle|P_D^*\rangle \oplus \frac{i}{2}(|C\uparrow;C\downarrow\rangle|P_C^*\rangle|P_D\rangle \oplus |D\uparrow;D\downarrow\rangle|P_C\rangle|P_D^*\rangle),$$

where $\{|P_C\rangle,|P_D\rangle\}$ and $\{|P_C^*\rangle,|P_D^*\rangle\}$ are the unexcited and excited (corresponding to detection of one or more particles) detector states, respectively. In the above, $\oplus$ has been used to indicate the lack of *coherence* between orthogonal detector states. When the detectors are found in the state $|P_C^*\rangle|P_D^*\rangle$ (coincidence), the state of the particles is projected onto $\frac{1}{\sqrt{2}}(|D\uparrow;C\downarrow\rangle \pm |D\downarrow;C\uparrow\rangle)$. The spin part of this state can be rewritten in the first quantized notation (using the paths as particle labels) as $|\psi^{\pm}\rangle_{CD} = \frac{1}{\sqrt{2}}(|\uparrow\rangle_D|\downarrow\rangle_C \pm |\downarrow\rangle_D|\uparrow\rangle_C)$ (spin-entangled state). It is fully legitimate to use the paths as particle labels because the particles are *identical* (the same labeling is used for photon pairs exiting a parametric down-converter [2]).

The above method produces entangled states successfully only 50 per cent of the time. Now we describe how to make this process arbitrarily efficient. The basic idea is to feed the pair back into the beamsplitter in case the particles bunched. In the set-up of figure 1, when P_C and P_D do not click in coincidence, it is fed back again into the beamsplitter. The incident state is $|A\uparrow;A\downarrow\rangle$ or $|B\uparrow;B\downarrow\rangle$ during such a feedback. On passing through the beamsplitter, if there is anti-coincidence in the detectors P_C and P_D, the state generated is $\frac{1}{\sqrt{2}}(|\uparrow\rangle_D|\downarrow\rangle_C \mp |\downarrow\rangle_D|\uparrow\rangle_C)$. For coincidence, the feedback procedure is again repeated. The procedure is repeated till the pair anti-bunches. Here, the probability of failure decreases exponentially (2^{-N}) with the number N of feedback rounds, while the required resources (a single beamsplitter and a pair of detectors) remain unchanged. Thus by increasing the number N of feedback rounds, the efficiency of our entanglement generation procedure can be increased arbitrarily.

As our entangling method is applicable to any degree of freedom of any species of particles, we have illustrated the entangling of molecular vibrations in figure 2. C_{60} (buckyball) molecules have been recently interfered [8]. If one chooses two identical buckyball molecules in distinct vibrational levels at the two input ports of a beamsplitter, as shown in figure 2, then on detecting one molecule at each exit port, an entangled state of the molecular vibrations is obtained.

2.1 *Robustness of the entangling set-up to randomness of the initial states*

Imagine the case that we are unable to carefully choose the initial states of the variable to be entangled. Assuming the worst scenario, imagine that the initial state of the variable to be entangled is completely random for either particle. This means an incident state of

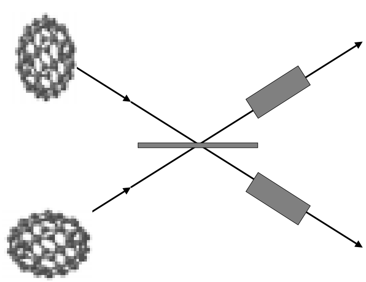

Figure 2. Application of our scheme to entangling the vibrational degrees of freedom of two buckyball molecules.

$$\frac{1}{\sqrt{2}}(a^{\dagger}_{A\uparrow}|0\rangle\langle 0|a_{A\uparrow} + a^{\dagger}_{A\downarrow}|0\rangle\langle 0|a_{A\downarrow}) \otimes \frac{1}{\sqrt{2}}(a^{\dagger}_{B\uparrow}|0\rangle\langle 0|a_{B\uparrow} + a^{\dagger}_{B\downarrow}|0\rangle\langle 0|a_{B\downarrow}).$$

In the above state, the particles impinge on the beamsplitter in a spin aligned way half of the time, while the particles impinge on the beamsplitter in a spin anti-aligned way the rest half of the time. If the particles were *bosons*, the first half of the cases would always lead to the particles exiting through the same port. Thus when you find coincidence in the detectors P_C and P_D, which will happen in a quarter of the cases, the entangled state $|\psi^+\rangle_{CD}$ will be generated.

In the case of fermions, we will first have to select out the subcases in which only one detector (either P_C or P_D) clicks. This will happen a quarter of the time. In this case, a pair of the particles exit together through the same path, and being fermions, are guaranteed to have opposite spins. Now this pair is fed back to the beamsplitter and the incident state now becomes $|A\uparrow;A\downarrow\rangle$ or $|B\uparrow;B\downarrow\rangle$. This state is now recursively fed back into the beamsplitter till the particles exit through separate paths. When they do so, they are in the entangled state $|\psi^+\rangle_{CD}$.

From the above discussion it is clear that for both bosons and fermions, our scheme works with 25 per cent efficiency when the initial state of the variable to be entangled is completely random.

3. A curious complementarity exhibited by our scheme

Our scheme exhibits a complementarity between particle distinguishability and the amount of entanglement produced. This complementarity involving 'which particle' information in two-particle interference differs fundamentally from the usual form involving 'which way' information in single-particle interference [9–12]. In two-particle interferometry, 'which channel' information is naturally replaced by 'which particle' information. The particles impinging on our setup (figure 1) are indistinguishable apart from their spins (which we choose not to measure as we intend to create a spin-entangled state). Now suppose, the particles were partially or fully distinguishable through some other observable such as energy or momentum or any non-spin internal degree of freedom. For example, suppose the incident state is $|A\uparrow S_1;B\downarrow S_2\rangle$, with $|\langle S_1|S_2\rangle| = a \le 1$. Then the two-particle state produced due to detector coincidence is $\frac{1}{\sqrt{2}}(|D\uparrow S_1;C\downarrow S_2\rangle \pm |D\downarrow S_2;C\uparrow S_1\rangle)$. The spin state of the particles (in the first quantized notation) is

$$\rho = \frac{1}{2}(|\uparrow_C\downarrow_D\rangle\langle\uparrow_C\downarrow_D| + |\downarrow_C\uparrow_D\rangle\langle\downarrow_C\uparrow_D| \pm |a|^2|\uparrow_C\downarrow_D\rangle\langle\downarrow_C\uparrow_D| \pm |a|^2|\downarrow_C\uparrow_D\rangle\langle\uparrow_C\downarrow_D|). \tag{2}$$

Note that in the second quantized notation all the degrees of freedom belong to the *same* Hilbert space (are created from the same vacuum). But they become elements of *distinct* Hilbert spaces when we proceed to the first quantized notation. For the above state, a certain entanglement measure called concurrence [13] is $\mathbf{E} = |a|^2$. The probability of successful discrimination between the states $|S_1\rangle$ and $|S_2\rangle$ (which is a measure of particle distinguishability) is $\mathbf{D} = 1 - |a|^2$. Thus we have, in analogy with Englert's relation in single particle interference [12], the following testable complementarity relation

$$\mathbf{E} + \mathbf{D} = 1. \tag{3}$$

The concurrence $\mathbf{E}$ for ρ can be inferred by measuring the expectation value of the Bell-CHSH operator $\hat{a}\hat{b} + \hat{a}\hat{b}' + \hat{a}'\hat{b} - \hat{a}'\hat{b}'$ on the two particles (labeled by their paths C and D) with $\hat{a} = \sigma_x^C, \hat{a}' = \sigma_y^C, \hat{b} = \frac{1}{\sqrt{2}}(\sigma_x^D + \sigma_y^D)$, $\hat{b}' = \frac{1}{\sqrt{2}}(\sigma_x^D - \sigma_y^D)$ and dividing the result by $\pm 2\sqrt{2}$.

Equation (3) also helps us to estimate the amount of entanglement generated by our scheme if $|S_1\rangle$ and $|S_2\rangle$ are Gaussian wave-packet states of the incident particles arriving at the beamsplitter. If the wave packets have width σ, velocity v, and a time delay Δt with respect to each other, the entanglement is $\mathbf{E} = \exp(-v^2\Delta t^2/2\sigma^2)$.

4. Summary and future

We have presented a scheme which entangles any variable of two identical particles by exploiting quantum indistinguishability. This entangling process is a feature of quantum indistinguishability which goes beyond quantum statistics. In other words, this phenomenon would happen for bosons, fermions and anions. Nonetheless, bosonic and fermionic statistics can both impart robustness to the protocol for mixedness of the original states. We have discussed the feasibility issues of the protocol elsewhere [14]. If a flippable spin is placed at the site of the beamsplitter, then, for fermions, the need for non-absorptive which-way detection can be avoided. This process has been explained in a separate publication in the context of the scattering of ballistic electrons from a magnetic impurity in a semiconductor quantum wire [15]. Our work also suggests the potential for further applications of quantum indistinguishability and statistics in quantum information processing. Indeed, applications of quantum statistics to entanglement concentration (an essential quantum information processing task) have recently been found [16].

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