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Information flow due to controlled interference in entangled systems

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Abstract. We point out that controlled quantum interference corresponds to measurement in an incomplete basis and implies a nonlocal transfer of classical information. A test of whether such a generalized measurement is permissible in quantum theory is presented.

Keywords. Controlled interference; incomplete measurement; causality.

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1. Introduction

The highly nonclassical phenomenon of quantum entanglement [1], its implications [2] and their tests [3,4] have vastly improved our understanding of an aspect of fundamental physics. Here we present a brief description of an experimental plan involving the process of spontaneous parametric downconversion (SPDC) [4], in which the effect of *controlled interference* (here: interference by focusing) of entangled light shows up in fourth-order correlations. Ours is a modification of the experiment reported in [5].

2. The experiment

Figure 1 presents a 'folded out' plan of an experiment in which two observers, designated Alice and Bob, share entangled photons produced via SPDC from a nonlinear crystal. The main difference between this and the experiment in ref. [5] is that here Bob is equipped with a direction filter, consisting of juxtaposed convex lenses shielded from each other except at their shared focus, where there is a small hole permitting only horizontal modes to reach his interferometer.

In figure 1, suppose p and q are the possible regions where downconversion could have occurred in the crystal, giving rise to three possible pairs of signal and idler modes (0, 1, 2 and 3, 4, 5, respectively). On account of Bob's direction filter, only modes 1 and 4 are relevant for writing the biphoton state originating from the nonlinear crystal

$$|\Psi\rangle = \frac{1}{\sqrt{2}}\left(\mathrm{e}^{ik\xi_1}|\psi_1\rangle|\psi_1\rangle + \mathrm{e}^{ik\xi_4}|\psi_4\rangle|\psi_4\rangle\right), \tag{1}$$

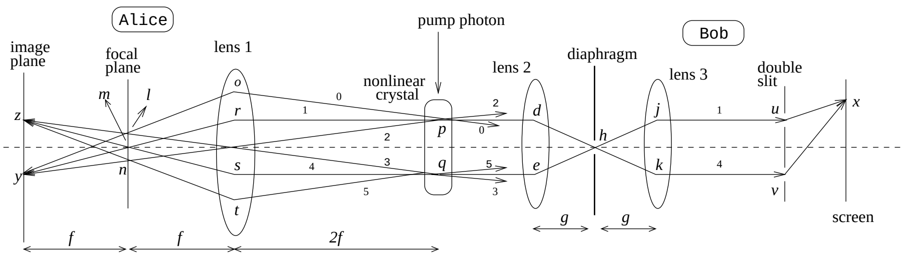

Figure 1. Light produced in a nonlinear crystal via spontaneous parametric downconversion (SPDC) is shared by Alice and Bob. By observing her photons at the focal or image plane of her lens, Alice leaves Bob's photons, respectively, as a wave-like object with a definite momentum, or a particle-like object with a definite point of origin. Access to Bob's interferometer is restricted to horizontal modes, by means of two juxtaposed lenses with an intervening diaphragm perforated at the shared focus.

where the first (second) ket register refers to Alice (Bob). ξ_1 (ξ_4) is the path length for the entire mode 1 (4), from Alice's detector to Bob's through the point of origin p (q) in the source, and k is the wave number. Natural units are adopted in this article.

Because of the narrowness of aperture h, both $|\psi_1\rangle$ and $|\psi_4\rangle$ diffract towards the double slit. Let the unitary action of h on the two modes be given by $|\psi_\alpha\rangle \longrightarrow \sin(\phi_\alpha/2)|u\rangle + \cos(\phi_\alpha/2)|v\rangle$, where ϕ_α ($\alpha = 1,4$) can be thought of as the angle of incidence of mode $|\psi_\alpha\rangle$ on the diaphragm. Let x be some point on Bob's screen, and $|x\rangle$ the eigenstate corresponding to detection at x. The transformation from the slit to the screen basis can be formally written as: $|u\rangle \longrightarrow \int_x \mathrm{e}^{ik\overline{ux}}|x\rangle\mathrm{d}x$ and $|v\rangle \longrightarrow \int_x \mathrm{e}^{ik\overline{vx}}|x\rangle\mathrm{d}x$, where $\overline{ux}$ ($\overline{vx}$) is the distance from slit u (v) to point x.

Substituting the above into eq. (1) for the second register, we find

$$\begin{aligned}|\Psi\rangle &= \frac{1}{\sqrt{2}}\sum_{\alpha=1,4}\left(\mathrm{e}^{ik\xi_\alpha}|\psi_\alpha\rangle\left[\sin(\phi_\alpha/2)|u\rangle + \cos(\phi_\alpha/2)|v\rangle\right]\right)\\ &= \frac{1}{\sqrt{2}}\sum_{\alpha=1,4}\int_x \mathrm{e}^{ik\xi_\alpha}|\psi_\alpha\rangle\left[\sin(\phi_\alpha/2)\mathrm{e}^{ik\overline{ux}} + \cos(\phi_\alpha/2)\mathrm{e}^{ik\overline{vx}}\right]|x\rangle\mathrm{d}x. \qquad (2)\end{aligned}$$

(a) *Alice measures position*: In figure 1, Alice's detection at some point on the image plane (say, y) means that the modes 0, 1 and 2 are selected from the signal and idler beams. Because of Bob's direction filter, only the two terms in eq. (2) containing $|\psi_1\rangle$ (corresponding to the two paths $yrpdhjux$ and $yrpdhkvx$) contribute to Bob's observed pattern.

Therefore, the amplitude for Bob to detect a photon at point x on his screen conditioned on Alice's detection at y is the sum of amplitudes for these two paths

$$A_{\phi\alpha}(x) \propto \mathrm{e}^{i\gamma}\left(\sin(\phi_\alpha/2)\mathrm{e}^{ik\overline{ux}} + \cos(\phi_\alpha/2)\mathrm{e}^{ik\overline{vx}}\right), \qquad (3)$$

($\alpha = 1$). Here γ is the phase factor corresponding to the distance along the path segments $zsqehju$ or $zsqehkv$. Proceeding along similar lines, if Alice detects her photon at z on the image plane, then the amplitude for Bob to observe at point x on his screen is eq. (3) with $\alpha = 4$.

Therefore, the interference pattern seen by Bob in his single counts assuming Alice detects a photon at y or z equally often, is given by

$$\begin{aligned} I(x) &\propto |A_{\phi 1}(x)|^2 + |A_{\phi 4}(x)|^2 \\ &= 2 + [\sin(\phi_1) + \sin(\phi_4)]\cos^2(ik[\overline{ux} - \overline{vx}]). \end{aligned} \tag{4}$$

(b) *Alice measures momentum*: In figure 1, Alice's detection at some point on the focal plane (say, m) means that the modes 1 and 4 are selected from the signal and idler beams. Because of Bob's direction filter, other detections by Alice produce no coincidental detections, but all four terms in eq. (2) (corresponding to the four paths $mrpdhjux$, $mrpdhkvx$, $msqehjux$ and $msqehkvx$) contribute to Bob's observed pattern:

$$\begin{aligned} A_m(x) &\propto \mathrm{e}^{iM}\left([\sin(\phi_1/2) + \sin(\phi_4/2)]\mathrm{e}^{i\overline{ux}} + [\cos(\phi_1/2) + \cos(\phi_4/2)]\mathrm{e}^{i\overline{vx}}\right) \\ &= \mathrm{e}^{i(M-\gamma)}\left(A_{\phi 1}(x) + A_{\phi 4}(x)\right), \end{aligned} \tag{5}$$

where segments $mrpdhju$, $mrpdhkv$, $msqehju$ and $msqehkv$ have equal path lengths M. So, the interference pattern seen by Bob in his single counts if Alice observes at the focal plane

$$\begin{aligned} I(x) &\propto |A_{\phi 1}(x) + A_{\phi 4}(x)|^2 \\ &= 2 + 2[\cos(\phi_1 - \phi_4)] \\ &\quad + \left[\sin(\phi_1) + \sin(\phi_4) + 2\sin\left(\frac{\phi_1 + \phi_4}{2}\right)\right]\cos^2(ik[\overline{ux} - \overline{vx}]), \end{aligned} \tag{6}$$

which may be contrasted with the interference pattern eq. (4) by the *visibility* (both configurations show an interference pattern). Note that Alice may delay her choice of whether to measure at the image or focal plane, until after the photons have left the source, engendering a potentially noncausal scenario if $4f < d_{\mathrm{B}}$, the distance from the source to Bob's double slit, for the effect of Alice's action on Bob's observation constitutes a classical communication, even though the two events are space-like separated. (A more detailed derivation of the experiment is given in [6].)

3. Understanding the nonlocality

Measurements that can determine along which path a photon traversed are represented by the usual von Neumann projection operators $\hat{P}_\alpha \equiv |\psi_\alpha\rangle\langle\psi_\alpha|$, for $\alpha \in \{0, \cdots, 5\}$ [7]. These operators form a complete set in that $\sum_\alpha \hat{P}_\alpha = \hat{I}$, where $\hat{I}$ is the unit operator in Alice's 6-dimensional Hilbert space.

But Alice's measurement at the focal or image plane represents a coarse-grained (or incomplete) measurement, e.g., detection at m, represented by operator $\hat{P}_m$, cannot distinguish between paths 1 and 4. In a non-interferometric setting, the probability for this outcome would be the expectation value of the projector $\hat{P}_1 + \hat{P}_4$. But, lacking cross-talk terms, this would imply that modes 1 and 4 do not interfere at m and x. Invoking field theoretic arguments, we can show that the desired operator is in fact $\hat{P}_m \equiv (|\psi_1\rangle + |\psi_4\rangle)(\langle\psi_1| + \langle\psi_4|)$, which is a rank 1 positive operator valued measure (POVM) element corresponding to a coarse-grained interferometric measurement (cf. [6] for details). Clearly, $\hat{P}_m \neq \hat{P}_1 + \hat{P}_4$.

One forms similar operators corresponding to measurements at l, m and y, z. The key point about coarse-grained interferometric measurements is that they are realized at path singularities of optical beams. Such 'singular measurements' do not form a complete set: $\hat{P}_l + \hat{P}_m + \hat{P}_n \neq \hat{P}_y + \hat{P}_z \neq \hat{I}$. This has two consequences. First, the joint probability for Alice and Bob is to be generalized to $P(\alpha, x) = \langle\Psi|\hat{P}_\alpha \otimes \hat{P}_x|\Psi\rangle/\beta$, where $\beta \equiv \sum_{\alpha,x}\langle\Psi|\hat{P}_\alpha \otimes \hat{P}_x|\Psi\rangle$ is the 're-normalization factor'. Here α runs over $\{l, m, n\}$ or $\{y, z\}$. If Alice's measurement is complete, then $\beta = 1$, and we recover the usual trace formula. Second, tracing over Alice, we find Bob's single counts probability will also depend on β, which is determined by the completeness of Alice's distant measurements. This is the origin of the present nonlocality. If we restrict ourselves to complete measurements (cf. [8]), then we recover causal nonlocality in the sense of [9]. By this reasoning, coarse-grained interferometric measurements can help distill classical communication from quantum correlations. The present experiment is therefore a test of whether such generalized measurements are permissible in quantum theory.

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References

[1] A Einstein, B Podolsky and N Rosen, *Phys. Rev.* **47**, 777 (1935)
[2] J S Bell, *Physics* **1**, 195 (1964)
F Clauser, M A Horne, A Shimony and R A Holt, *Phys. Rev. Lett.* **23**, 880 (1969)
[3] A Aspect, P Grangier and G Roger, *Phys. Rev. Lett.* **49**, 91 (1982)
W Tittel, J Brendel, H Zbinden and N Gisin, *Phys. Rev. Lett.* **81**, 3563 (1998)
G Weihs, T Jennewein, C Simon, H Weinfurter and A Zeilinger, *Phys. Rev. Lett.* **81**, 5039 (1998)
[4] R Ghosh and L Mandel, *Phys. Rev. Lett.* **59**, 1903 (1987)
C K Hong, Z Y Ou and L Mandel, *Phys. Rev. Lett.* **59**, 2044 (1987)
Y H Shih and C O Alley, *Phys. Rev. Lett.* **61**, 50 (1988)
P G Kwiat, A M Steinberg and R Chiao, *Phys. Rev.* **A47**, 2472 (1993)
D V Strekalov, A V Sergienko, D N Klyshko and Y H Shih, *Phys. Rev. Lett.* **74**, 3600 (1995)
T B Pittman, Y H Shih, D V Strekalov and A V Sergienko, *Phys. Rev.* **A52**, R3429 (1995)
[5] A Zeilinger, *Rev. Mod. Phys.* **71**, S288 (1999)
A Dopfer, Ph.D. Thesis (Univ. of Innsbruck, 1998)
[6] R Srikanth, eprints quant-ph/0101023, quant-ph/0110132
[7] D Home, *Conceptual foundations of quantum mechanics* (Plenum, New York, 1994)
[8] G C Ghirardi, R Grassi, A Rimini and T Weber, *Europhys. Lett.* **6**, 95 (1988)
[9] P H Eberhard, *Nuovo Cimento* **B46**, 392 (1978); *Europhys. Lett.* **6**, 95 (1988)
G C Ghirardi, A Rimini and T Weber, *Lett. Nuovo Cimento* **27**, 293 (1980)
P H Eberhard and R R Ross, *Found. Phys. Lett.* **2**, 127 (1989)
D Bohm and B J Hiley, *The undivided universe: An ontological interpretation of quantum theory* (Routledge, London and New York, 1993)
P J Bussey, *Phys. Lett.* **A90**, 9 (1982)
T F Jordan, *Phys. Lett.* **A94**, 264 (1983)

A J M Garrett, *Found. Phys.* **20**, 381 (1990)
C D Cantrell and M O Scully, *Phys. Rep.* **43**, 499 (1978)
C Simon, V Buzek and N Gisin, *Phys. Rev. Lett.* **87**, 170405 (2001)
R Srikanth, *Phys Lett.* **A292**, 161 (2002)