PRAMANA — journal of physics

Vol. 58, No. 3
March 2002
pp. 551–561

Semileptonic ($\Lambda_b \to \Lambda_c\ e\nu$) decay in a field theoretic quark model

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MS received 11 October 2000; revised 27 June 2001

Abstract. The semileptonic decay width of heavy baryons such as $(\Lambda_b \to \Lambda_c e\nu)$ has been estimated in the framework of a nonrelativistic field theoretic quark model where four component quark field operators along with a harmonic oscillator wave function are used to describe translationally invariant hadronic states. The present estimation does not make an explicit use of heavy quark symmetry and has a reasonable agreement with the experimentally measured decay width, polarisation ratio and form factors with the harmonic oscillator radii and quark momentum distribution inside the hadron as free parameters.

Keywords. Semileptonic decay; quark model; field theory.

PACS Nos 13.30.Ce; 12.39.Jh; 14.20.Lq; 14.20.Mr

1. Introduction

Even though quantum chromodynamics (QCD) [1] has been a very successful theory of hadrons, it does not explain completely the low energy hadronic phenomena such as semileptonic decay of baryons $(\Lambda_b \to \Lambda_c e^- \overline{\nu})$ at its first principle application level. For such type of low energy phenomena one resorts to phenomenological models incorporating the basic features of QCD at a structural level. We calculate here the semileptonic decay width and branching ratio for the process $\Lambda_b \to \Lambda_c e^- \overline{\nu}$ based on the framework of a field theoretic quark model [2,3] which has been successful in its applications to a variety of hadronic phenomena [4–9]. As such semileptonic decay process $\Lambda_b \to \Lambda_c e^- \overline{\nu}$ is very interesting to the effect that it being a weak process can be extremely useful in obtaining the weak structure of hadrons and for studying CP-violation effect with respect to the estimation of CKM matrix element V_{cb} involved in the process.

There is not much work found in the literature with regard to heavy quark semileptonic decays. However, in very recent works [10,11] which were based on nonrelativistic quark models [12,13] the decay width and branching ratio for $\Lambda_b \to \Lambda_c e^- \overline{\nu}$ has been estimated. In view of the earlier success of the present field-theoretic quark model [2,3], the semileptonic process $\Lambda_b \to \Lambda_c e^- \bar{\nu}$ is considered here as an application.

The present article is organised as follows. In §2, we describe briefly the field-theoretic quark model to be applied. Section 3 considers the usual kinematics and in §4, the estimation of the decay width, branching ratio and form factors for the process $\Lambda_b \to \Lambda_c e^- \bar{\nu}$ are considered. Section 5 discusses the results of the present estimation and further prospects in this regard.

2. Field theoretic quark model

The field theoretic quark model considers a quark field operator $\Psi_q(\vec{x})$ of a quark of flavor q written as

$$\Psi_q(\vec{x}) = q(\vec{x}) + \tilde{q}(\vec{x}), \tag{2.1}$$

where $q(\vec{x})$ and $\tilde{q}(\vec{x})$ represent annihilation of a quark and the creation of an antiquark respectively. For $x^0 = t = 0$, in the Fourier space, the quark field operators are written as

$$q(\vec{x}) = \frac{1}{(2\pi)^{3/2}} \int u(\vec{k}) q_I(\vec{k}) e^{i\vec{k}\cdot\vec{x}} d\vec{k} \tag{2.2}$$

and

$$\tilde{q}(\vec{x}) = \frac{1}{(2\pi)^{3/2}} \int v(\vec{k}) \tilde{q}_I(\vec{k}) e^{-i\vec{k}\cdot\vec{x}} d\vec{k} \tag{2.3}$$

with

$$q_I(\vec{k}) = \sum_{r\pm 1/2} q_{Ir}(\vec{k}) u_{Ir}, \quad \tilde{q}_I(\vec{k}) = \sum_{r=\pm 1/2} \tilde{q}_{Ir}(\vec{k}) v_{Ir}, \tag{2.4}$$

where the two component spinors u_{Ir} and v_{Ir} are explicitly written as

$$u_{I1/2} = \begin{pmatrix} 1 \\ 0 \end{pmatrix}, \quad u_{I-1/2} = \begin{pmatrix} 0 \\ 1 \end{pmatrix}, \quad v_{I1/2} = \begin{pmatrix} 0 \\ -i \end{pmatrix}, \quad v_{I-1/2} = \begin{pmatrix} i \\ 0 \end{pmatrix}.$$

In two component forms $u(k)$ and $v(k)$ are also written as

$$u(\vec{k}) = \begin{pmatrix} f_q(\vec{k}^2) \\ g_q(\vec{k}^2)(\vec{\sigma}\cdot\vec{k}) \end{pmatrix}, \quad v(\vec{k}) = \begin{pmatrix} g_q(\vec{k}^2)\vec{\sigma}\cdot\vec{k} \\ f_q(\vec{k}^2) \end{pmatrix}, \tag{2.5}$$

where the relationship between the arbitrary functions $f_q(\vec{k}^2)$ and $g_q(\vec{k}^2)$ for a quark of particular flavor q as

$$f_q^2(\vec{k}^2) + g_q^2(\vec{k}^2)\vec{k}^2 = 1, \tag{2.6}$$

yields to the equal time algebra for the constituent quark field operators in the form of

$$\{\Psi_{q\alpha}(x), \quad \Psi^+_{q\beta}(y)\}_{x^0=y^0} = \delta_{\alpha\beta}\delta(\vec{x}-\vec{y}) \tag{2.7}$$

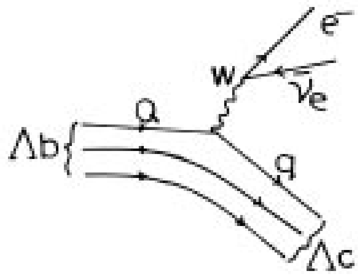

Figure 1. The semileptonic decay of a heavy quark Q into a lighter quark q and a virtual W, which becomes a charged lepton and neutrino.

with α and β as Dirac quantum numbers.

The baryon states $|\Lambda_b(\vec{0})\rangle$ and $|\Lambda_c(\vec{K})\rangle$ in the rest and moving frames respectively are described in the present model in terms of quark field operators consistent with their SU(6) quantum numbers along with the ground state harmonic oscillator wave function as

$$|\Lambda_b(\vec{0})\rangle = \frac{\varepsilon_{ijk}}{2\sqrt{3}}\int \delta(\vec{k}_1+\vec{k}_2+\vec{k}_3)\tilde{U}_{\Lambda_b}(\vec{k}_1,\vec{k}_2,\vec{k}_3)\mathrm{d}\vec{k}_1\mathrm{d}\vec{k}_2\mathrm{d}\vec{k}_3 \times\left[\left\{u^{i+}_{I1/2}(\vec{k}_1)d^{j+}_{I-1/2}(\vec{k}_2)-u^{i+}_{I-1/2}(\vec{k}_1)d^{j+}_{I1/2}(\vec{k}_2)\right\}b^{k+}_{I1/2}(\vec{k}_3)\right]|vac\rangle, \tag{2.8}$$

$$|\Lambda_c(\vec{K})\rangle = \frac{\varepsilon_{ijk}}{2\sqrt{3}}\int \delta(\vec{k}'_1+\vec{k}'_2+\vec{k}'_3)\tilde{U}_{\Lambda_c}(\vec{k}'_1,\vec{k}'_2,\vec{k}'_3)\mathrm{d}\vec{k}'_1\mathrm{d}\vec{k}'_2\mathrm{d}\vec{k}'_3 \times\left[\left\{u^{i+}_{I1/2}(\vec{k}'_1+\lambda_1\vec{K})d^{j+}_{I-1/2}(\vec{k}'_2+\lambda_2\vec{K})-u^{i+}_{I-1/2}(\vec{k}'_1+\lambda_1\vec{K})d^{j+}_{I1/2}(\vec{k}'_2+\lambda_2\vec{K})\right\} \times c^{k+}_{I1/2}(\vec{k}'_3+\lambda_3\vec{K})\right]|vac\rangle, \tag{2.9}$$

with λ_1,λ_2 and λ_3 being fractions of momentum carried by the constituent quarks satisfying the constraint $\lambda_1+\lambda_2+\lambda_3=1$. Here, $\tilde{U}_{\Lambda_b}(\vec{k}_1,\vec{k}_2,\vec{k}_3)$ and $\tilde{U}_{\Lambda_c}(\vec{k}'_1,\vec{k}'_2,\vec{k}'_3)$ are the baryon wave functions for Λ_b and Λ_c respectively and is written for any baryon B in general as

$$\tilde{U}_B(\vec{k}_1,\vec{k}_2,\vec{k}_3)=\left(\frac{3R_B^4}{\pi^2}\right)^{3/4}\exp\left[-\frac{R_B^2}{6}\sum_{i<j}(\vec{k}_i-\vec{k}_j)^2\right], \tag{2.10}$$

where R_B is the harmonic oscillator radius for the baryon B.

3. Kinematics of semileptonic decay of baryons

In the present case of the semileptonic decay process Λ_b decays into Λ_c, e^- and $\bar{\nu}$ as shown in figure 1, where b-flavored quark of the parent baryon Λ_b decays into c-flavored quark and a charged W-boson which subsequently decays in the leptonic channel through an electron and an antineutrino, the other two constituent quarks of the baryon Λ_b acting as spectators and finally a Λ_c is formed from the c and the two spectators. Such a process has calculational simplicity while considering it in the parent baryon (Λ_b) rest frame. Nevertheless, lepton center of mass (CM) frame of reference is worth considering in this regard for the sake of comparison of estimations with the experimental observations. The natural variables such as initial baryon energy (E_{Λ_b}), final baryon energy (E_{Λ_c}) and momentum ($\vec{k}$) in terms of the dimensionless variable $y = q^2/m^2_{\Lambda_b}$ in the center of mass frame are expressed as [11]

$$E_{\Lambda_b} = \frac{m_{\Lambda_b}}{2\sqrt{y}}\left(1 - \frac{m^2_{\Lambda_b}}{m^2_{\Lambda_c}} + y\right), \tag{3.1a}$$

$$E_{\Lambda_c} = \frac{m_{\Lambda_b}}{2\sqrt{y}}\left(1 - \frac{m^2_{\Lambda_c}}{m^2_{\Lambda_b}} - y\right), \tag{3.1b}$$

$$|\vec{k}| = \frac{|\vec{K}|}{\sqrt{y}} \tag{3.1c}$$

while in the parent rest frame the energy and three momenta of daughter baryon Λ_c are

$$\tilde{E}_{\Lambda_c} = \frac{m_{\Lambda_b}}{2}\left(1 + \frac{m^2_{\Lambda_c}}{m^2_{\Lambda_b}} - y\right), \tag{3.2a}$$

$$|\vec{K}| = \frac{m_{\Lambda_b}}{2}\left[\left(1 - \frac{m^2_{\Lambda_c}}{m^2_{\Lambda_b}} - y\right)^2 - \frac{4m^2_{\Lambda_c}}{m^2_{\Lambda_b}}y\right]^{1/2}. \tag{3.2b}$$

The S-matrix element of the decay process is written in the standard form as

$$S_{fi} = \langle f|S|i\rangle = \delta_4(p_f - p_i)M_{fi}, \tag{3.3}$$

where the transition matrix element of the decay process is written as

$$M_{fi} = \frac{1}{(2\pi)^2} \times \frac{1}{4\sqrt{M_i E_f E_e E_\nu}}\mathbf{m}_{fi} \tag{3.4a}$$

with

$$\mathbf{m}_{fi} = \frac{G_{\mathrm{F}} V_{cb} H_\mu L^\mu}{\sqrt{2}}, \tag{3.4b}$$

where $M_i = m_{\Lambda_b}$ is the mass of the initial baryon and $E_f = E_{\Lambda_c}, E_e$ and E_ν are the energies of the final baryon, electron and neutrino respectively, G_{F} is the effective Fermi-coupling constant, V_{cb} is the CKM matrix element and H_μ and L_μ are the hadronic and leptonic matrix elements respectively and are explicitly written as

$$H_\mu = \sqrt{4M_i E_f}\langle\Lambda_c(\vec{k})|\overline{\Psi}_{\Lambda_c}(\vec{0})\gamma_\mu(1-\gamma_5)\Psi_{\Lambda_b}(\vec{0})|\Lambda_b(\vec{p})\rangle. \tag{3.5}$$

Now, with baryon states of §2, the hadronic matrix element is explicitly written in initial parent rest frame as

$$H_\mu = \sqrt{4M_i E_f}\langle \Lambda_c(\vec{K})|J_5^\mu(0)|\Lambda_b(\vec{0})\rangle \tag{3.6}$$

and for the leptonic part

$$L^\mu = \bar{u}_r \gamma^\mu (1-\gamma_5) u_s, \tag{3.7}$$

where u_r and u_s are the leptonic spinors, i.e. Dirac spinors.

The matrix element of the hadronic current can be constructed from Lorentz-invariant form factors and the four vectors involved in the process. The quark current when explicitly written becomes, $J_5^\mu = V^\mu - A^\mu$, where V^μ and A^μ are the vector and axial vector parts of the quark current respectively. Taking this, one defines,

$$\begin{aligned}\langle \Lambda_c(\vec{k})|V^\mu(0)|\Lambda_b(\vec{p})\rangle = \bar{u}^c_{Ir}\Big[&g(q^2)\gamma^\mu + g_+(q^2)(\vec{p}+\vec{k})^\mu \\ &+g_-(q^2)(\vec{p}-\vec{k})^\mu\Big]u^b_{Is},\end{aligned} \tag{3.8}$$

$$\begin{aligned}\langle \Lambda_c(\vec{k})|A^\mu(0)|\Lambda_b(\vec{p})\rangle = \bar{u}^c_{Ir}\Big[&a(q^2)\gamma^\mu\gamma_5 + a_+(q^2)(\vec{p}+\vec{k})^\mu\gamma_5 \\ &+a_-(q^2)(\vec{p}-\vec{k})^\mu\gamma_5\Big]u^b_{Is}.\end{aligned} \tag{3.9}$$

Here, u^c_{Ir} and u^b_{Is} are the spinors associated with Λ_c and Λ_b respectively. Further, the four momentum transfer q in the present case is in terms of the initial and final momenta of hadrons in the leptonic center of mass frame and is written as $q^2 = (p-k)^2$. The form factors are also conventionally expressed by introducing dimensionless variables through a scaling with respect to the initial baryon mass m_{Λ_b}, i.e. $y = q^2/m^2_{\Lambda_b}$. Neglecting the lepton mass, the kinematic range of the dimensionless variable y is expressed in terms of a constraint relation as

$$0 \le y \le \left(1 - \frac{m_{\Lambda_c}}{m_{\Lambda_b}}\right)^2. \tag{3.10}$$

The differential decay width for the exclusive semileptonic decays with a baryon in the final state is written in general form as [10,11]

$$\frac{\mathrm{d}\Gamma}{\mathrm{d}y} = \frac{G_\mathrm{F}^2|V_{cb}|^2 K m^2_{\Lambda_b} y}{96\pi^3}\,|\vec{H}|^2, \tag{3.11}$$

where the net helicity amplitude H is expressed in terms of helicity basis contributions as

$$|\vec{H}|^2 = |\vec{H}_+|^2 + |\vec{H}_-|^2 + |\vec{H}_0|^2, \tag{3.12}$$

with $|\vec{H}_+|^2 + |\vec{H}_-|^2$ and $|\vec{H}_0|^2$ referring to the transverse and to the longitudinal polarisation modes respectively. The helicity amplitudes when expressed in terms of invariant form factors become

$$\vec{H}_{\pm} = \pm(aF_0 \mp gF_-), \tag{3.13}$$

and

$$\vec{H}_0 = \left\{ \left[2a\left(1-\frac{1}{2}F_0\right) - \frac{2k}{\sqrt{y}}a_+F_-\right]^2 + \left[\frac{2k}{\sqrt{y}}g_+F_0 + gF_+\right]^2 \right\}^{1/2} \tag{3.14}$$

with the $e\bar{\nu}$ center of mass frame momentum $k = |\vec{k}|$, and

$$F_{\pm} = \left[\frac{(E_{\Lambda_b}+m_{\Lambda_b})(E_{\Lambda_c}+m_{\Lambda_c})}{4m_{\Lambda_b}m_{\Lambda_c}}\right]^{1/2} \left[\frac{k}{\sqrt{y}(E_{\Lambda_c}+m_{\Lambda_c})} \pm \frac{k}{\sqrt{y}(E_{\Lambda_b}+m_{\Lambda_b})}\right] \tag{3.15}$$

and

$$F_0 = \left[\frac{(E_{\Lambda_b}+m_{\Lambda_b})(E_{\Lambda_c}+m_{\Lambda_c})}{4m_{\Lambda_b}m_{\Lambda_c}}\right]^{1/2} \left[1 - \frac{k^2}{y(E_{\Lambda_b}+m_{\Lambda_b})(E_{\Lambda_c}+m_{\Lambda_c})}\right]. \tag{3.16}$$

Subsequently one also obtains the longitudinal to transverse contribution ratio as

$$\frac{\Gamma_{\mathrm{L}}}{\Gamma_{\mathrm{T}}} = \frac{\int |\vec{H}_0|^2 ky\, \mathrm{d}y}{\int (|\vec{H}_+|^2 + |\vec{H}_-|^2) ky\, \mathrm{d}y}, \tag{3.17}$$

where Γ_{L} and Γ_{T} are the longitudinal and transverse decay widths respectively.

4. Semileptonic decay of baryons and form factors

Two light quarks being spectators the hadronic current when calculated, the weak hadronic current arises only due to the heavy quarks with the form $V^{\mu} - A^{\mu}$. The corresponding quark current when sandwiched between the two baryon states of the field-theoretic model [2,3] described earlier, yields to

$$\begin{aligned} H_{\mu} = \sqrt{4m_{\Lambda_b}E_{\Lambda_c}} \int \mathrm{d}\vec{k}_1 \mathrm{d}\vec{k} \tilde{U}^*_{\Lambda_c}(\vec{k}'_1,\vec{k}'_2,\vec{k}'_3) \tilde{U}_{\Lambda_b}(\vec{k}_1,\vec{k}_2,\vec{k}_3) \\ \times\, \bar{u}^c_{1/2}(\vec{k}')\gamma^{\mu}(1-\gamma_5)u^b_{1/2}(\vec{k}), \end{aligned} \tag{4.1}$$

where $u^c_{1/2}(\vec{k}')$ and $u^b_{1/2}(\vec{k})$ are the spinors of the model for c and b quarks respectively, and the momenta after δ-function integrations are,

$$\begin{aligned} &\vec{k}'_1 = \vec{k}_1 - \lambda_1\vec{K},\ \vec{k}'_2 = \vec{k}_2 - \lambda_2\vec{K},\ \vec{k}'_3 = \vec{k} + \vec{K} - \lambda_3\vec{K}, \\ &\vec{k}_2 = -\vec{k}_1 - \vec{k},\ \vec{k}_3 = \vec{k},\ \vec{k}' = \vec{k} + \vec{K}. \end{aligned} \tag{4.2}$$

In fact we will use these momenta identities while deducing the temporal and spatial components of the matrix element H_{μ}.

Thus, the temporal and the spatial components of the matrix element of the hadronic current of eq. (4.1) becomes

$$\begin{aligned} H_\mu &= [(H_V^0 - H_A^0,\ H_V^S - H_A^S)] \\ &= \sqrt{4m_{\Lambda_b}E_{\Lambda_c}} \int \mathrm{d}\vec{k}\mathrm{d}\vec{k}_1 \tilde{U}^*_{\Lambda_c}\left(\vec{k}_1 - \lambda_1\vec{K},\ -(\vec{k}_1+\vec{k}+\lambda_2\vec{K}),\ (\vec{k}+\vec{K}-\lambda_3\vec{K})\right) \\ &\quad \times \tilde{U}_{\Lambda_b}\left(\vec{k}_1,\ -(\vec{k}_1+\vec{k}),\ \vec{k}\right) u^+_{I1/2}\Big[\Big\{ f_c(\vec{k}'^2) f_b(\vec{k}^2) + (\vec{\sigma}\cdot\vec{k}')\ (\sigma\cdot\vec{k}) g_c\ (\vec{k}'^2) g_b(\vec{k}^2) \\ &\quad - f_b(\vec{k}^2) g_c(\vec{k}'^2)(\vec{\sigma}\cdot\vec{K})\Big\}\Big\{ f_b(\vec{k}^2) g_c(\vec{k}'^2)(\vec{K}+i(\vec{\sigma}\times\vec{K})) \\ &\quad - \left(f_b(\vec{k}^2) f_c(\vec{k}'^2) - \frac{g_b(\vec{k}^2) g_c(\vec{k}'^2)}{3}\vec{k}^2\right)\Big\}\vec{\sigma}\Big] u_{I1/2}, \end{aligned} \tag{4.3}$$

where the notations (H_V^0, H_V^S), are for temporal and spatial components for the matrix element of the vector part of the hadronic current while (H_A^0, H_A^S), are for the axial vector part of the current and $\vec{k}' = \vec{k}+\vec{K}$.

However, for the sake of further simplifications we approximate here the parameters of the constituent quark field operators through expansions as

$$\begin{aligned} f_{q_i}(\vec{k}^2) &= (1-g_{q_i}^2 k^2)^{1/2} \cong 1 - \frac{1}{2} g_{q_i}^2 k^2; \\ f_{q_i}(\vec{k}'^2) &= (1-g_{q_i}^2 \vec{k}'^2)^{1/2} \cong 1 - \frac{1}{2} g_{q_i}^2 \vec{k}'^2. \end{aligned} \tag{4.4}$$

We may note here that such an approximation has been quite reasonable in the earlier applications of the model as the other higher order terms of the infinite series has been found to be highly converging and thus contribute negligibly. Thus, using such a simplifying approximation and integrating analytically the simple Gaussian integrations we write the relevant matrix elements for the temporal and spatial components of the quark currents separately as

$$\begin{aligned} H_V^0 = u^+_{I1/2}\Bigg[A\Bigg\{ 1 - \frac{1}{2}g_c^2\vec{K}^2 - \left(\frac{2(g_c-g_b)^2 - g_c^2 g_b^2 \vec{K}^2}{4}\right)\left(\frac{2}{R^2_{\Lambda_c}+R^2_{\Lambda_b}} + \beta^2\vec{K}^2\right) \\ + \frac{g_c^2 g_b^2}{4}\left(\frac{20}{3(R^2_{\Lambda_c}+R^2_{\Lambda_b})^2} + \frac{12\beta^2 K^2}{(R^2_{\Lambda_c}+R^2_{\Lambda_b})} + \beta^2\vec{K}^4\right)\Bigg\}\Bigg] u_{I1/2}, \end{aligned} \tag{4.5}$$

$$H_V^S = u^+_{I1/2}\left[A g_c \left\{ 1 - \frac{g_b^2}{2}\left(\frac{2}{R^2_{\Lambda_c}+R^2_{\Lambda_b}} + \beta^2\vec{K}^2\right)\right\}\{\vec{K}+i(\vec{\sigma}\times\vec{K})\}\right] u_{I1/2}, \tag{4.6}$$

$$H_A^0 = u^+_{I1/2}\left[A g_c \left\{ 1 - \frac{g_b^2}{2}\left(\frac{2}{R^2_{\Lambda_c}+R^2_{\Lambda_b}} + \beta^2\vec{K}^2\right)\right\}(\vec{\sigma}\cdot\vec{K})\right] u_{I1/2}, \tag{4.7}$$

$$H_A^S = u_{I1/2}^+ \left[A \left\{ 1 - \frac{1}{2} g_c^2 \vec{K}^2 - \left(\frac{g_c g_b}{3} + \frac{1}{2}(g_c^2 + g_b^2) - \frac{1}{4} g_c^2 g_b^2 \vec{K}^2 \right) \left(\frac{2}{R_{\Lambda_c}^2 + R_{\Lambda_b}^2} + \beta^2 \vec{K}^2 \right) \right.\right.$$
$$\left.\left. + \frac{g_c^2 g_b^2}{4} \left(\frac{20}{3(R_{\Lambda_c}^2 + R_{\Lambda_b}^2)^2} + \frac{12\beta^2 \vec{K}^2}{(R_{\Lambda_c}^2 + R_{\Lambda_b}^2)} + \beta^4 \vec{K}^4 \right) \right\} \vec{\sigma} \right] u_{I1/2}, \tag{4.8}$$

with

$$A = \sqrt{4 m_{\Lambda_b} E_{\Lambda_c}} \left(\frac{4 R_{\Lambda_c}^2 R_{\Lambda_b}^2}{(R_{\Lambda_c}^2 + R_{\Lambda_b}^2)^2} \right)^{3/2}$$
$$\times \exp \left[- \left(\frac{R_{\Lambda_c}^2 R_{\Lambda_b}^2}{(R_{\Lambda_c}^2 + R_{\Lambda_b}^2)} \right) (\lambda_1^2 + \lambda_2^2 + \lambda_1 \lambda_2) \vec{K}^2 \right], \tag{4.9}$$

$$\beta = \frac{R_{\Lambda_c}^2}{(R_{\Lambda_c}^2 + R_{\Lambda_b}^2)} (\lambda_1 + \lambda_2). \tag{4.10}$$

However, conventionally, in the parent rest frame the temporal and spatial components become [12,13],

$$H_V^0 = u_{I1/2}^+ \left[g(q^2) + (m_{\Lambda_b} + m_{\Lambda_c}) g_+(q^2) + (m_{\Lambda_b} - m_{\Lambda_c}) g_-(q^2) \right] u_{I1/2}, \tag{4.11}$$

$$H_V^S = u_{I1/2}^+ \left[\left\{ \frac{g(q^2)}{2 m_{\Lambda_c}} + g_+(q^2) - g_-(q^2) \right\} \vec{K} + i \left(\frac{\vec{\sigma} \times \vec{K}}{2 m_{\Lambda_c}} \right) g(q^2) \right] u_{I1/2}, \tag{4.12}$$

$$H_A^0 = u_{I1/2}^+ \left[\left\{ a(q^2) - (m_{\Lambda_b} + m_{\Lambda_c}) a_+(q^2) \right.\right.$$
$$\left.\left. - (m_{\Lambda_b} - m_{\Lambda_c}) a_-(q^2) \right\} \left(\frac{\vec{\sigma} \cdot \vec{K}}{2 m_{\Lambda_c}} \right) \right] u_{I1/2}, \tag{4.13}$$

$$H_A^S = u_{I1/2}^+ \left[a(q^2) \vec{\sigma} \left(1 + \frac{\vec{K}^2}{8 m_{\Lambda_c}^2} \right) - (a_+(q^2) - a_-(q^2)) \left(\frac{\vec{\sigma} \cdot \vec{K}}{2 m_{\Lambda_c}} \right) \vec{K} \right] u_{I1/2}, \tag{4.14}$$

which in fact when compared with eqs (4.5)–(4.8) yields explicitly to the form factors as

$$g(q^2) = 2 A m_{\Lambda_c} g_c \left(1 - \frac{g_b^2}{(R_{\Lambda_c}^2 + R_{\Lambda_b}^2)} - \frac{1}{2} g_b^2 \beta^2 \vec{K}^2 \right), \tag{4.15}$$

$$g_+(q^2) = \frac{A}{2m_{\Lambda_b}}\left[1 - \frac{1}{2}g_c^2\vec{K}^2 - \left(\frac{2(g_c-g_b)^2 - g_c^2 g_b^2 \vec{K}^2}{4}\right)\left(\frac{2}{R_{\Lambda_c}^2 + R_{\Lambda_b}^2} + \beta^2\vec{K}^2\right)\right.$$
$$+ \frac{g_c^2 g_b^2}{4}\left(\frac{20}{3(R_{\Lambda_c}^2 + R_{\Lambda_b}^2)^2} + \frac{12\beta^2\vec{K}^2}{(R_{\Lambda_c}^2 + R_{\Lambda_b}^2)} + \beta^4\vec{K}^4\right)$$
$$\left. -2m_{\Lambda_c} g_c\left(1 - \frac{g_b^2}{(R_{\Lambda_c}^2 + R_{\Lambda_b}^2)} - \frac{1}{2}g_b^2\beta^2\vec{K}^2\right)\right], \tag{4.16}$$

$$a(q^2) = A\left(1 + \frac{\vec{K}^2}{8m_{\Lambda_c}^2}\right)^{-1}\left[1 - \frac{1}{2}g_c^2\vec{K}^2 - \left(\frac{1}{2}(g_c^2 + g_b^2) + \frac{g_c g_b}{3} - \frac{1}{4}g_c^2 g_b^2 \vec{K}^2\right)\right.$$
$$\left. \times\left(\frac{2}{R_{\Lambda_c}^2 + R_{\Lambda_b}^2} + \beta^2\vec{K}^2\right) + \frac{g_c^2 g_b^2}{4}\left(\frac{20}{3(R_{\Lambda_c}^2 + R_{\Lambda_b}^2)^2} + \frac{12\beta^2\vec{K}^2}{(R_{\Lambda_c}^2 + R_{\Lambda_b}^2)} + \beta^4\vec{K}^4\right)\right], \tag{4.17}$$

$$a_+(q^2) = \frac{A}{2m_{\Lambda_b}}\left\{\left(1 + \frac{\vec{K}^2}{8m_{\Lambda_c}^2}\right)^{-1}\left[1 - \frac{1}{2}g_c^2\vec{K}^2 - \left(\frac{1}{2}(g_c^2 + g_b^2) + \frac{g_c g_b}{3} - \frac{1}{4}g_c^2 g_b^2 \vec{K}^2\right)\right.\right.$$
$$\left. \times\left(\frac{2}{R_{\Lambda_c}^2 + R_{\Lambda_b}^2} + \beta^2\vec{K}^2\right) + \frac{g_c^2 g_b^2}{4}\left(\frac{20}{3(R_{\Lambda_c}^2 + R_{\Lambda_b}^2)^2} + \frac{12\beta^2\vec{K}^2}{(R_{\Lambda_c}^2 + R_{\Lambda_b}^2)} + \beta^4\vec{K}^4\right)\right]$$
$$\left. -2m_{\Lambda_c} g_c\left(1 - \frac{g_b^2}{(R_{\Lambda_c}^2 + R_{\Lambda_b}^2)} - \frac{1}{2}g_b^2\beta^2\vec{K}^2\right)\right\}. \tag{4.18}$$

Next, using the conventional expression for the two body final state decay width as

$$\Gamma_{(\Lambda_b \to \Lambda_c e\bar{\nu})} = \int_{y_{\min}}^{y_{\max}} \frac{G_{\mathrm{F}}^2 |V_{cb}|^2 \vec{K}^2 m_{\Lambda_b}^2 y}{96\pi^3}\left(|H_+|^2 + |H_-|^2 + |H_0|^2\right) \mathrm{d}y, \tag{4.19}$$

one can have the estimation for the same. However, we may note here that while estimating the y-integral in eq. (4.19), one is to express the helicity amplitudes described in eqs (3.13)–(3.16) in terms of the form factors of (4.15)–(4.18) as functions of the lab-frame Λ_c momentum K which subsequently generates a y dependence of the integrand through eq. (3.2b) in the kinematically allowed range. Further, such an integrand does not favor an analytical integration and so to obtain the decay widths one performs a numerical integration.

5. Results and discussions

With the expressions in eqs (4.15)–(4.18) of earlier section, we calculate here dimensionless form factors with the model parameters such as constituent quark masses, harmonic oscillator radii of the baryons taken from earlier applications of the model [2–9] as

$$m_c = 1.8\ \text{GeV}, \quad m_b = 4.5\ \text{GeV}, \quad R^2_{\Lambda_c} = R^2_{\Lambda_b} = 14\ \text{GeV}^{-2},$$
$$g_c = \frac{1}{2m_c} = 0.277\ \text{GeV}^{-1} \quad \text{and} \quad g_b = \frac{1}{2m_b} 0.111\ \text{GeV}^{-1}. \tag{5.1}$$

We take baryon masses and CKM matrix element as the central values of their experimental measurements [14]. The momentum fractions λ_i's in the present calculation taken as the free parameters to be

$$\lambda_1 = \lambda_2 = 0.0725, \quad \lambda_3 = 0.855, \tag{5.2}$$

where SU(2) flavor symmetry has been utilised. With these parameters when we calculate weak decay form factors in dimensionless forms at $q^2 = q^2_{\text{max}}$ and tabulate them in table 1 along with those estimated in other quark models [10,11], we observe a reasonable agreement. We may note that the present model explicitly derives the q^2 dependence of form factors in the allowed kinematic range unlike the pole dominance model [11,13] where one extrapolates from their values at q^2_{max} to obtain a q^2 dependence. However, we have not described them explicitly in the present brief report, rather, we have reported their values at $q^2 = q^2_{\text{max}}$ in table 1 for comparison with other estimations [10,11,15,16]. With the estimated form factors, we evaluate numerically the integral in eq. (4.19) to obtain the decay rate $(\Lambda_b \to \Lambda_c e^- \bar{\nu})$ and the polarisation ratio Γ_L/Γ_T and report them in table 2 along with the experiment [14] and those of refs [10,11].

The present calculation which does not make an explicit use of heavy quark symmetry [17] derives the weak form factors by taking into account the standard model interaction and a nonrelativistic field-theoretic quark model. Like its earlier success, the present model [2,3] has explained this exclusive semileptonic decay in a much simplified and tractable manner due to the harmonic oscillator hadronic wave functions. We may note that even though higher energy is available in the final state, it is surprising to note the success of the nonrelativistic models in explaining such a process. However, in this regard we believe it

Table 1. Form factors for the process $\Lambda_b \to \Lambda_c e^- \bar{\nu}$.

Form factors	Present estimation	Ref. [10]	Ref. [11]
$g(q^2_{\text{max}})$ (GeV)	9.1	8.02	9.0
$g_\pm(q^2_{\text{max}})$	-0.17	-0.15	-0.17
$a(q^2_{\text{max}})$ (GeV)	7.15	7.07	7.1
$a_\pm(q^2_{\text{max}})$	-0.17	0.008	-0.17

Table 2. Decay rate $\Gamma_{\Lambda_b \to \lambda_c e^- \bar{\nu}}$ and polarisation ratio Γ_L/Γ_T.

Decay rate	Present estimation	Ref. [10]	Ref. [11]	Expt.
Γ	3.745×10^{-14} GeV	3.75×10^{-14} GeV	3.88×10^{-14} GeV	3.75×10^{-14} GeV
Γ_L/Γ_T	1.19	-	1.2	1.1

could probably be due to the fact that heavy quark in heavy hadron acts as a source of static color field [10]. Further, we have considered in this brief report only a single process that too with very limited observations. Nevertheless, the extended version of all such baryonic processes is being considered and is to be published elsewhere.

Acknowledgements

The authors are thankful to Prof. N Barik for suggestions and discussions. They are also thankful to K C Roy, P Das and B P Roy for many discussions.

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