Scaling in the Bombay stock exchange index

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Abstract. In this paper we study Bombay stock exchange (BSE) index financial time series for fractal and multifractal behaviour. We show that BSE index time series is monofractal and can be represented by a fractional Brownian motion.

1. Introduction

A lot of activity has been witnessed in recent times to study the nature of financial time series. Econophysics [1–4] is an interdisciplinary field of research in which methods of physics and mathematics are applied to analyse economic systems. Applying physics to financial problems offers fresh look to existing theories of finance. On the other hand, the availability of long term financial data and its behaviour on both at short and long time scale offers new interesting challenges to physicists. Indeed, attempts have been made to look for long range [3,5] and short range power-law correlation [6]. In this paper we will study scaling behaviour of Bombay stock exchange (BSE) index of last one year from 1st January 2000 to 31st December 2000 (shown in figure 1). We will use R/S analysis to look for future trends and search for multifractal or Brownian motion features in its time dependence.

Fractals have been applied in many fields of science like physics, biology, chemical sciences, astrophysics and engineering sciences. However, it is not well known that the concept of fractal originated from the field of economics when Mandelbrot was investigating the price changes dynamics of an open market in 1963 [7]. He found similarity between various charts of market price changes (of cotton price) with different time resolution. He came to the conclusion that such scale invariance could help to characterize many complex phenomena seen in physical sciences. Mandelbrot [2] observed that price movements follow a family of distributions which have high peaks and fat tails. Such distributions are known as stable Paretian which have infinite or undefined variance.
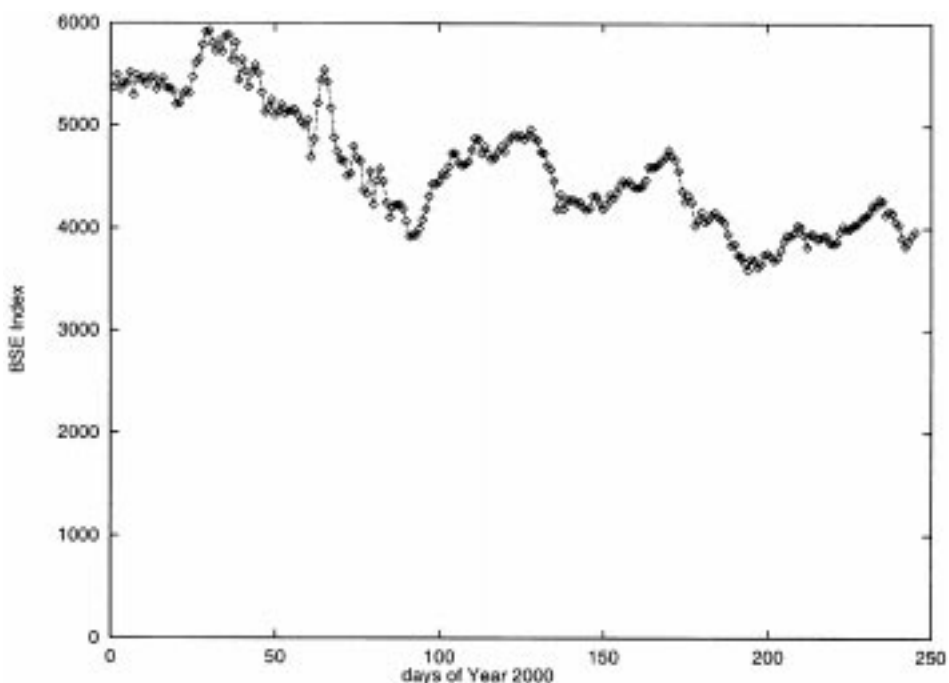
Figure 1. BSE index for the period from 1st January 2000 to 31st December 2000 [27].

2. Stock market returns: A new paradigm

Earlier it has been argued that genuinely competitive stock market, returns, follow random walk model and are normally distributed [8]. Since stock market is a large system and has large degrees of freedom (or investors), there is an underlying assumption that today's change in price is caused only by today's unexpected 'new' information. This means that today's returns have nothing to do with yesterday's behaviour and there are no 'memory' effects i.e. the returns are independent. This leads to the argument that the data of stock prices and returns should follow normal distribution with stable mean and finite variance. Because of this argument, capital market efficiency theories are mainly based on random walk model or classical Brownian motion concepts. This approach means that information arrives to an investor linearly and reaction of an investor to 'linear' information is instantaneous. This is based on the assumption that yesterday's information has been already folded into yesterday's price.

However, the actual market data shows that returns are not normally distributed but have higher peak than theoretically predicted around the mean and have fatter tails. Dow Jones Industrial Index from 1963 to 1993 shows leptokurtic distribution [8]. Apart from Dow Jones other stock exchanges of western countries also show non-normal behaviour [8–11]. The presence of fatter tails indicate 'memory' effects which arise due to nonlinear stochastic processes. Actually the information flow to an investor is clustered and its arrival is irregular rather than continuous and smooth in nature. This clustered and/or irregular

arrival of 'new' information results in periods of low and high volatility [2] which results in 'leptokurtic' distribution instead of normal. This brings in a new paradigm in which reaction of investor or trader to new information is 'nonlinear'. To investigate the validity of this new paradigm the concepts of chaos theory and fractals have been used extensively. Models like ARCH and GARCH [12,13] have been used to include memory effects but these models are not popular (mainly from physics point of view [3]) because they do not take into account scaling property of the process.

3. Brownian motion and fractional Brownian motion concepts

In economics concept of classical Brownian motion [3,9] have been widely used to take into account 'memory effects' which get revealed in the power law behaviour based on random walk model.

Let $x(t)$ be the position of particle (which is random function of time), then for Brownian motion

$$x(t) - x(0) = \zeta |t - t_0|^h. \tag{1}$$

The position $x(t)$ is obtained when $x(t_0)$ is known and by choosing a random number ζ from a Gaussian distribution. Here, $h = \frac{1}{2}$ for classical Brownian motion (bm). In Brownian motion it is not the position of the particle which is independent of its position at another time but it is displacement of a given particle at one time which is independent of its displacement at another time interval. Brownian motion is 'self-affine' by nature [2]. A transformation that scales time and distance by different factors is called affine and behaviour that reproduces itself under affine transformation is called self-affine [2]. Again it was Mandelbrot ([2] and references therein) who introduced the concept of fractional Brownian motion (fbm). Exponent h varies from 0 to 1 in the above equation for fractional Brownian motion. For $h = \frac{1}{2}$, the time series is independent and un-correlated but the distribution may not be Gaussian.

4. Hurst analysis: Search for fractal behaviour

Hurst invented a new statistical method called re-scaled range analysis (R/S analysis) [2,14]. He was interested in developing the design of an ideal reservoir based upon the given record of observed water discharges. Hurst developed a new exponent called Hurst exponent (H) which can classify time series into random or non-random. Hurst exponent is also related to fractal dimension. A measure of a signal 'roughness' is also given by Hurst exponent. The 'roughness' of a profile can be defined by observing how signal amplitudes vary in time (and in space if necessary), in particular the correlation between various amplitude fluctuations. R/S analysis is a method for distinguishing completely random time series from a correlated time series. The analysis begins by finding an average over a chosen time period say τ,

$$\langle Z \rangle_\tau = \frac{1}{\tau} \sum_{t=1}^{\tau} Z(t). \tag{2}$$

Let $X(t)$ be the accumulated departure of the influx $Z(t)$ from the mean $\langle z \rangle_\tau$.

$$X(t,\tau) = \sum_{u=1}^{t} (z(u) - \langle z \rangle_\tau). \tag{3}$$

The difference between the maximum and the minimum accumulated influx X is the range R which is given as

$$R(\tau) = \max X(t,\tau) - \min X(t,\tau). \tag{4}$$

Hurst used the dimensionless ratio R/S, where S is the standard deviation which is given as

$$S = \left(\frac{1}{\tau} \sum_{t=1}^{\tau} (Z(t) - \langle z \rangle_\tau)^2 \right)^{\frac{1}{2}}. \tag{5}$$

Hurst found that the observed re-scaled range, R/S ratio for a time series is given as

$$\frac{R}{S} = \left(\frac{\tau}{2} \right)^{H}, \tag{6}$$

where H is the Hurst exponent. Using the above equation we can find exponent H. If H is between 0.5 and 1, the trend is persistent which indicates long memory effects. This also means that the increasing trend in the past implies increasing trend in the future also or decreasing trend in the past implies decreasing trend in the future also. In contrast to this, if H is between 0 and 0.5 then an increasing trend in the past implies a decreasing trend in the future and decreasing trend in past implies increasing trend in the future. It is important to note that persistent stochastic processes have little noise whereas anti-persistent processes show, presence of high frequency noise.

The relationship between fractal dimensions D_f and hurst exponent H can be expressed as [11]

$$D_f = 2 - H. \tag{7}$$

So by finding Hurst exponent of a financial time series, we can find out the fractal dimension of the time series. When $D_f = 1.5$, there is normal scaling. When D_f is between 1.5 and 2, time series is anti-persistent and when $D-f$ is between 1 and 1.5 the time series is persistent. For $D_f = 1$, time series is a smooth curve and purely deterministic in nature and for $D_f = 1.5$ time series is purely random. Long term correlations of indexes in developed and emerging markets have been studied by using Hurst analysis and detrended fluctuation analysis (DFA) as investigating tools [15]. However, it has been argued by Vandewalle and Ausloos [16] that DFA analysis is better than Hurst scaling for short term time series.

5. Global Hurst exponent: Search for multifractal nature

In general, box counting method is being used for studying multifractal features [17–20] but this method is mainly preferred for problems of spatial nature. However, to study

multifractal nature of financial time series, alternative methods have been suggested [21]. This technique involves calculating qth order height–height correlation function or qth order structure function [21,22] of a normalised time series $y(t_i)$

$$c_q(\tau) = \langle |y(t_{i+r}) - y(t_i)|^q \rangle, \tag{8}$$

where only non-zero terms are considered in the average, taken over all pairs (t_{i+r}, t_i) such that

$$\tau = |t_{i+r} - t_i| \tag{9}$$

and

$$c_q(\tau) \sim \tau^{\eta(q)}, \tag{10}$$

where $q \geq 0$ is the order of moment and $\eta(q)$ is the scale invariant structure function exponent. For $q = 1, H = \eta(1)$ is the 'Hurst' exponent characterising the scaling non conservation of mean. For $q = 2$, we obtain $\eta(2) = \beta - 1$, where β is the slope of the Fourier power spectrum. In general $\eta(q)$ is given by

$$\eta(q) = qH - \frac{C_1}{\alpha - 1}(q_\alpha - q), \tag{11}$$

where $C_1 \leq d$ is an intermittency parameter, d is the dimension of space (here $d = 1$) and α varies between 0 and 2. α is the Levy index. A multifractal process is characterised by a nonlinear behaviour of $\eta(q)$ [23] because of multiplicative cascades whereas those processes which are additive in nature, $\eta(q)$ is linear or bi-linear. For Brownian motion (bm), $\eta(q) = \frac{q}{2}$ and for fractional Brownian motion (fbm), $\eta(q) = qH$ [24,25]. Thus for a purely bm or fbm, $\eta(q)$ is linear, whereas for multifractal nature $\eta(q)$ is nonlinear.

6. Data analysis and results

Following the above discussion, we analyse BSE index data of year 2000 from 1st January to 31st December which gives us 245 data points. Data from time financial series are one dimensional and more simple to analyse than spatial one. For a financial time series there are no holidays or weekends. In order to do R/S analysis data has been divided into 20, 40, ..., 220, 240 parts. The next step is to calculate R/S statistics and plot it against the corresponding sample length on double logarithmic plot as shown in figure 2. The Gaussian asymptotic behaviour of R/S which represents independent random process with finite variances is given in line marked 'b' in figure 2 and can be written as

$$\frac{R}{S} = \left(\frac{\pi \tau}{2}\right)^{\frac{1}{2}}. \tag{12}$$

It is clear from this figure that data fits to the Hurst exponent $H = 0.915$. This value has been obtained by fitting $R/S = (A\tau)^H$ to the observed R/S value. The parameter $A = 0.61$ for the fit given in the line marked 'a' in figure 2. This value of H is an example of persistent behaviour. Fractal dimension for BSE index time series is $D_f = 1.085$. Large values of H,
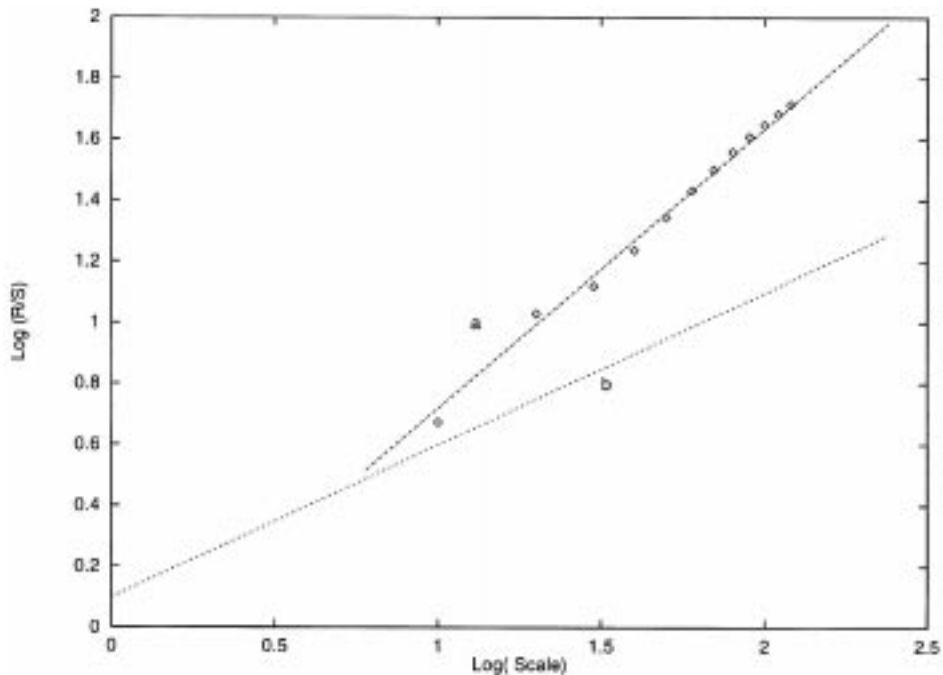
Figure 2. Log (R/S) with log (τ). Curve 'a' corresponds to actual data and curve 'b' to asymptotic theoretical expectation.

have been obtained for many naturally occurring phenomena like monthly sunspot activity. In figure 3 we have plotted $\eta(q) = qH(q)$, with q which has been obtained by calculating structure function. We find that $\eta(q)$ has a linear relationship with q which as discussed above shows that BSE index does not have multifractal nature but it represents a fractional Brownian motion.

7. Conclusions

By using R/S analysis, we are able to find fractal dimension of BSE index. We also observed that trend in BSE index is Persistent for the Year 2000. This means if market is not doing well in the year 2000, persistent trend will continue i.e. in future, market will continue to give low returns. If we see behaviour of BSE from January 2001 onwards, market performance has not been good. The linear behaviour of $qH(q)$ values of BSE index with q shows that signal is monofractal and data follows simple scaling behaviour for these values of q. But it is important to note that we have used data of only one year comprising 240 data points. It will be interesting to look for multifractal features in short term (single day data, but intra-day behaviour) or long term (may be decade or more) data of BSE index. In other stock exchanges such studies have shown multifractal features [26]. It is important to make such studies because market returns have been correlated to [26] multifractal
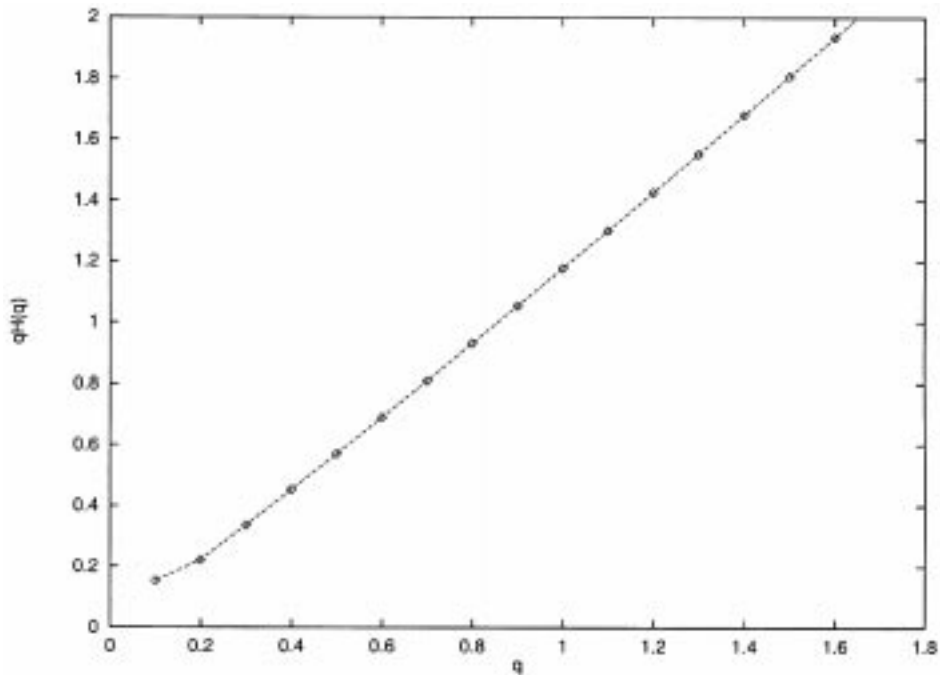
Figure 3. $\eta(q) = qH(q)$, has been plotted against q for BSE index. It shows linear behaviour indicating fractional Brownian motion.

features in the index data, more so when functioning of BSE will be more transparent from July 2001 onwards, with the introduction of new market mechanisms like 'OPITIONS' and 'FUTURES'.

Acknowledgement

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