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High spin properties of ^{124}Ba

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Abstract. The ^{124}Ba nucleus is investigated on the basis of the method of statistical mechanics by assuming the nucleons to move in triaxially deformed Nilsson potential. The variation in the Fermi energies of protons and neutrons is studied as a function of spin and temperature. The Fermi energies determined as a function of angular momentum is used to study the dependence of shell correction on angular momentum using the Strutinsky smoothing procedure. The most important observation is that the shell correction is almost the same for all spins for ^{124}Ba. The spin cutoff parameter and the single particle level density parameter are studied as a function of spin and temperature. Constant entropy lines drawn by plotting the excitation energy against angular momentum are found to be roughly at constant energy above the yrast line and are almost equally spaced. It is observed that no yrast traps are present for ^{124}Ba.

Keywords. Statistical theory; nuclear structure; high spin; shape change.

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1. Introduction

The effect of rotation on nuclear structure [1–3] under extreme conditions of temperature and deformation is very important for studying the intrinsic properties of nuclei. In the last few years considerable interest [4–6] has been shown to study the high spin properties of nuclei in the region $A = 120$. The experimental searches [7–9] for a high spin shape transition in the barium nuclei are inconclusive. A dynamical triaxial deformation is observed [10] in the mass region $A = 120$–140 in particular for xenon and barium nuclei.

In this work hot rotating ^{124}Ba is studied and its structural properties are presented using statistical theory [11]. The Strutinsky smoothing procedure [12] is followed to calculate the shell correction in ^{124}Ba by taking into account the deformation, temperature and spin degrees of freedom. It has been found that the shell nonuniformities in the energy distribution of the nucleons do not disappear in deformed nuclei.

2. The partition function method

The grand canonical partition function [13] for a system of N neutrons and Z protons in a state of total angular momentum M along the direction of the rotation axis is

$$Q(\alpha_Z, \alpha_N, \beta, \gamma) = \sum \exp(-\beta\varepsilon_i + \alpha_Z Z_i + \alpha_N N_i + \gamma M_i). \quad (1)$$

The Lagrange multiplier α_Z, α_N and γ conserve the proton number, neutron number and total angular momentum M along the Z axis for a given temperature $T = 1/\beta$.

In terms of single particle energies ε_i and the spin projections m_i the conservation equations are

$$\begin{aligned}
\langle Z\rangle &= \sum n_i^z = \sum[1+\exp(-\alpha_z + \beta\varepsilon_i^z + \gamma m_i)]^{-1},\\
\langle N\rangle &= \sum n_i^n = \sum[1+\exp(-\alpha_n + \beta\varepsilon_i^n + \gamma m_i)]^{-1},\\
\langle E\rangle &= \sum(n_i^n\varepsilon_i^n + n_i^z\varepsilon_i^z),\\
\langle M\rangle &= \sum(n_i^n m_i^n + n_i^z m_i^z),
\end{aligned} \quad (2)$$

where n_i is the occupation probability of the ith shell. These equations are solved for a given temperature T to obtain the Lagrange multipliers. The applicability of thermodynamical concepts at very high temperatures is known from [14]. It is assumed that all the states with the same excitation energy E are equally populated.

3. Triaxially deformed Nilsson oscillator potential

The single particle energies and the spin projections [15,16] used here correspond to the triaxially deformed Nilsson harmonic oscillator potential [17,18]

$$V = (m/2)(w_x^2x^2 + w_y^2y^2 + w_z^2z^2) - k\hbar\dot{\omega}_0[2l_t\cdot S + (l_t^2 - \langle l_t^2\rangle_N)]. \quad (3)$$

The three oscillator frequencies are given by the equations

$$\begin{aligned}
\omega_x &= \dot{\omega}_0[1-(2\delta/3)\cos(\theta - 2\pi/3)],\\
\omega_y &= \dot{\omega}_0[1-(2\delta/3)\cos(\theta + 2\pi/3)],\\
\omega_z &= \dot{\omega}_0[1-(2\delta/3)\cos\theta],
\end{aligned}$$

with the constraint that the total volume remains constant such that $\omega_x\omega_y\omega_z = \dot{\omega}_0^3 =$ a constant. The constant $\dot{\omega}_0$ is the value of $\omega_0(\delta)$ at $\delta = 0$. The intrinsic nuclear spin is represented by S while l_t represents the orbital angular momentum in the stretched coordinate basis. The k, μ dependence is related to the main oscillator quantum number N rather than A. The k, μ pair used in generating the single particle levels and spins are as in [18] and they are different for different oscillator shells. These parameters are appropriate since they reproduce the experimental proton band head energies [19] more accurately than the standard parameter set given in [20]. However for neutrons the standard parameter set given in [20] is used.

The deformation parameter δ is varied in the range $\delta = 0.0$ to 0.6 in steps of 0.1 for $\theta = +60°$ (oblate shape rotating around the symmetry axis [21]).

The cranking frequency ω is taken to be zero. The required angular momenta are generated by means of statistical theory [11,13] by introducing the Z projection of the angular momentum as a constant of motion through the Lagrange multiplier corresponding to the single particle spins. The levels generated up to $N = 8$ are found to be sufficient for the range of temperatures used in this calculation. Though a complete description of low spin spectrum is possible only with the inclusion of pairing correlation into the picture, we have not included pairing correlation in our calculation since no traps have been found experimentally so far in ^{124}Ba [22].

4. Strutinsky smoothing procedure

The Fermi energies for protons and neutrons are determined for ^{124}Ba as a function of temperature and spins from the Lagrange multipliers α_z and α_n, respectively. In this work the protons have been treated as if they are independent of the neutrons. The Fermi energies determined as a function of angular momentum is used to study the variation of shell correction with angular momentum since the shell effects not only vary with deformation but also with angular momentum for many nuclei. For this study the Strutinsky smoothing procedure is employed. The shell effect may be considered as a small deviation from a uniform distribution and the energy difference between the two nucleon distributions may be expressed as a difference of the single particle energies [12, 23] given as

$$\delta U = U - \tilde{U}. \tag{4}$$

The sum of the single particle energies for the equilibrium deformation is

$$U = \sum_k 2E_k \tag{5}$$

with the sum being over all occupied states. The uniform distribution or the smoothed out shell model energy

$$\tilde{U} = 2\int_{-\alpha}^{\alpha} E g(E) \mathrm{d}E, \tag{6}$$

where the Gaussian smoothed single particle level density is

$$g(E) = (1/\gamma\sqrt{\pi}) \sum_k f_k \exp\{-(E-E_k)/\gamma\}^2. \tag{7}$$

E_k are single particle levels and γ is the width of the smoothing function. The function f_k is retained up to the 6th order as in [23].

5. Spin cutoff parameter

The spin cutoff parameter [11] is estimated from the rotational energy $E_{\rm rot}$ using the relation

$$\sigma^2 = T(I/\hbar^2) \tag{8}$$

as a function of angular momentum for different temperatures. The moment of inertia I in the above expression is calculated from the relation

$$(I/\hbar^2) = M/(\mathrm{d}E_{\mathrm{rot}}/\mathrm{d}M). \tag{9}$$

The rotational energies are calculated using the relation

$$E_{\mathrm{rot}} = E(M,T) - E(0,T) \tag{10}$$

by minimizing the free energy for different δ and θ for a given angular momentum at a particular temperature T.

6. Single particle level density parameter

The single particle level density parameter a is calculated using the expression

$$a(M,T,\delta,\theta) = S^2(M,T,\delta,\theta)/4E^*(M,T,\delta,\theta), \tag{11}$$

where the entropy is obtained from

$$S = -\sum [n_i \ln n_i + (1-n_i)\ln(1-n_i), \tag{12}$$

where n_i is the occupation probability of the ith shell.

7. Excitation energy as a function of angular momentum

The excitation energy as a function of angular momentum for equilibrium deformation is calculated using the relation

$$E^*(M,T,\delta,\theta) = \sum n_i E_i - \sum_{i=1}^{z} E_i \tag{13}$$

with a similar equation for neutron. The summation in the first term in eq. (13) is carried over all the levels generated up to $N = 8$. By subtracting rigid body rotational energy (which increases as a function of angular momentum) from the excitation energy, we define the excitation energy without the rotational energy as

$$E_{\mathrm{exc}} = E(M,T) - E(M,0). \tag{14}$$

This excitation energy without rigid body rotational energy is important in the calculation of shell correction [11].

8. Occupation probability

The occupation probability for the ith shell is given as

$$n_i = 1/1 + \{\exp(-\alpha + \beta E_i + \gamma m_i)\} \tag{15}$$

for protons and neutrons.

In our calculation the temperature is varied from 0.2 to 3 MeV and for each temperature the excitation energy E_{exc} and the entropy are computed by minimizing the free energy for different δ and θ for each spin.

9. Results and discussion

Figure 1 shows the change in neutron Fermi energy with temperature. It is found that at high temperatures the Fermi energy becomes constant above the normal value. At low temperatures the increase of neutron Fermi energy is more for high spins than for low spins. The increase in neutron Fermi energy above the normal value with increasing spin is due to the large energy gap of about 0.9 MeV between the last partially filled level $d_{3/2}$ with two neutrons and the next higher level $h_{11/2}$ for the oblate deformed equilibrium shape corresponding to $\delta = 0.2$ for ^{124}Ba. This shows an interaction between the partially filled $d_{3/2}$ level and the next higher level $h_{11/2}$.

From figure 2 it is evident that at temperature 0.1 MeV the proton Fermi energy increases above the normal value for low spins. As the temperature and the spin increase there is a depression of proton Fermi energy which becomes more pronounced for higher temperatures, which shows an interaction between the $g_{7/2}$ level with 6 protons and the next lower level $g_{9/2}$.

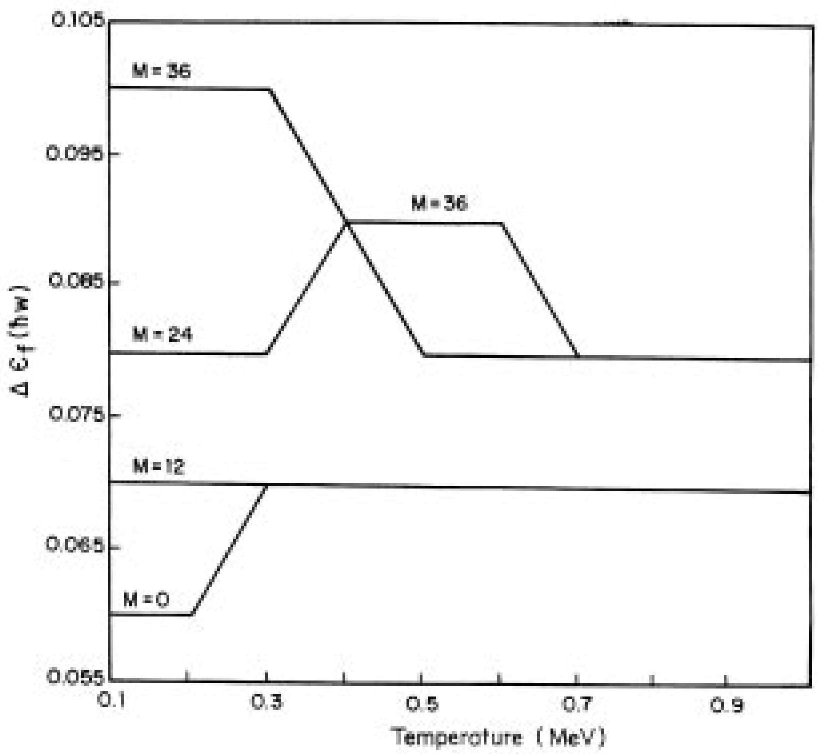

Figure 1. Change in Fermi energy of neutrons as a function of temperature and spin for ^{124}Ba for the equilibrium deformation $\delta = 0.2$ and $\theta = +60°$ corresponding to free energy minimum.

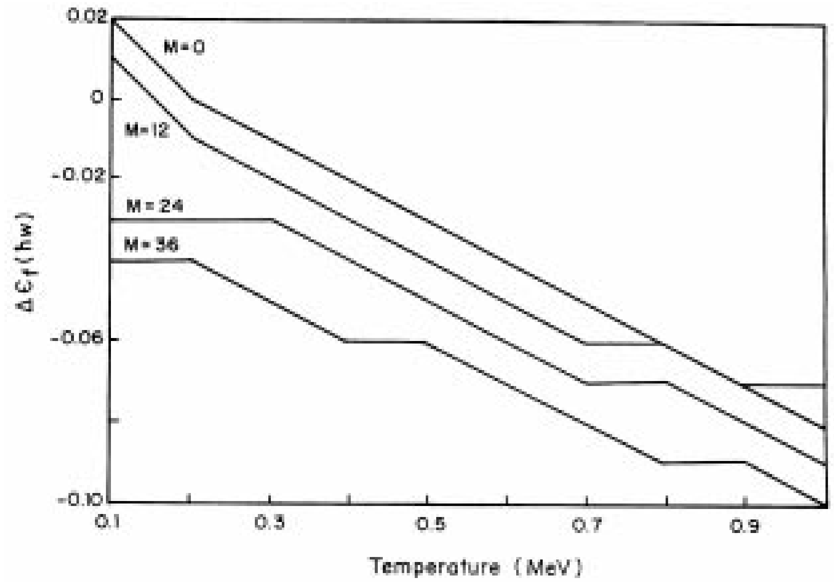

Figure 2. Change in Fermi energy of protons as a function of temperature and spin for ^{124}Ba for the equilibrium deformation $\delta = 0.2$ and $\theta = +60°$ corresponding to free energy minimum.

It is evident that for low temperatures with increase in spin the neutron Fermi energy increases above the normal value whereas the proton Fermi energy decreases below the normal value. As the temperature increases the neutron Fermi energy reaches a steady value whereas the proton Fermi energy continues to be affected. The most economical way of generating a total angular momentum will occur when both neutrons and protons have the same slope of the Fermi surface [22].

The shell correction for ^{124}Ba evaluated by the Strutinsky method as a function of shell smearing parameter γ for neutrons and protons is shown in figure 3. It also shows the total shell correction for this nucleus. It is found that the proton shell correction is almost constant up to $\gamma = 1.4\hbar w$. For γ values greater than $1.4\hbar w$ the shell correction for protons decreases. The neutron shell correction is found to be constant between values 0.8 and 1.2 and is found to increase for higher values. Larger values of γ will lead to contribution to the shell correction from too high levels. Whereas for smaller values of γ it is not possible to define a unique value for the shell level density [12]. This means that the choice of the parameter γ must not be too high and too low and the shell correction as a function of γ is valid only for the intermediate values. It is evident from figure 3 that the shell correction is almost stable for the smoothing parameter values between 0.8 and $1.4\hbar w$. It is to be stressed that the shell correction is independent of angular momentum. The curves depicted in figure 3 correspond to spins from 0 to $36\hbar$. The independence of shell correction on angular momentum is due to the fact that the change in the Fermi energies of protons and neutrons with angular momentum is very small because of the large energy gap between the last filled level and the next higher level.

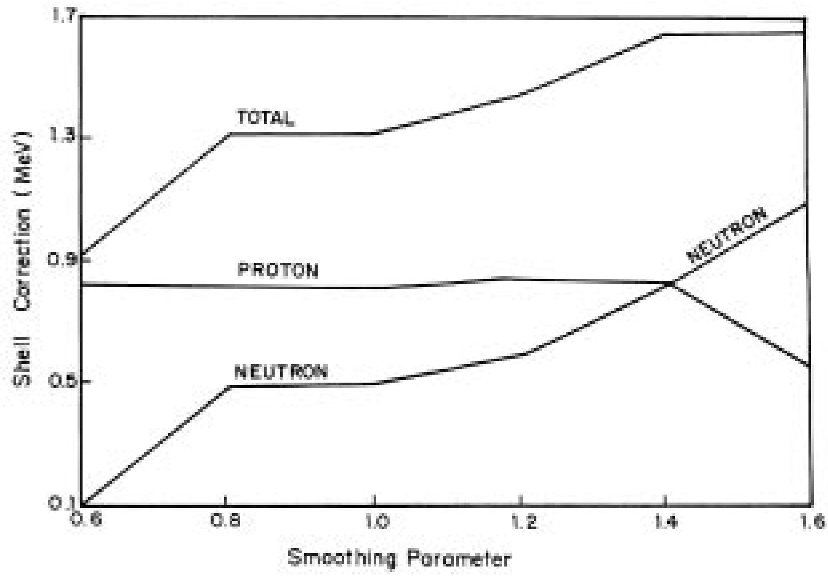

Figure 3. Shell correction as a function of Strutinsky smoothing parameter for protons and neutrons for the equilibrium deformation $\delta = 0.2$ and $\theta = +60°$ for free energy minimum of ^{124}Ba. Total shell correction for ^{124}Ba is also shown. These curves corresponding to spin from 0 to 36$\hbar$, since shell correction is independent of angular momentum for ^{124}Ba.

The spin cutoff parameter as a function of angular momentum for various temperatures is represented in figure 4. It is evident that for low temperatures with increase in spin the spin cutoff parameter decreases up to a spin of 15$\hbar$ and then slightly increases. The spin cutoff parameter at $T = 0.6$ MeV shows fluctuation in the region 30$\hbar$ and 35$\hbar$. This may be suggestive of the favorable shape transition pointed out by Ragnarsson *et al* [4] at and around spin 35$\hbar$. However for higher temperatures the spin cutoff parameter shows a steady decrease with increasing spin.

Figure 5 shows the single particle level density parameter as a function of temperature for various spins in the case of ^{124}Ba. It is found that for high spins the single particle level density parameter increases steeply at low temperatures and reaches the constant value $a \simeq A/10$ predicted experimentally [24,25] at higher temperatures. At zero spin the single particle level density parameter remains almost constant without any fluctuations. It is also found that for a given temperature the single particle level density parameter decreases with increasing spin. However at a very high temperature of 3 MeV it is almost constant for all spins. The nuclear level density is found to increase with excitation energy for all spins and to build up higher spins at given nuclear level density a higher excitation energy is needed.

The excitation energy without rigid body rotational energy as function of angular momentum for constant entropy values is displayed in figure 6. Since the rigid body rotational energy increases with angular momentum, it has been subtracted from excitation energy. It is observed that no yrast traps are present for ^{124}Ba. As the temperature increases the entropy as well as the excitation energy increases and these constant entropy lines are useful in determining the phase space available for the nucleus. These lines are found to be

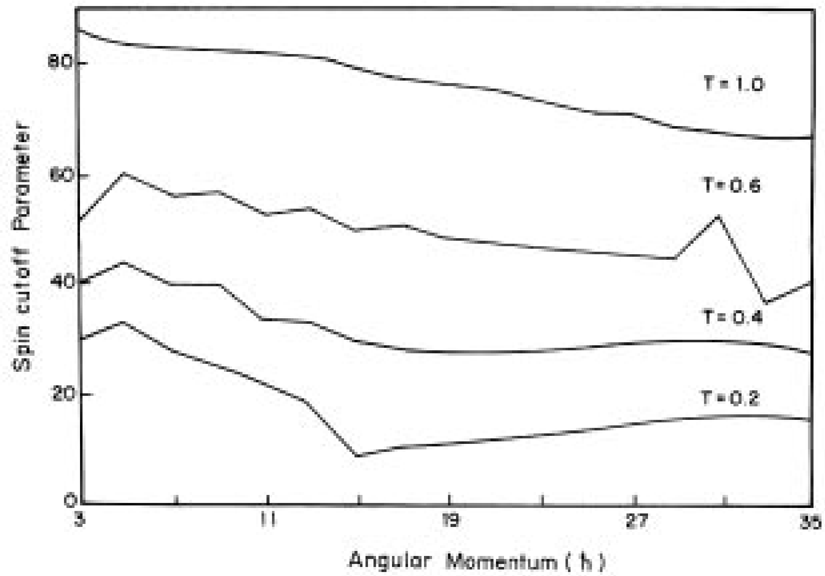

Figure 4. Spin cutoff parameter as a function of angular momentum for different temperatures in ^{124}Ba for the equilibrium deformation $\delta = 0.2$ and $\theta = +60°$ corresponding to free energy minimum.

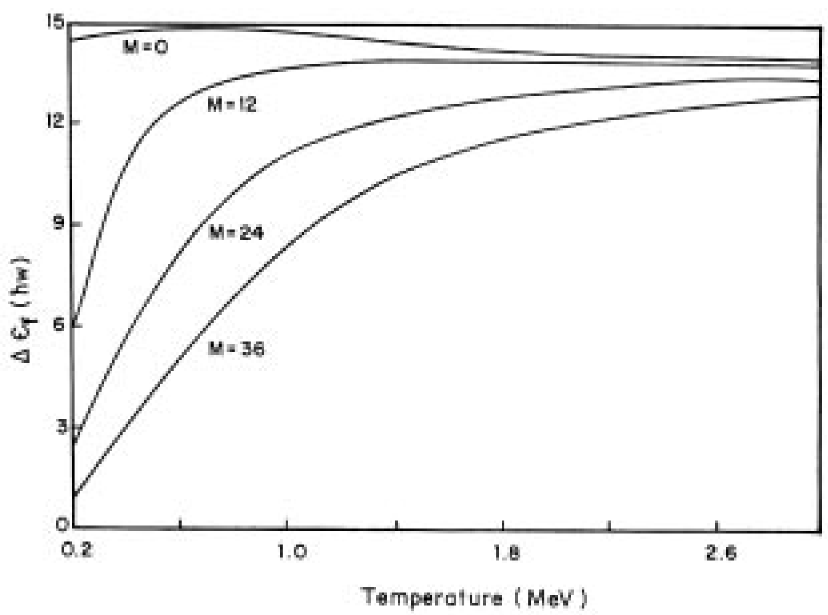

Figure 5. Single particle level density parameter as a function of temperature for various spins for ^{124}Ba. The numbers on the curves refer to the angular momentum of the nucleus for equilibrium deformation corresponding to free energy minimum.

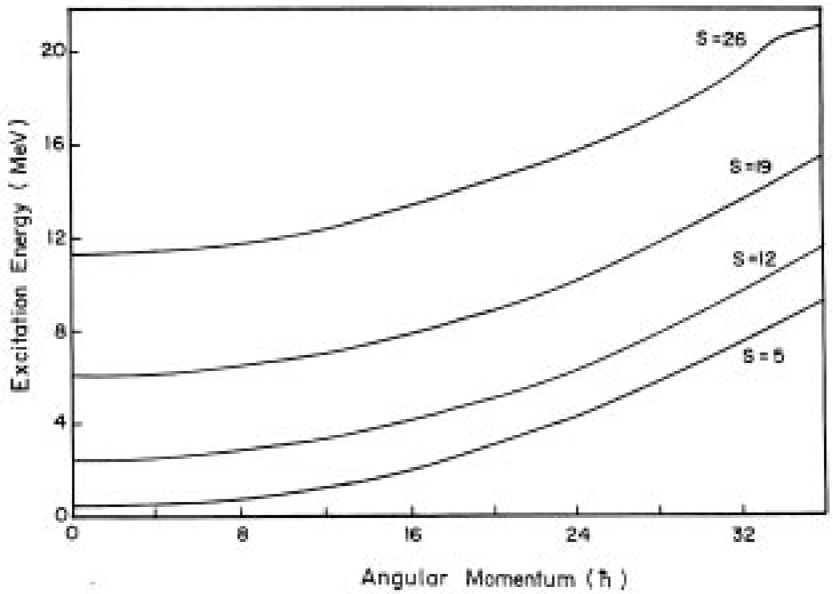

Figure 6. Excitation energy without rigid body rotational energy as a function of angular momentum for constant entropy values for ^{124}Ba for equilibrium deformation corresponding to free energy minimum.

roughly at constant energy above the yrast line as in [26,27]. The absence of yrast traps and the associated backbending effects is mainly due to the fact that the proton number is 56 with 50 protons up to $g_{9/2}$ shell and 6 protons in the next partially filled $g_{7/2}$ shell. This is combined with neutron number 68 with 66 neutrons up to $s_{1/2}$ shell and 2 neutrons in the next partially filled $d_{3/2}$ shell.

10. Conclusion

The ^{124}Ba nucleus is found to have an oblate equilibrium shape at the deformation $\delta = 0.2$ for $\theta = +60^\circ$ rotating around the symmetry axis for all spins at all temperatures.

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