

## Transport in quantum wires

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**Abstract.** With a brief introduction to one-dimensional channels and conductance quantization in mesoscopic systems, we discuss some recent experimental puzzles in these systems, which include reduction of quantized conductances and an interesting *odd–even* effect in the presence of an in-plane magnetic field. We then discuss a recent non-homogeneous Luttinger liquid model proposed by us, which addresses and gives an explanation for the reduced conductances and the *odd–even* effect. We end with a brief summary and discussion of future projects.

**Keywords.** Quantum wires; contact resistance; Luttinger liquids.

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### 1. Introduction

Recent advances in the fabrication of gating of the two-dimensional electron gases formed at the inversion layer of high mobility GaAs–AlGaAs heterostructures have enabled the experimental study [1–7] of electron transport through very few channels or even a single channel, not all of which have been theoretically well-understood. This has caused a tremendous upsurge in the study of quantum wires and Luttinger liquids (LLs) [8].

Early experiments [1] found that the dc conductance through narrow constrictions were quantized with the steps separated by  $\Delta g = \tilde{g}(2e^2/h)$ , where  $\tilde{g} \sim 1$ . This could be understood within the Landauer–Buttiker picture of transport through the scattering matrix approach [9]. In fact, when electron–electron interactions were included via the LL theory, initial calculations [10] expected a reduction in quantized conductance –  $\tilde{g}$  was expected to be the Luttinger parameter, which depended on the strength of the interactions. Later on it was realized [11] that the Fermi liquid leads play a role, and when that was taken properly into account,  $\tilde{g} \sim 1$ , even for a LL wire.

However, in the last few years it has been found in several experiments [2–7] that  $\tilde{g} < 1$  and in fact, varies, depending on the temperature  $T$  and the length of the wire  $L$ . However, conductance quantization or plateaux were still seen, indicating that the reduction was not due to impurities, and moreover, the reduction was found to be uniform in all the channels. Also, the flatness of the plateaux appeared to indicate an insensitivity to the electron density in the channel. Besides this, when an external magnetic field was placed in-plane and parallel to the channels [5], a splitting of the conductance steps was observed together with an odd–even effect of the renormalization of the plateau heights, with the odd

and even plateau heights being renormalized by smaller and larger amounts respectively. This motivated us to study the LL model of a quantum wire, with Fermi liquid leads and the unusual feature of additional lengths of short LLs modeling the contacts between the leads and the wire.

This paper is organized as follows: In §2, we review conductance quantization of one-dimensional channels in mesoscopic systems. In §3, we give a brief description of some recent experimental results and some of the puzzles mentioned above, such as reduction of quantized conductance even in clean quantum wires and a novel odd–even effect in the presence of a magnetic field. In §4, we give a brief introduction to Luttinger liquids (LLs) and bosonization and transport in these models. In §5, we describe our model, which is essentially an inhomogeneous LL model, with five distinct regions, the two *leads* and *contacts* and the central *wire* and having two barriers at the boundaries between the contacts and the leads and explain how this model is motivated by the experimental results. We then describe the results of our calculations with the model, showing reduced conductances as a function of temperature, and lengths of the wire and the contacts. In §6, we show that in the presence of an in-plane magnetic field, the barriers show different renormalizations for the odd and even plateau leading to the odd–even effect. Finally, we end in §7, with a brief summary and possible future extensions of our work.

## 2. Introduction to one-dimensional channels and conductance quantization

Semiconductor mesoscopic conductors are fabricated by confining electrons to a two-dimensional conducting layer formed at the interface between GaAs and GaAlAs. The electrons are free to move in the  $x$ - $y$  plane, but are confined by some potential in the  $z$ -direction. Their dispersion is given by  $E = E_n + \frac{\hbar^2}{2m}(k_x^2 + k_y^2)$ . The index  $n$  labels different sub-bands. At low temperatures and low carrier densities, only the lowest sub-band  $n = 0$  is occupied. So we can ignore the  $z$ -dimension altogether and treat the electron gas as a two-dimensional electron gas (2DEG) in the  $x$ - $y$  plane.

Now, we apply strong confining potential in the  $y$ -direction. If the dimensions of the conductor were large, then the conductivity of the sample would be given by  $g = \sigma W/L$ , where  $\sigma$  is the conductivity of the material and  $W$  and  $L$  are the widths and lengths of the sample. But as the length is reduced, experimentally, it was found that the conductance reaches a limiting value  $g_c$  instead of increasing indefinitely. The reason for this is that there exists a non-zero resistance at the interface between the sample and the leads, which is unavoidable. This is called the contact resistance and for very clean samples, at low temperatures, it was found to be a universal quantity.

This can be very easily understood, using Landauer's picture [9] of electron transport as a scattering problem. Consider applying a strong confining potential in the  $y$ -direction, so that the dispersion is given by  $E = E_0 + \epsilon_m + \hbar^2 k_x^2/2m$ . For each value of  $m$ , we have a different sub-band. The spacing between sub-bands is fixed by the confining potential; the stronger the confinement, the farther apart the sub-bands. In actual experiments, confinement in the  $y$ -direction is controlled by a gate voltage. We have a quasi-one-dimensional wire when only a few sub-bands are occupied.

Let us now calculate the conductance through each sub-band or channel of the wire for an applied voltage bias  $\mu_2 - \mu_1$ . The current  $I$  is given by  $I = (1/L) \sum_k e v = (e/L) \sum_k (\partial E / \partial \hbar k)$  because the density of the electrons for each  $k$  state is  $1/L$ . Since

$\Sigma_k = 2 \times (L/2\pi) \int dk$ , we get

$$I = \frac{2e}{h} \int_{\mu_1}^{\mu_2} dE = \frac{2e^2}{h} \frac{(\mu_2 - \mu_1)}{e}. \quad (1)$$

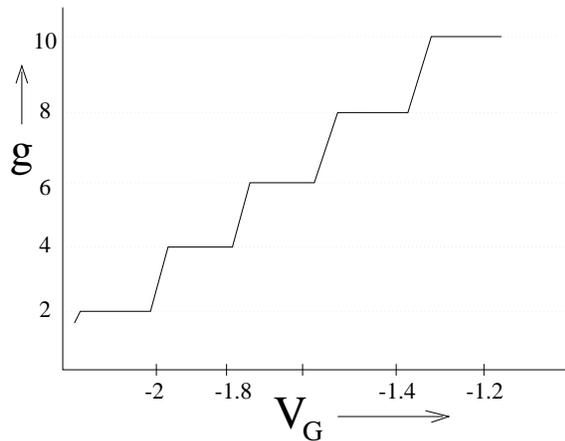
Hence, the minimum contact resistance per channel is given by  $h/2e^2$ . For  $M$  channels, the same argument gives  $I = (2e^2/h)M((\mu_2 - \mu_1)/e)$ , which implies that the contact resistance =  $h/2e^2M$ . Since a macroscopic conductor has a very large number of channels, for macroscopic conductors the contact resistance is negligible. But for a single channel,  $R_c = 12.9 \text{ k}\Omega$ , and is certainly not negligible.

This contact resistance has been experimentally measured [1]. A gate voltage controls the density of electrons in the channel and narrows down the constriction progressively. The current is plotted as a function of the gate voltage. Although the width of the constriction changes continuously, the current goes down in steps as seen in figure 1. So starting with current through a single channel, the current increases each time a sub-band comes below the Fermi level. When  $M$  sub-bands are filled,  $g = 2Me^2/h$  – i.e., the steps are quantized at multiples of  $2e^2/h \equiv g_0$ . For very precise quantization, one needs ultra-clean samples where the only source of resistance is the contact resistance.

This quantization can be shown to be true even when the electrons are interacting. That is not a surprising result when one realises that the resistance that is being measured here is purely a contact resistance and does not depend upon the interior of the wire.

### 3. Brief description of recent experimental results

Here, we give a brief summary of the experimental results on transport in quasi-one-dimensional channels in the last few years. One of the main features that has been found is the reduction of quantized conductances from  $N \times g_0$ . Flat plateaux, independent of



**Figure 1.** Quantized conductance of a ballistic wire. A negative gate voltage is applied to deplete the electrons in a narrow channel. The conductance  $g$  is a measure in units of  $e^2/h$ .

the gate voltage are still seen in each channel, but the quantization is now at some value below  $N \times g_0$  (see, for instance, figures 2 and 3 in [3]). This has been noticed by several groups [2–6] and the variation of the reduction as a function of the temperature and the length of the quantum wire has also been measured. Tarucha *et al* [2] performed experiments with wires of lengths  $2 \mu\text{m}$  to  $10 \mu\text{m}$  fabricated using split-gate methods at temperatures from 0.3 K to 1.1 K, and found deviations from the perfect quantization of the steps. Yacoby *et al* [3] made measurements on a  $2 \mu\text{m}$  wire and at temperatures ranging from  $T = 0.3 \text{ K}$  to  $T = 25 \text{ K}$  and they found that the reduction is surprisingly uniform for each channel. They also found that the step heights were increasingly renormalized at lower temperatures. Similar experiments [4–6] studied the variation in the reduction as a function of both length and temperature and confirmed that the step heights were increasingly renormalized when either the temperature was lowered (for a fixed length of quantum wire) or the length of the quantum wire was increased at a fixed temperature. Such renormalizations would require backscattering of electrons. If these backscatterings were due to impurities within the quantum wires, the conductance corrections would be gate-voltage dependent as shown in our calculations. This certainly can not lead to flat conductance plateaus as seen in the experiments.

In the presence of a magnetic field, Liang *et al* [5] found that as they turned up the external magnetic field (kept in plane and aligned along the direction of the channel) from 0 to 11 T, the spin degeneracy gets lifted and each conductance step splits into two steps, with the heights of both being less than  $g_0$ . At a magnetic field strength of 11 T, they found that the difference between the conductance of successive pairs of spin-split sub-bands alternates. This shows that the conductance of the odd numbered spin-split sub-bands containing the moments aligned with the magnetic field undergoes little renormalization, while the conductance of the even numbered spin-split sub-bands containing the moments anti-aligned with the magnetic field undergoes a large renormalization. This is the odd–even effect.

The naive non-interacting Landauer explanation for the renormalization of the plateaus would require back-scattering of electrons due to impurities within the quantum wires, which could lead to  $g = g_0 \times N \times T$  where  $T$  is the transmission coefficient. But  $T$  is a function of the energy  $E$  of the electrons, which in turn, is related to the density of the electrons in the channel (which is controlled by the gate-voltage  $V_G$ ). Thus, the conductance corrections would be gate-voltage dependent. This certainly can not lead to flat conductance plateaus as seen in the experiments. To obtain flat plateau renormalizations, one would have to postulate that  $T$  is independent of  $E$ , which is very unlikely. Moreover, the conductance corrections depend on temperature. This is hard to arrange within the usual non-interacting Landauer model of conductance corrections. Neither is the odd–even effect easy to see within the standard picture.

#### 4. Introduction to Luttinger liquids and bosonization

The standard paradigm for many-fermion systems is the Fermi liquid theory. The Fermi gas is a collection of non-interacting fermions, filled up to the Fermi level. Excitations over the ground state are quasiparticles (above the Fermi surface) and quasiholes (below the Fermi surface), which have the same quantum numbers as that of the original electrons or holes. The idea behind the Fermi liquid (FL) theory is that interactions can change

the ground state, modify the excitations and their energies and so on, but essentially, one continues to have single-particle fermion like excitations even after the inclusion of the interactions. These excitations (called Landau quasiparticles) can have their masses, couplings, etc, renormalized, but basically each state is in one-to-one correspondence with the non-interacting states.

In three dimensions, most electronic phenomena (with some exceptions like the two-channel Kondo problem) can be understood within the framework of FL theory. In two dimensions, there exist some phenomena, where it is not clear whether FL theory is really applicable. For instance, many people believe that high  $T_c$  superconductivity needs non-FL behavior. Also, the  $\nu = 1/2$  state in FQHE is probably an example of a non-FL.

But in one dimension, it is well-known that the FL theory breaks down, and the ground state is a non-trivial state called the *Luttinger liquid* (LL) state, which no longer has quasiparticles similar to that of the non-interacting case. Why is one dimension different? The reason is that the Fermi surface is reduced to two points. Hence, low-energy particle-hole pair excitations have their energy fixed by momentum and vice-versa (see figure 2) and consequently, the particle-hole mode propagates coherently as a new bosonic particle. All low energy modes are exhausted by these bosonic modes; hence, the fermionic theory can be rewritten in terms of the bosonic fields using a procedure called bosonization.

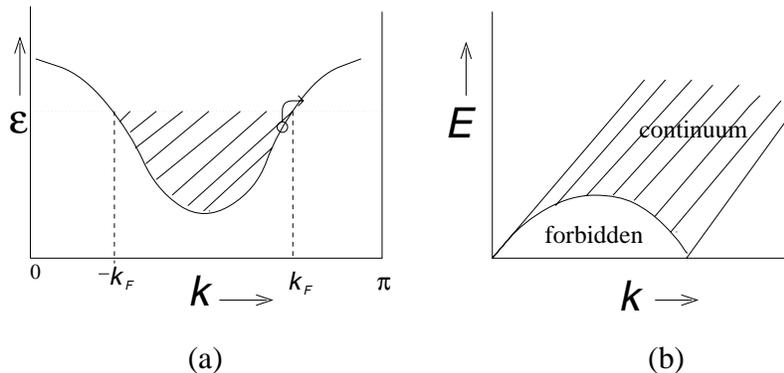
An interesting point is that it is possible to solve a non-trivial fermion model by recasting in boson language. In fact, even when the bosonic model is not exactly solvable, bosonization turns out to be a useful tool to compute correlation functions, which can then be supplemented with renormalization group (RG) arguments to get useful information.

#### 4.1 Bosonization

The explicit relation relating bosons and fermions is given by

$$\psi_R \sim \frac{\eta_R}{\sqrt{2\pi}} e^{-2i\sqrt{\pi}\phi_R} \quad (2)$$

for a right-moving fermion  $\psi_R$  and a right-moving boson  $\phi_R$  and similarly for left-movers.  $\eta_R$  is a Klein factor which takes care of the anti-commutation property of fermions.



**Figure 2.** (a) Single particle spectrum of the Fermi gas in one dimension. (b) Particle-hole pair spectrum with the forbidden region and the continuum.

For any interaction, one can show that the velocities of the spin and charge modes are different; so generically, a LL state has spin-charge separation. Also, instead of a pole in the single particle propagator, even when interactions are included, as one would expect for a FL, here one finds that all correlation functions have a power law fall-off, with the power being dependent on the interaction parameter. These anomalous exponents in various correlation functions or response functions are the hallmark of the LL behavior.

A generic LL model is given by

$$H = \frac{v}{2} \int dx \left[ K\Pi^2 + \frac{1}{K}(\partial_x\phi)^2 \right]. \quad (3)$$

For non-interacting fermions  $K = 1$  and  $v = v_F$ . For repulsive interactions  $K < 1$ . All low energy properties can be found in terms of  $K$  and  $v$ . But one problem with LL models is that contact with the microscopic theory is not obvious.

#### 4.2 Transport and conductances in LL

In this subsection, we study transport, in particular the dc (or zero frequency) conductivity in clean one-dimensional wires with no impurities or barriers.

*Conductance of a clean LL.* First, we shall perform a calculation to compute the conductance of a LL without any consideration of contacts or leads. By computing the conductance using the current-current correlation functions in the Kubo formula, we can show that the conductance of a clean Luttinger wire (with no leads) is given by

$$g \equiv \frac{I}{V} = \frac{Ke^2}{2\pi}. \quad (4)$$

But when we include leads modeled as semi-infinite Fermi liquids on either side (i.e., we study the same bosonic model, but with  $K$  varying spatially –  $K = K_L = 1$  for  $x < 0$  and  $x > D$ , where  $D$  is the length of the wire, and  $K = K_W$  for  $0 < x < D$ ), then we find that

$$g = \frac{K_L e^2}{2\pi} = \frac{e^2}{2\pi}. \quad (5)$$

This is not a surprising result. As we have explained before, the resistance is only due to the contacts. So whether the electrons in the wire interact or not, it is not relevant to the conductance.

*Transport through single impurity.* Here, we study the conductance through a single impurity. We will see that interactions change the picture dramatically. For a non-interacting one-dimensional wire, from just solving usual one-dimensional quantum mechanics problems, we know that we can get both transmission and reflection depending on the strength of the scattering potential. But for an interacting wire, we find that for any scattering potential, however small, for repulsive interactions between the electrons, there is zero transmission and full reflection (implies conductance is zero, or that the wire is ‘cut’) and for attractive interactions between electrons (which is of course possible only for some renormalized ‘effective’ electrons), there is full transmission and zero reflection (implying perfect conductance or ‘healing’ of the wire).

*Note:* The above is true for  $T = 0$  and for  $L \rightarrow \infty$ , where  $T$  is the temperature and  $L$  is the length of the wire. For finite temperature and lengths, there will be calculable corrections to the above results, which, in fact, is what we shall explicitly compute in our model.

Let us introduce a single impurity (a barrier or constriction, or even an experimentally introduced strong bias at a point along the wire) in the quantum wire. We can model this by a potential  $V(x)$  around origin and write the Hamiltonian for the system as

$$\begin{aligned} \delta H &= \int dx V(x) \psi^\dagger(x) \psi(x) = -\lambda [\psi_R^\dagger(x=0) \psi_L(x=0) + \text{h.c.}] \\ &= \lambda \cos 2\sqrt{\pi} K \phi(0). \end{aligned} \quad (6)$$

In the weak barrier limit, the renormalization group (RG) equation for the barrier strength gives

$$\frac{d\lambda}{dl} = (1 - K)\lambda. \quad (7)$$

Thus, the perturbation is relevant when  $K < 1$ , and the strength of the potential grows as we renormalize down to lower temperatures or longer lengths. At  $T = 0$  or  $L \rightarrow \infty$ , this implies that the wire is cut.

Since the RG equation is perturbative in  $\lambda$ , it is necessary to look at the strong barrier limit and ask what happens when we start from a cut wire and allow for hopping. Here, the perturbation term or the hopping term is given by

$$\delta H = -t [\psi_\lessdot^\dagger(x=0) \psi_\gtrdot(x=0) + \text{h.c.}] = t \cos \frac{2\sqrt{\pi}}{K} \theta(0), \quad (8)$$

and the RG equation (perturbative in  $t$ ) is given by

$$\frac{dt}{dl} = \left(1 - \frac{1}{K}\right)t. \quad (9)$$

For  $K < 1$ ,  $t \rightarrow 0$  – i.e., the hopping term is irrelevant and the wire remains ‘cut’.

In either case, the conductance is zero. For finite  $T$  and  $L$ , there are temperature- and length-dependent corrections.

## 5. Our one-dimensional LL model

One might expect that the simplest model to obtain renormalizations of the quantized conductances would be a LL model between two FL leads, with barriers at the junctions between the leads and the wires. However, the problem in such a model is that the velocity of the electron in the wire is set by  $v_w = \sqrt{v_F^2 - 2E_s/m}$  where  $E_s$  are the discrete energy levels of the confinement potential set by the gate voltage  $V_G$ . As  $V_G$  changes, the density of electrons in the channel varies and  $E_s$  clearly depends on the channel number  $s$ . These velocities are, hence, density and channel dependent. The interaction parameter is also expected to be density and channel dependent. Hence, we cannot expect flat and channel independent renormalizations of the conductance plateau, using such a model.

Instead, in our model for the quantum wire, we include two additional contact regions modeling the change from the 2DEG to the 1DEG. The contacts themselves are chosen to be short lengths of LL wire. Since, interactions are expected to be important in this region, we take  $K = K_C$ . But the important point is that this region is independent of the gate voltage, so no 1D bands are formed and  $v_C$  is set by  $v_F$  and interactions in the contact, and is not influenced by the gate voltage that controls the density in the wire.

In fact, Yacoby *et al* [7] established the existence of  $l_{2D-1D}$ , which is the length of the scattering region between the 2D leads and the 1D wire. They found that a length  $l_{2D-1D} \sim 2 - 6 \mu\text{m}$  was needed to cause backscattering. Such a scattering length existed in their original experiment as well and was responsible for reduction from ideal quantized conductance value. Hence, our model that a finite contact region is needed to obtain the smooth transition from the 2DEG to the 1D wire is physically quite reasonable.

So our model for the quantum wire, motivated both by the way the wire is constructed as well as by the experimental results, is a LL model, but in five pieces. The external 2DEG reservoirs are modeled as two semi-infinite FL leads, while the contacts are modeled as short quantum wires with the junctions at either end modeled as  $\delta$ -function barriers. The inter-electron interactions in the system, and hence the parameter  $K$  which characterizes the interactions, vary abruptly at each of the junctions. Hence, we study a  $K_L$ - $K_C$ - $K_W$ - $K_C$ - $K_W$  model. The barriers at the junctions are because changes in the geometry (if not adiabatic) and also in the interaction parameters do lead to barrier-like potentials at the junctions. The important point is that  $v_C$  and  $K_C$  are independent of gate voltage. We assume weak barriers at the contact-lead junction and negligible barriers at the contact-wire junction, because the change in geometry and interaction strengths are likely to be stronger at the contact-wire junction.

Hence, the model can be written as

$$S_0 = \int dt \left[ \int_{-\infty}^0 dx \mathcal{L}_1 + \int_0^d dx \mathcal{L}_2 + \int_d^{l+d} dx \mathcal{L}_3 + \int_{l+d}^{l+2d} dx \mathcal{L}_2 + \int_{l+2d}^{\infty} dx \mathcal{L}_1 \right],$$

where

$$\begin{aligned} \mathcal{L}_1 &= \mathcal{L}(\phi_\rho; K_L, v_L) + \mathcal{L}(\phi_\sigma; K_L, v_L), \\ \mathcal{L}_2 &= \mathcal{L}(\phi_\rho; K_{C\rho}, v_{C\rho}) + \mathcal{L}(\phi_\sigma; K_{C\sigma}, v_{C\sigma}), \\ \mathcal{L}_3 &= \mathcal{L}(\phi_\rho; K_{W\rho}, v_{W\rho}) + \mathcal{L}(\phi_\sigma; K_{W\sigma}, v_{W\sigma}) \end{aligned}$$

with

$$\mathcal{L}(\phi; K, v) = \frac{1}{2Kv} (\partial\phi/\partial t)^2 - \frac{v}{2K} (\partial\phi/\partial x)^2. \quad (10)$$

The action for the coupling to the gate voltage, as well as the junction-barrier terms, are given by

$$\begin{aligned} S_{\text{gate}} + S_{\text{barrier}} &= \frac{eV_G}{\sqrt{\pi}} \int dt [\phi_{3\rho} - \phi_{2\rho}] + \frac{V}{2\pi\alpha} \sum_i \int dt [\cos(2\sqrt{\pi}\phi_{1i}) \\ &\quad + \cos(2\sqrt{\pi}\phi_{4i} + \eta)], \end{aligned} \quad (11)$$

where  $i$  is summed over  $\uparrow, \downarrow$ ,  $\alpha$  is a short distance cutoff and  $\eta$  in terms of the wave numbers in the contact and wire regions is given by  $\eta = 2k_{\text{C}}d + k_{\text{W}}l$ .

Once we have the above action, there exists a straightforward method to study it. We integrate out the bosonic degrees of freedom everywhere except at  $x = 0, d, l + d$  and  $x = l + 2d$  and study the effective action in various limits. Here, we just quote the results [12,13] that we obtain in the various limits.

• *High temperature or high frequency limit*

The Matsubara frequencies  $\bar{\omega}_n$  are quantized in multiples of the temperature as  $\bar{\omega}_n = 2\pi n k_{\text{B}}T$  and hence the temperature is equivalent to frequency. When  $T \gg T_d \equiv v_{\text{C}\rho}/(2\pi k_{\text{B}}d)$  and  $T_l \equiv v_{\text{W}\rho}/(2\pi k_{\text{B}}l)$ , the conductance correction is given by

$$g = \frac{2e^2}{h} K_{\text{L}} [1 - c_1 T^{2(K_{\text{eff}} - K_{\text{L}})} (|V(0)|^2 + |V(l + 2d)|^2)]. \quad (12)$$

Here,  $c_1$  is a dimensionful constant depending on  $v_{\text{C}\rho}$  and the cutoff  $\alpha$  (but independent of all factors depending on gate voltage  $V_{\text{G}}$ ) and

$$K_{\text{eff}} = \frac{K_{\text{L}} K_{\text{C}\rho}}{K_{\text{L}} + K_{\text{C}\rho}} + \frac{K_{\text{L}} K_{\text{C}\sigma}}{K_{\text{L}} + K_{\text{C}\sigma}} = \frac{K_{\text{C}\rho}}{1 + K_{\text{C}\rho}} + \frac{1}{2}. \quad (13)$$

The only parameters are  $c_1$  and the interaction parameter  $K_{\text{C}\rho}$  because  $K_{\text{C}\sigma} = 1$  in the absence of a magnetic field. For the Yacoby *et al* experiment,  $l_{2\text{D}-1\text{D}} = 6 \mu\text{m}$ , which means that  $T_d, T_l \sim 0.1 - 0.2 \text{ K}$ , so that all their results are in this regime. Thus, if we assume that their  $l_{2\text{D}-1\text{D}}$  is what we are calling the contact length  $d$ , then, this result explains why they get flat plateau renormalizations and renormalizations independent of the channel, since  $K_{\text{c}}$  and  $v_{\text{c}}$  are independent of the gate voltage. From this analysis, one expects a similar order of magnitude for the contact length  $l_{2\text{D}-1\text{D}}$  even in the Tarucha *et al* and Liang *et al* experiments. In fact, our results also tell us that as the temperature  $T$  is raised, the conductance corrections get smaller and approaches integer multiples of  $g_0$ , as indeed seen in the experiments.

Note that our results crucially depend on introduction of contact region and placing of the barriers between the contact and the lead. In fact, the barriers can be anywhere in the contact, as long as they are not too close to the wire region, and we still get the same result. Essentially, by modeling the contacts as short LL's, we have included many-body effects at the contacts and shown that they are responsible for the observed conductances.

• *Intermediate temperatures*

Here, there are two possible scenarios. Either, we have (a)  $T_l \ll T \ll T_d$  (if  $d \ll l$ ), or we have (b)  $T_d \ll T \ll T_l$  (if  $l \ll d$ ).

For scenario (a), which we call the wire regime,

$$g = \frac{2e^2}{h} K_{\text{L}} [1 - c_2 T_d^{2(K_{\text{eff}} - \tilde{K}_{\text{eff}})} T^{2(\tilde{K}_{\text{eff}} - K_{\text{L}})} (|V(0)|^2 + |V(l + 2d)|^2)], \quad (14)$$

where

$$\tilde{K}_{\text{eff}} = \frac{K_{\text{L}} K_{\text{W}\rho}}{K_{\text{L}} + K_{\text{W}\rho}} + \frac{K_{\text{L}} K_{\text{W}\sigma}}{K_{\text{L}} + K_{\text{W}\sigma}} = \frac{K_{\text{W}\rho}}{1 + K_{\text{W}\rho}} + \frac{1}{2}. \quad (15)$$

Here, clearly, the conductance depends on the wire parameters and hence on the gate voltage; in this regime, we would not expect flat and channel independent plateau renormalizations.

Scenario (b), which we call the quantum point contact limit, however, has channel independent flat plateau renormalizations both for the high and intermediate temperature regimes.

• *Low temperatures*

Here,  $T \ll T_d$  and  $T_l$  and in this regime, the conductance is given by

$$g = \frac{2e^2}{h} K_L [1 - c_3 T^{2(K_L-1)} T_d^{2(K_{\text{eff}}-\bar{K}_{\text{eff}})} T_l^{2(\bar{K}_{\text{eff}}-K_L)} |V(0) + V(l+2d)|^2] \\ = \frac{2e^2}{h} [1 - c_3 T_d^{2(K_{\text{eff}}-\bar{K}_{\text{eff}})} T_l^{2(\bar{K}_{\text{eff}}-1)} |V(0) + V(l+2d)|^2]. \quad (16)$$

In this regime, there is no temperature dependence. The conductance correction only depends on  $d$  and  $l$ , since they take over the role of the infra-red cutoff. As  $d$  and  $l$  decrease, corrections become smaller and  $g$  approaches  $g_0$ . In this case, there also exists the possibility of resonant transmission, since the barriers are seen coherently. The conductance corrections in this regime depends on wire parameters and hence gate voltage, and so one would not expect flat plateau renormalizations. So a concrete prediction of our model would be that at very low temperatures, the conductance corrections will not be flat and moreover, there could be resonances in the transmission due to the two barriers being seen coherently. A study of the experimental graphs of Reilly *et al* [14] leads us to speculate that perhaps such resonances have already been seen.

## 6. Model in a magnetic field

In this section, we will study the effects of an in-plane magnetic field on the conductivity of a quantum wire. Since, orbital motion is not possible in an in-plane magnetic field, we will only consider the effect of the Zeeman term. This term couples differently to spin-up and spin-down electrons; here up and down are defined with respect to the direction of the magnetic field which may or may not be parallel to the quantum wire. Thus the  $SU(2)$  symmetry of rotations is explicitly broken and the spin and charge degrees of freedom do not decouple any longer.

For very strong magnetic fields ( $\gg 16$  T) (Zeeman energies  $\gg$  inter-sub-band energies), all spins are polarized and conductances are quantized as  $e^2/h$  and the wire is essentially described as a spinless LL.

But when the magnetic field is not that large, but still large ( $8 \text{ T} \ll b \ll 16 \text{ T}$ ), each sub-band splits into two – a spin  $\uparrow$  sub-band and spin  $\downarrow$  sub-band. We might expect barrier renormalizations for both the sub-bands to be the same. But it turns out that the barrier renormalization for spin parallel (moments aligned with the magnetic field) band is much smaller than that for the spin anti-parallel (or anti-aligned moments) band. Hence, the conductance correction is much more for even bands and negligible for odd bands.

In fact, this odd–even effect even exists when the magnetic field is not large enough to completely spin-split the bands. Hence, one would expect that the electrons with moments

aligned with the magnetic field go through the quantum wire more easily than the electrons which are anti-aligned.

This odd–even effect, in fact, has been experimentally seen [5], and perhaps could be instrumental in constructing a spin-valve using these wires [13].

## 7. Summary and future prospects

In this paper, we have introduced a non-homogeneous LL model for the quantum wire, motivated by the experimental results. The main features of the experiments that we have explained using our model are:

- *Flat plateau renormalization*

This was essentially done by modeling the contact regions as LLs with barriers between the contacts and the leads and by keeping the contacts independent of gate voltage. The results were found to be in qualitative agreement with experiments.

The main point to note here is that, in the absence of scatterers in the wire, conductances are only dependent on the contacts. The internal wire region can only be experienced if it has scatterers.

- *Odd–even effect in the presence of a magnetic field*

We explained the odd–even effect in the presence of an in-plane magnetic field by showing that the barrier renormalization due to interactions is different for  $\uparrow$  and  $\downarrow$  bands in the presence of the magnetic field.

By explicitly fitting a power law form to the conductance corrections as a function of temperature as given in the inset of figure 3 in ref. [3], we find that

$$\delta g = -0.3512T^{-0.1058-0.0345T}. \quad (17)$$

In future, we would like to understand the reasons for the discrepancy of this law from the simpler power law form that our model predicts and perhaps make a more realistic model, which could agree with the data more quantitatively. We would also like to understand several features which are observed on the rise between two successive plateaus, such as the ‘0.7 effect’. For this, one needs to study the model when some sub-band is partially opened.

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