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Numerical investigation of space charge electric field for a sheet electron beam between two conducting planes

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Abstract. Analytical and numerical study of the stability of sheet electron beam in periodically cusped magnetic field (PCM) is made. The beam has been considered as having diffused density profile. The conditions for beam focusing are discussed.

Keywords. Periodic cusped magnetic field (PCM); sheet electron beam; beam focusing.

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1. Introduction

One of the problems in scaling high power vacuum and plasma microwave sources to higher frequencies is the need to transport beams with higher space charge density, since the radio frequency circuit transverse dimensions tend to decrease with wavelength. The use of sheet electron beams can alleviate this difficulty since large amounts of current can be transported at reduced space charge density by increasing the width of the beam. Sheet beams permit reduction in space charge effects on the efficiency of the devices in which electron energy is recovered. However, due to unneutralized charges, the electron beam erosion can occur. This can be overcome by using an axial magnetic field. This is an unstable configuration due to $E \times B$ drift forces arising from the guiding magnetic field B and the transverse electric space charge field E of the beam. Due to shearing effects a diocotron instability is created in the beam. Physical consequence of the diocotron instability is that the beam forms localized kinks and vortices with the eventual result of the filamentation, 'heating', breakup, and possibly interception on the confining vessel walls. A finite length of propagation is required before this instability becomes unacceptably pronounced. A lower bound estimate for the diocotron instability growth length can be obtained as

$$L_d(\mathrm{cm})\rangle v_z \frac{\omega_c}{\omega_p^2} \approx 800\gamma^3\beta_z^2 \frac{B_z(\mathrm{kG})}{J_b(\mathrm{A/cm^2})},$$

where $\omega_c = eB/m$ is the electron cyclotron frequency, $\omega_p^2 = ne^2/\gamma^3 m\varepsilon_0$ is the square of the relativistic beam plasma frequency (n is the beam density in the laboratory frame),

$\gamma = (1 - \beta_z^2)^{-1/2}$ is the relativistic energy parameter, and $\beta_z = v_z/c$ is the beam axial velocity normalized to the speed of light in vacuum. Clearly, the diocotron instability can be suppressed by large beam energies (large γ) or low-beam current densities J_b. This is just a consequence of a space-charge instability being reduced by reducing the effect of space charge. In the present paper we have depicted analytically that by proper tailoring of the beam density profile the space charge effect can substantially be reduced. We are considering periodic cusped magnetic field [1] for focusing purpose.

Section 2 of this paper presents an analysis of the low frequency stability of the sheet electron beams in periodic magnetic field. Expressions for magnetic and space charge fields are derived in §3 for the sheet beam with density profile in periodic cusped magnetic field (PCM) focusing configuration. In §4 of this paper the conditions of focusing in case of the beam with diffused density profile are discussed. Results and discussions are presented in §5.

2. Analysis of the stability of sheet electron beam in periodic magnetic focusing

In the magnetic system such as wiggler or PCM, focusing of rectilinear electron beams is the result of a ponderomotive force effect that arises from coupled $V \times B$ forces in two component periodic magnetic fields. We consider a configuration such as that shown in figure 1 with rectangular cross-section and periodic cusps in both transverse planes. The near axis field for this configuration can be approximated by the following expressions, which are derived from a scalar potential χ_m:

$$\vec{B} = -\vec{\nabla}\chi_m,$$

where

$$\chi_m = \left(\frac{B_0}{k_m}\right)[\cosh(k_x x)\cosh(k_y y)\cos(k_m z)].$$

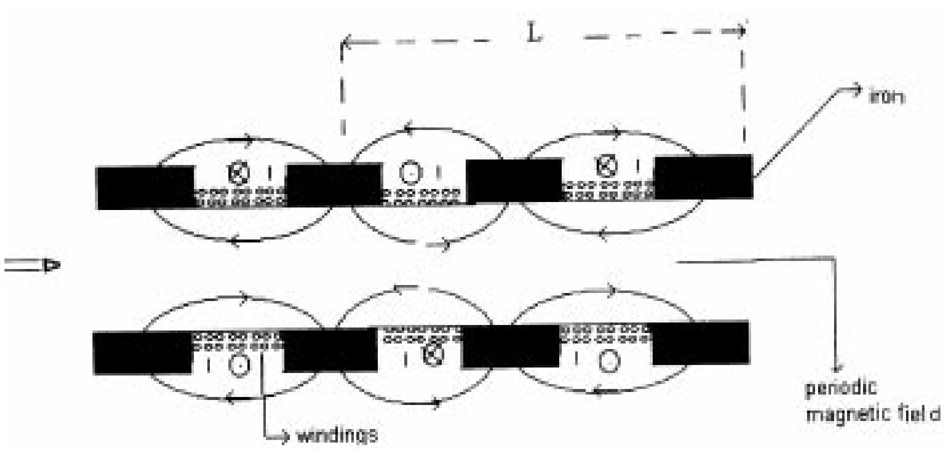

Figure 1. The two plane rectangular PCM focusing configuration.

From this scalar potential the magnetic field can be expressed as

$$B_x \approx -(k_x^2/k_m) x B_0 \cos(k_m z), \tag{1}$$

$$B_y \approx -(k_y^2/k_m) y B_0 \cos(k_m z), \tag{2}$$

$$B_z \approx B_0 \sin(k_m z), \tag{3}$$

where $k_m^2 = k_x^2 + k_y^2$.

Our model of sheet electron beam assumes a thin beam with charge density per unit area N and the electron density per unit volume n that are related as $Nq = ntq$, where t is the thickness of the beam and q denotes the charge. We consider the harmonic perturbation of sort given in figure 3b. To find stability criteria our model assumes a uniform solenoidal field $B = B_0\hat{Z}$. We assume that $z \approx u_0 t$ and $u_0 \gg u|\dot{y}|$, where u_0 is the beam axial velocity. And focusing on the dynamics of electron at the beam center, the x and y component of Lorentz force equations of motion are

$$\ddot{x} = \frac{qE_x}{m\gamma^3} - \left(\frac{\omega_{c0}}{\gamma}\right)\dot{y}, \tag{4a}$$

$$\ddot{y} = \frac{qE_x}{m\gamma^3} + \left(\frac{\omega_{c0}}{\gamma}\right)\dot{x}, \tag{4b}$$

where γ is relativistic factor and $\omega_{c0} = qB_0/m$. In eqs (4a) and (4b) we implicitly incorporated self-magnetic field through a $1/\gamma^2$ reduction of the self-electric fields, and have ignored bunching effects. For the harmonic perturbation of the sort $\mathrm{e}^{i(kx-\omega t)}$ of the beam, the linearized equations of motion are

$$-\omega^2\tilde{x} = \frac{q\tilde{E}_x}{m\gamma^3} - i\omega\frac{\omega_{c0}}{\gamma}\tilde{y}, \tag{5a}$$

$$-\omega^2\tilde{y} = \frac{q\tilde{E}_y}{m\gamma^3} - i\omega\frac{\omega_{c0}}{\gamma}\tilde{x}, \tag{5b}$$

where $\tilde{x}, \tilde{y}$ denote the perturbed quantities. In the perturbed state the electrons in the beam center experiences an electric field $\tilde{E}$ that is approximately the average of the field at the top $\tilde{E}_+$ and the bottom $\tilde{E}_-$ (figure 3b), and is given by

$$\tilde{E} = \frac{1}{2}(\tilde{E}_+ + \tilde{E}_-). \tag{6}$$

For the electrons at the center of the beam, the perturbed equation of continuity is approximated as

$$\tilde{N} = -ikN\tilde{x}. \tag{7}$$

Assuming that $kt \ll k|\tilde{y}| \ll 1$ (figure 3a), continuity of tangential field and the jump in the normal field components results in

$$\tilde{E}_{x+} - \tilde{E}_{x-} = -i(Nq/\varepsilon_0)k\tilde{y}, \tag{8}$$

$$\tilde{E}_{y+} - \tilde{E}_{y-} = (\tilde{N}q/\varepsilon_0). \tag{9}$$

Next by assuming the time scale of the perturbation to be slow (i.e. $\omega \ll \omega_{c0}$) equation (5)–(9) can be combined to yield

$$\frac{\tilde{E}_{x+} + \tilde{E}_{x-}}{\tilde{E}_{x+} - \tilde{E}_{x-}} = \frac{\omega}{k}\frac{2\varepsilon_0}{Nq\gamma}B_0, \tag{10}$$

$$\frac{\tilde{E}_{y+} + \tilde{E}_{y-}}{\tilde{E}_{y+} - \tilde{E}_{y-}} = -\frac{\omega}{k}\frac{2\varepsilon_0}{Nq\gamma}B_0. \tag{11}$$

Boundary conditions at the beam vacuum edges are obtained by assuming a quasistatic limit $k \ll \omega/2\pi c$ such that fields in this vacuum region are separable in their dependence on x and y. For a curl-free electric field this results in the approximation

$$\tilde{E}_{x\pm} \approx \mp i\tilde{E}_{y\pm}. \tag{12}$$

Finally we can combine eqs (10), (11), and (12) to yield

$$\frac{\omega}{k} = \pm i(Nq\gamma/2\varepsilon_0 B_0), \tag{13}$$

i.e. a purely growing mode.

The above model is applied to the case of the sheet electron beam in periodic magnetic focusing field. Again, we are assuming the perturbation as $\mathrm{e}^{i(kx-\omega t)}$ and that the perturbation evolves on a time sufficiently long to justify averaging dynamics over a magnet period. Hence, the equations of motion are

$$-\omega^2\tilde{x} = \frac{q\tilde{E}_x}{m\gamma^3} + \frac{\omega_{c0}^2}{2\gamma}C_x\tilde{x}, \tag{14}$$

$$-\omega^2\tilde{y} = \frac{q\tilde{E}_y}{m\gamma^3} + \frac{\omega_{c0}^2}{2\gamma}\tilde{y}. \tag{15}$$

Using eqs (8), (9) and (12) we obtain for this case the following two equations:

$$\omega^2 = \left(\frac{Nq^2}{2m\gamma^3\varepsilon_0}\right)k + \frac{\omega_{c0}^2}{2\gamma}C_x, \tag{16}$$

$$\omega^2 = -\left(\frac{Nq^2}{2m\gamma^3\varepsilon_0}\right)k + \frac{\omega_{c0}^2}{2\gamma}. \tag{17}$$

Normal mode solutions exist only for simultaneous solution of eqs (16) and (17). Upon substituting this yields

$$\omega^2 = \frac{\omega_{c0}^2}{4\gamma}(1+C_x), \tag{18}$$

i.e. bounded harmonic betatron motion (no growing instability). This is valid for perturbations for which $(\omega/u_0) \ll k_m$ or

$$\omega_{c0}^2(1+C_x) \ll 4\gamma k_m^2 u_0^2, \tag{19}$$

where u_0 is the beam axial velocity. That is one can expect to achieve stabilization (to diocotron modes) of the sheet beam in PCM focusing provided the magnetic field fluctuation frequency $k_m u_0$ is larger than the growth rate given in eq. (18).

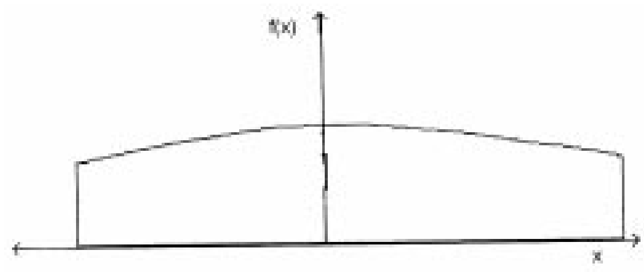

Figure 2. Horizontal density profile of diffused parabolic beam for which the space charge fields are calculated.

3. Analytic expression for space charge electric field

In this section of the paper we obtain the space charge electric fields for the case of a wide sheet beam when the beam has diffused edges along its width (figure2). We consider the propagation of sheet electron beam of thickness t and width w (where $t \ll w$) into a conducting waveguide of cross section dimensions as a and b along x and y directions respectively. We are considering the diffused density profile. The Poisson's equation for scalar potential ϕ_E in case of a wide rectangular sheet electron beam confined between parallel conducting plates is given by

$$\nabla^2\phi_E(x,y) = \frac{-\varphi(x,y)}{\varepsilon_0}, \quad |x| \le a/2, \quad |y| \le b/2 \tag{20}$$

with boundary condition $\phi_E(|x| = a/2) = \phi_E(|y| = b/2) = 0$. A Green's function solution to the relevant equation

$$\left(\frac{\partial^2}{\partial x'^2} + \frac{\partial^2}{\partial y'^2}\right) G(x,x';y,y') = -\frac{\delta(x'-x)\delta(y'-y)}{\varepsilon_0},$$

with the boundary conditions

$$G(|x'| = a/2) = G(|y'| = b/2) = 0,$$

is obtained as

$$G(x,x';y,y') = \sum_{j=0}^{\infty} g_j \cos(k_j y)\cos(k_j y) \times \sinh\left[k_j\left(\frac{a}{2} \pm x\right)\right] \sinh\left[k_j\left(\frac{a}{2} \mp x'\right)\right], \quad x' \gtrless x,$$

where

$$g_j \equiv \frac{2}{\varepsilon_0 b k_j \sinh(k_j a)}, \quad k_j \equiv \frac{(2j+1)\pi}{b}.$$

In the limit $a \to \infty$, the above Green's function is

$$G(x,x';y,y') = \sum_{j=0}^{\infty} \frac{1}{\varepsilon_0 b k_j} \cos(k_j y') \cos(k_j y) \mathrm{e}^{\pm k_j (x-x')}, \quad x' \gtrless x. \tag{21}$$

The solution to eq. (20) is therefore

$$\phi_E(x,y) = \iint_{swg} \mathrm{d}x' \mathrm{d}y' G(x,x';\, y,y') \varphi(x',y'),$$

where *swg* represents the waveguide cross section.

The formulas for electrostatics space charge fields are investigated when beam has the diffused density profile

$$n(x,y) = n_0 f(x) g(y),$$

where

$$f(x) = \begin{cases} 1 - \frac{1}{2} \exp\left\{ \frac{-(w/2+x)}{L} \right\}, & -w/2 < x \le 0 \\ 1 - \frac{1}{2} \exp\left\{ \frac{-(w/2-x)}{L} \right\}, & 0 \le x < w/2 \end{cases} \tag{22}$$

and

$$g(y) = \begin{cases} \frac{1}{2} \exp\left\{ \frac{(t/2+y)}{l} \right\}, & -b/2 \le y \le -t/2 \\ 1 - \frac{1}{2} \exp\left\{ \frac{-(t/2+y)}{l} \right\}, & -t/2 < y \le 0 \\ 1 - \frac{1}{2} \exp\left\{ \frac{-(t/2-y)}{l} \right\}, & 0 < y \le t/2 \\ \frac{1}{2} \exp\left\{ \frac{(t/2-y)}{l} \right\}, & t/2 < y \le b/2 \end{cases}. \tag{23}$$

When the density varies sufficiently slowly in x we can expect

$$\frac{\partial^2 \phi_E}{\partial x^2} \ll \frac{\partial^2 \phi_E}{\partial y^2}. \tag{24}$$

Poisson's equation thus gives

$$\frac{\partial^2 \phi_E}{\partial y^2} = (q/\varepsilon_0) n(x,y). \tag{25}$$

By solving the above equation, the potential can be obtained as

$$\phi_E = \frac{q n_0 f(x)}{2\varepsilon_0} \begin{cases} y^2 - l^2 \exp\left\{ \frac{-(t/2-y)}{l} \right\} + ly + 2l^2 \\ \qquad -l^2 \exp\left\{ \frac{-(b-t)}{2l} \right\} - \frac{tb}{2} + \frac{t^2}{4}, & 0 < y \le t/2 \\ l^2 \exp\left\{ \frac{t}{2l} - \frac{y}{l} \right\} + l^2 - ly + lb + (t-l)\left(y - \frac{b}{2} \right), & t/2 \le y \le b/2 \end{cases} \tag{26}$$

and for the electric field component in the region $|y| \le t/2$,

$$E(x) = -\frac{qn_0}{2\varepsilon_0}\left[y^2 - l^2\exp\left\{\frac{-(t/2-y)}{l}\right\} + ly + 2l^2 \right.$$
$$\left. -l^2\exp\left\{\frac{-(b-t)}{l}\right\} - \frac{tb}{2} + \frac{t^2}{4}\right]\frac{\mathrm{d}f(x)}{\mathrm{d}x}, \tag{27a}$$
$$E(y) = -qn_0 f(x)\left[2y - l\exp\left\{\frac{-(t/2-y)}{l}\right\} + l\right]. \tag{27b}$$

To establish the validity of the above equations we calculated the electric fields by Green's function solution with a prescribed diffused density variation as given in eqs (22) and (23). The horizontal electric component can be expressed as

$$E_x(x,y) = \int_{-\infty}^{\infty}\mathrm{d}x' qn_0 f(x')\int_{-b/2}^{b/2}\mathrm{d}y' g(y')\frac{\partial G(x,x';y,y')}{\partial x},$$

where $G(x,x';y,y')$ is as given in eq. (21),

$$E_x(x,y) = \frac{qn_0}{\varepsilon_0 bk_j}\sum_{j=0}^{\infty}\cos(k_j y)\int_{-\infty}^{\infty}(\pm)f(x')\exp(\pm k_j(x-x')\mathrm{d}x'$$
$$\times\int_{-b/2}^{b/2}\cos(k_j y')g(y')\mathrm{d}y'. \tag{28}$$

Let

$$\int_{-\infty}^{\infty}(\pm)f(x')\exp(\pm k_j(x-x')\mathrm{d}x' = A_j \;\; x \gtrless x', \text{ and } \int_{-b/2}^{b/2}\cos(k_j y')g(y')\mathrm{d}y' = Y_j,$$

then by substituting the values of $f(x')$ and $g(y')$, and evaluating the above integrals we get

$$A_j = \frac{\exp(k_j(x-w/2)) - \exp(-k_j(x+w/2)) + k_j L\exp(-w/2L - k_j x) - k_j L\exp(-w/2L - x/L)}{k_j(k_j^2L^2-1)}$$

and

$$Y_j = \frac{2}{k_j(1+k_j^2l^2)}[\{k_j l\cos(k_j t/2) - k_j^2 l^2\sin(k_j t/2)\}$$
$$+\exp((t-b)/2l)\{-k_j l\cos(k_j b/2) + k_j^2 l^2\sin(k_j b/2)\}$$
$$+2\sin(k_j t/2) + k_j l\exp(-t/2l) - k_j l\cos(k_j t/2)].$$

Substituting these values in eq. (28) we have

$$E_x(x,y) = \frac{qn_0}{\varepsilon_0}\sum_{j=0}^{\infty}\frac{\cos(k_j y)}{bk_j}$$
$$\times\left(\frac{\exp(k_j(x-w/2)) - \exp(-k_j(x+w/2)) + k_j L\exp(-w/2L - k_j x) - k_j L\exp(-w/2L - x/L)}{k_j(k_j^2L^2-1)}\right)$$

$$\times \left(\begin{array}{l} \frac{2}{k_j(1+k_j^2 l^2)}[\{k_j l\cos(k_j t/2) - k_j^2 l^2 \sin(k_j t/2)\} \\ + \exp((t-b)/2l)\{-k_j l\cos(k_j b/2) + k_j^2 l^2 \sin(k_j b/2)\} \\ +2\sin(k_j t/2) + k_j l\exp(-t/2l) - k_j l\cos(k_j t/2)] \end{array} \right). \tag{29}$$

Similarly the value of $E_y(x,y)$ can also be evaluated as

$$E_y(x,y) = \frac{qn_0}{\varepsilon_0 bk_j}\sum_{j=0}^{\infty}\sin(k_j y)\int_{-\infty}^{\infty} f(x')\exp(\pm k_j(x-x')\mathrm{d}x' \times \int_{-b/2}^{b/2}\cos(k_j y')g(y')\mathrm{d}y'. \tag{30}$$

By solving the above equation we have the value of $E_y(x,y)$ as

$$E_y(x,y) = \frac{qn_0}{\varepsilon_0}\sum_{j=0}^{\infty}\frac{\sin(k_j y)}{bk_j} \times \left(\frac{\exp(k_j(x-w/2)) - \exp(-k_j(x+w/2)) + k_j L\exp(-w/2L - k_j x) - k_j L\exp(-w/2L - x/L)}{k_j(k_j^2 L^2 - 1)} \right)$$
$$\times \left(\begin{array}{l} \frac{2}{k_j(1+k_j^2 l^2)}[\{k_j l\cos(k_j t/2) - k_j^2 l^2 \sin(k_j t/2)\} \\ + \exp((t-b)/2l)\{-k_j l\cos(k_j b/2) + k_j^2 l^2 \sin(k_j b/2)\} \\ +2\sin(k_j t/2) + k_j l\exp(-t/2l) - k_j l\cos(k_j t/2)] \end{array} \right). \tag{31}$$

4. Focusing of sheet electron beam with diffused parabolic density profile

The problem of matching a rectangular sheet beam with two-plane focusing periodic magnetic fields is a nonlinear, nonseparable problem involving the balance of magnetic focusing forces, space charge defocusing forces and transverse kinetic pressure associated with finite beam emittance. We first consider the guidelines for periodic cusped magnetic focusing of an infinitely wide sheet beam with uniform density. In this limit the problem depends only on the y dimension for which the equation of motion is

$$\frac{\mathrm{d}v_y}{\mathrm{d}t} \approx q\frac{q}{m\gamma^3}E_y - \frac{\omega_{c0}}{2\gamma}y \approx \frac{\omega_p^2}{\gamma^3} - \frac{\omega_{c0}^2}{2\gamma}y.$$

For beam focusing

$$\frac{\omega_{c0}^2}{\gamma} \geq \frac{2\omega_p^2}{\gamma^3}. \tag{32}$$

For the validity of the period averaged equations, as well as to ensure stability against diocotron modes, we require a spatial magnet period short enough to satisfy $k_\beta \ll k_m, k_p =$

$\omega_p/u_0 \ll k_m$ and $k_d = 2\pi/L_d \ll k_m$, where k_β is the betatron wave number and L_d is the diocotron stability growth length. Simultaneous satisfaction of these requirements leads to the following conditions for a stable confinement regime of periodically focused sheet electron beams,

$$2\omega_p^2 \leq \omega_{c0}^2. \tag{33}$$

We now consider bulk focusing of sheet beam in the horizontal dimension. For a closed magnetic configuration and a sheet beam near $y = 0$, the approximate equation of motion is

$$\frac{\mathrm{d}v_x}{\mathrm{d}t} \approx \frac{q}{m\gamma^3}E_x - \frac{\omega_{c0}^2}{2\gamma}\frac{k_x^4}{k_m^4}x, \tag{34}$$

where $E_x(x,y)$ is the electric field for the beam whose edge have a definite density variation. The expression for the electric field at beam edge $(x = w/2)$ and under the condition $k_j w \gg 1$ can be given as

$$E_x(x,y) = \frac{m\omega_p^2}{q}\sum_{j=0}^{\infty}\frac{\cos(k_j y)Y_j}{bk_j}[1 + k_j L\exp(-w/2L) - k_j L]. \tag{35}$$

The condition for focusing to exceed over defocusing is $\mathrm{d}\tilde{v}_x/\mathrm{d}t \leq 0$, one gets

$$\frac{q}{m\gamma^3}E_x \leq \frac{\omega_{c0}}{2\gamma}\frac{k_x^4}{k_m^4}x, \tag{36}$$

by substituting the value of $E_x(x,y)$ from eq. (35) in eq. (36) we have,

$$\frac{\omega_p^2}{\gamma^3}\sum_{j=0}^{\infty}\frac{\cos(k_j y)Y_j}{bk_j}[1 + k_j L\exp(-w/2L) - k_j L] \leq \frac{\omega_{c0}^2}{2\gamma}\frac{k_x^4}{k_m^4}\frac{w}{2}. \tag{37}$$

By taking a particular case we depicted the values of peak PCM magnetic fields for stable sheet beam confinement for different beam current densities j_b.

5. Results and discussion

Figure 3 displays variation of normalized electric field $\hat{E}_x$ with respect to the normalized horizontal position for a parabolic sheet beam between two conducting planes separated by distance b in the $\hat{y} = 0$ and 0.01 planes for $\hat{L} = 1.0$ and 3.0. Here normalized quantities are introduced as $\hat{L} = L/b, \hat{w} = w/b, \hat{x} = x/b$, $\hat{l} = l/b$ and $\hat{E}_x(x,y) = E_x(x,y)/E_0$. From the figure it is evident that space charge fields get substantially reduced for a diffuse edge sheet beam. For a given scale length of density variation of the diffused edge that is normally located at $|y| = t/2$, the space charge fields $\hat{E}_x$ decreases with increase in $\hat{L}$. Figure 4 represents the variation of the confining magnetic field as a function of the beam current density j_b, where j_b is related to the beam voltage as $j_b = P/Av_b^{3/2}$ where P/A represents perveance per unit area of the beam. To control emittance growth in periodically focused beams, a precise balance of space charge fields and magnetic focusing fields are needed. For a laminar sheet beam the y component of the space charge field decreases at $|x| = w/2$ due to fringing effects. When the condition (37) is satisfied, the beam matching in the center results in over focusing.

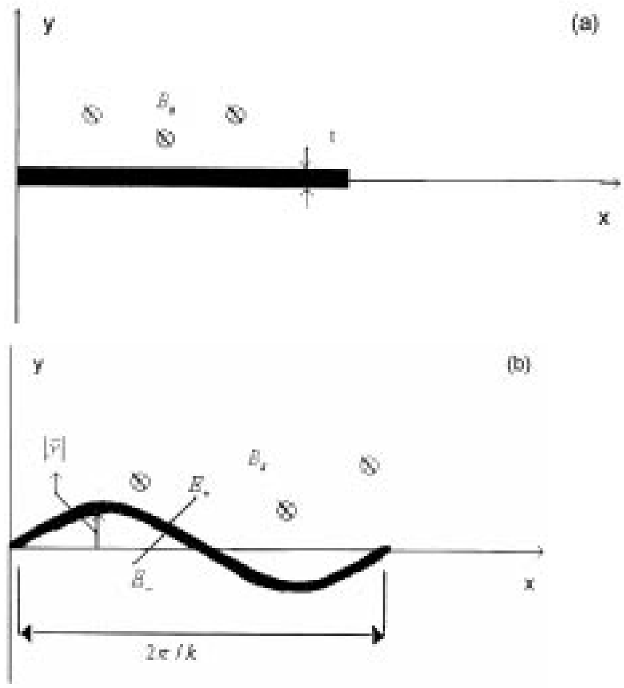

Figure 3. Sheet electron beam in (**a**) unperturbed state; (**b**) perturbed state.

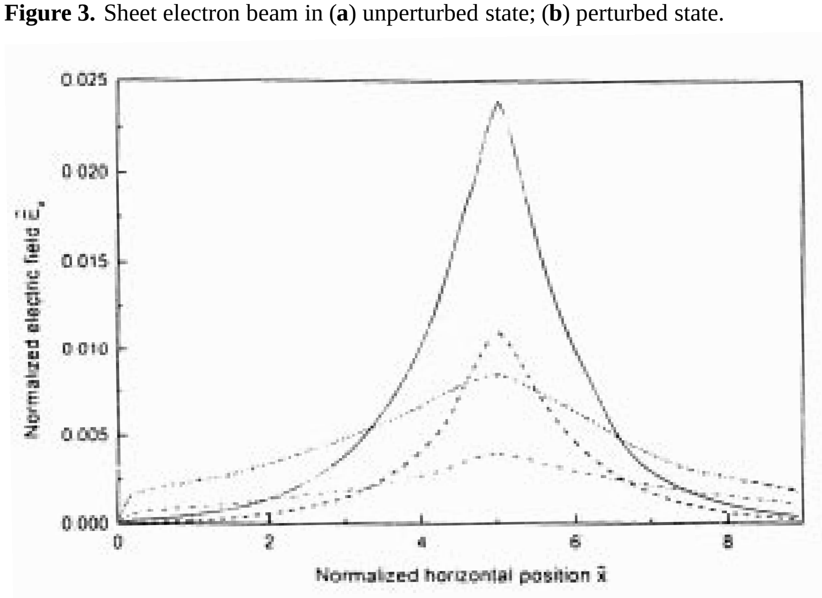

Figure 4. Variation of normalized space charge field for a beam parabolic density profile with respect to horizontal position. —— line is for $\hat{y} = 0.001$, $\hat{L} = 1$, - - - line is for $\hat{y} = 0.00$, $\hat{L} = 1$, ... line is for $\hat{y} = 0.00$, $\hat{L} = 3$, and -.-.- line is for $\hat{y} = 0.01$, $\hat{L} = 3$.

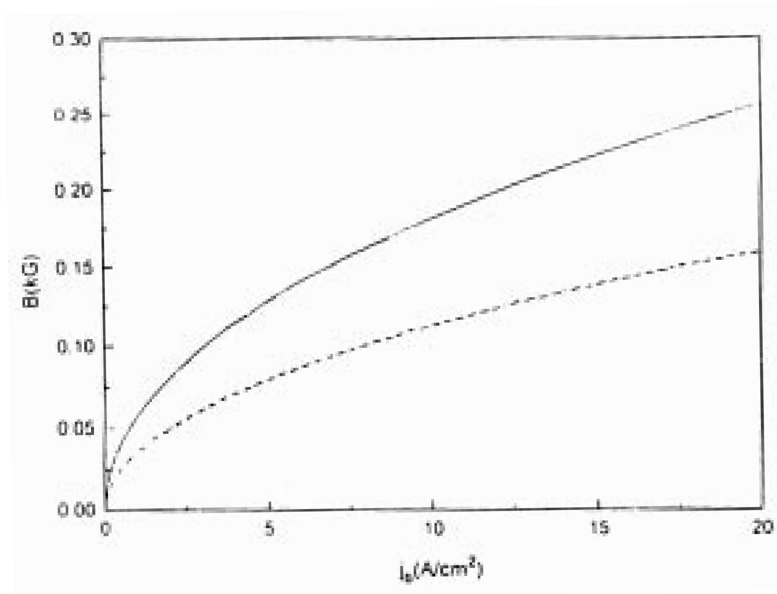

Figure 5. Variation of confining magnetic field for PCM focused sheet beam with respect to the beam current density J_b. Solid line shows the Booske's result and dotted line gives our results for a magnet period $l_m = 1$, $t/b = 1$ and $w/b = 6$.

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