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Soliton-based ultra-high speed optical communications

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Abstract. Multi-terabit/s, ultra-high speed optical transmissions over several thousands kilometers on fibers are becoming a reality. Most use RZ (Return to Zero) format in dispersion-managed fibers. This format is the only stable waveform in the presence of fiber Kerr nonlinearity and dispersion in all optical transmission lines with loss compensated by periodic amplifications. The nonlinear Schrödinger equation assisted by the split step numerical solutions is commonly used as the master equation to describe the information transfer in optical fibers. All these facts are the outcome of research on optical solitons in fibers in spite of the fact that the commonly used RZ format is not always called a soliton format. The overview presented here attempts to incorporate the role of soliton-based communications research in present day ultra-high speed communications.

Keywords. Optical soliton; optical communication; soliton.

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1. Introduction

The worldwide demand for ultra-high speed communications initiated by the explosive increase of internet traffic has resulted in remarkable progress in the capacity of optical communications both in the total bitrate and in the distance of propagation. This review starts from introduction of basic issues of ultra-high speed optical communications including methods of coding information in lightwaves and elementary concept of information transfer in optical fibers. The master equation that describes information transfer in fibers (the nonlinear Schrödinger equation) is then derived, followed by description of linear evolution of information and introduction of optical soliton solution as the stable nonlinear solution.

The paper then introduces problems in the use of ideal solitons as information carriers resulting primarily from the timing jitters and the solution by means of soliton control. Dispersion management techniques are then introduced as a means to overcome these problems and description of nonlinear pulse (the dispersion managed soliton) in a dispersion managed fiber is presented.

Examples of recent remarkable experimental results of transmission of dispersion managed solitons will be presented to demonstrate the effectiveness of dispersion management techniques both in time division multiplexing (TDM) and wavelength division multiplexing (WDM) long distance communications. The manuscript ends with a description of future issues in ultra-high speed optical communications.

2. How formation is coded in lightwaves

A lightwave, compared with a microwave or a radio wave, carries very high frequency of oscillation of over 100 THz. This means the lightwave can carry hundred thousands times more information than a microwave in unit time span. However, since a lightwave has a wavelength of order 1 μm, significantly shorter than that of a microwave, it may face serious problems when it propagates in a medium over a distance of human interest. Fortunately, optical fibers, which can guide lightwaves because it has an index of refraction higher than the air, have been developed having loss in the order of 0.2∼0.3 dB/km, limited only by theoretical limitation from Rayleigh scatterings and molecular resonance absorption over a wavelength range of 1300 nm to 1700 nm. Thus a lightwave of this wavelength range can propagate over tens of kilometers before it faces serious loss of energy. In addition, optical amplifiers in this wavelength regime have been developed and as a result the fiber based transmission has been effectively lossless.

For a lightwave to carry information it should be modulated since a single frequency wave carries no information. The information is carried by modulation. A modulated lightwave may be expressed by a modulation amplitude $\bar{E}(z,t)$ of the optical electric field $E(z,t)$.

$$E(z,t) = \frac{1}{2}\left[\bar{E}(z,t)\mathrm{e}^{i(k_0 z-\omega_0 t)} + c.c\right], \tag{1}$$

where ω_0 and k_0 are the frequency and wave number of the unmodulated lightwave and $\bar{E}$ represents the modulation which is in general a complex function of distance z of propagation and time t. Depending on the choice of modulation at the input of a fiber $z = 0$, $\bar{E}(0,t)$, various formats of information coding have been tried. They may be classified as an analog format and a digital format. An analog format takes advantage of the coherent nature of the laser generated lightwave and information is coded by either amplitude, frequency or phase modulation, similar to radio signals. A digital format utilizes light pulse intensity to represent '1' digit and its absence '0'. However as will be shown later, coherence of the lightwaves is important to minimize the pulse deformation even in case of the digital formats. There are variations among digital formats. Soliton format primarily uses one soliton to represent '1' digit and takes advantage of its robust nature.

Generally when one pulse is designated to represent '1' digit, the format is called RZ (Return to Zero). On the other hand if two (or more) pulses are connected when a sequence of '1' appears, the format is called NRZ (Not Return to Zero). In addition, if the '1' pulse is allowed to have two type of pulses with opposite phases, these format is called duo-binary. These formats are illustrated in figure 1.

Usually either NRZ or duo-binary format can carry more information for a give bandwidth.

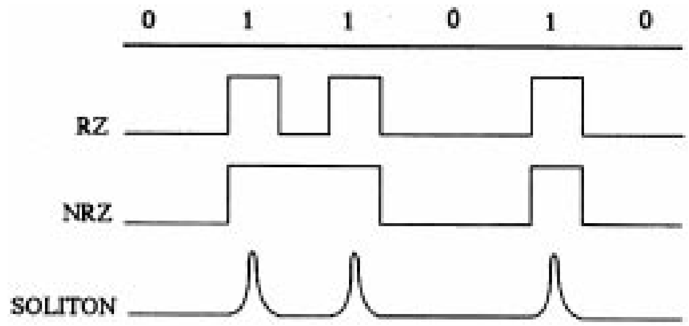

Figure 1. Various modulation formats for information transfer in fibers.

3. How information is transferred in fibers

To study how the information is transmitted it is essential to study how the information, or modulation, propagates in a fiber. Information may be lost not only by the fiber loss but also by its deformation, which arises during the propagation in fibers due to various properties of the fiber. As is well known, the information propagates at the group velocity, $\partial\omega/\partial k$. So $\bar{E}$ propagates at the group velocity. The loss of information originates from a deformation of the modulation $\bar{E}(z,t)$; i.e., if $\bar{E}(z,t)$ does not change in z, there is no loss of information.

Information can be put on the lightwave at $z=0$ by choosing a proper shape of $\bar{E}(0,t)$ in time. The amount of information depends on how rapidly $\bar{E}(t)$ varies, which can be designated by the width of the Fourier spectrum, $\bar{E}(\Omega)$,

$$\bar{E}(\Omega)=\int_{-\infty}^{\infty}\bar{E}(t)\mathrm{e}^{i\Omega t}\mathrm{d}t. \tag{2}$$

The amount of information (Bit/s) that the lightwave carries is approximately given by the spectral width $\Delta\Omega$. Therefore more the amount of information, the wider the spectral width becomes. Consequently to achieve an ultra-high speed of information transfer in a fiber, we should study the behavior of $\bar{E}(z,t)$ having a wide spectral width.

Deformed information may be reconstructed by a regeneration scheme. The scheme generally requires detecting of information, which may be done by rectification of the lightwave to extract $\bar{E}(z_R,t)$ at the repeater located $z=z_R$, by regenerating the original information $\bar{E}(0,t)$ from $\bar{E}(z_R,t)$ and by modulating a fresh lightwave by $\bar{E}(0,t)$. In an early stage of optical communication, this scheme has commonly been adopted. However it has soon become clear that the repeater cost is the major obstacle for the increase of transmission rate because it requires electronic circuits having increasingly higher speeds of operation. Consequently in 1990, Bell laboratory engineers have switched to an all optical transmission scheme, which employs optical amplifiers using EDFA, erbium doped fiber amplifier, which became available at that time. Actually an amplifier based all optical transmission scheme had been proposed earlier by Hasegawa [1] for an optical soliton transmission scheme [2] since a soliton maintains its shape during the transmission, it is quite an appropriate choice for an all optical system. The decision by the Bell Labs team was in fact influenced by this idea, although at that stage solitons were not adopted because of the reason explained later.

Most high speed transmission systems at present use an all optical scheme with loss compensated for by periodic optical amplifications. When the fiber loss is effectively eliminated by optical amplifiers what effects remain that limit the transfer of information? The major limitations come from the *group velocity dispersion* and *nonlinearity* of the fiber.

Let us first discuss the effect of the group dispersion. The group dispersion is the effect in which the group velocity varies as a function of the lightwave frequency. The group dispersion originates from the combination of the wave guide property and the material property of the fiber. In the presence of the group dispersion, information carried by a different frequency component of $\bar{E}(t)$ propagates at a different speed and thus arrives at a different time. The relative delay Δt of arrival time of information at frequencies ω_1, and ω_2 at distance z is given by

$$\Delta t_D = \frac{z}{v_g(\omega_1)} - \frac{z}{v_g(\omega_2)} = \frac{\left(\frac{\partial v_g}{\partial \omega}\right)(\omega_2 - \omega_1)z}{v_g^2}. \tag{3}$$

If we use

$$v_g = \frac{\partial \omega}{\partial k} = \frac{1}{\partial k/\partial \omega} = \frac{1}{k'},$$
$$\frac{\partial v_g}{\partial \omega} = \frac{\partial}{\partial \omega}\left(\frac{1}{k'}\right) = \frac{-k''}{(k')^2} = -k'' v_g^2. \tag{4}$$

Equation (3) becomes

$$\Delta t_D = k''(\omega_1 - \omega_2)z. \tag{5}$$

Equation (5) shows that the difference of arrival time of information is proportional to the group dispersion, k'', the difference of the frequency components $\omega_1 - \omega_2$ and the distance of propagation z.

We note that if $k'' < 0$ (called anomalous dispersion regime), the higher frequency component of information arrives earlier and for normal dispersion, $k'' > 0$, the other way around. If the information at different frequency components arrives at a different time, the information may be lost. The problem becomes more serious if the amount of information is large so that $\omega_1 - \omega_2$ is large.

Another important mechanism that could lead to a loss of information is the nonlinear effect. For an ordinary fiber, the lowest order nonlinearity originates from the Kerr effect where the index of refraction, n, changes in proportion to the electric field intensity $|\bar{E}|^2$. In the presence of the Kerr effect, the index of reaction n is given by

$$n = n_0(\omega) + n_2|\bar{E}|^2 \tag{6}$$

and the wave number,

$$k = \frac{\omega n}{c} = \frac{\omega n_0(\omega)}{c} + \frac{\omega n_2}{c}|\bar{E}|^2. \tag{7}$$

Equation (7) indicates that the Kerr effect induces a nonlinear phase shift $\Delta\Phi_N$ through the nonlinear part of the wave number k_N given by

$$\Delta\Phi_N = k_N z = \frac{\omega}{c} n_2 |\bar{E}|^2 z = \frac{2\pi z n_2 |\bar{E}|^2}{\lambda}. \tag{8}$$

Here n_0 is the linear index of refraction and n_2 is the Kerr coefficient having a value $\approx 10^{-22}$ (m/V)^2. For a lightwave with a peak power of 1 mW, $|\bar{E}|$ becomes about 10^5 V/m in a typical fiber. Thus $n_2|\bar{E}|^2$ has a value of 10^{-12}. Therefore, even if $n_2|\bar{E}|^2 \approx 10^{-12}$, since $z/\lambda \approx 10^{12}$ for $z = 10^3$ km and $\lambda \approx 1500$ nm, $\Delta\Phi_N$ between the high intensity portion and the low intensity portion can become 2π over this distance of propagation. This indicates that the phase information is lost over a distance of 10^3 km if the lightwave power is as low as a milliwatt. In addition it creates a mixture of information in amplitude and phase. This means that information transfer by means of coherent modulation is not appropriate for a lightwave in fibers for a propagation distance beyond 10^3 km. Similarly, the phase sensitive duo-binary format faces loss of information when adjacent pulses overlap during the transmission.

4. Master equation for information transfer in optical fibers

4.1 *Derivation of the master equation*

The information carried by the lightwave in fibers is expressed by the modulation $\bar{E}(z,t)$ shown in §2. We now derive the equation, which describes evolution $\bar{E}$ along the direction z of the propagation of information. The most convenient way to derive the envelope equation is to Taylor expand the wave number $k(\omega, |\bar{E}|^2)$, around the carrier frequency ω_0 and the electric field intensity $|\bar{E}|^2$,

$$k - k_0 = k'(\omega_0)(\omega - \omega_0) + \frac{k''(\omega_0)}{2}(\omega - \omega_0)^2 + \frac{\partial k}{\partial |E|^2}|\bar{E}|^2, \tag{9}$$

and to replace $k - k_0$ by the operator $i\partial/\partial z$ and $\omega - \omega_0$ by $-i\partial/\partial t$, and to operate on the electric field envelope, $\bar{E}(z,t)$. The resulting equation reads,

$$i\left(\frac{\partial \bar{E}}{\partial z} + k'\frac{\partial \bar{E}}{\partial t}\right) - \frac{k''}{2}\frac{\partial^2 \bar{E}}{\partial t^2} + \frac{\partial k}{\partial |E|^2}|\bar{E}|^2\bar{E} = 0. \tag{10}$$

As shown in §3, the index of refraction $n(k, \omega, |\bar{E}|^2)$ for a plane electromagnetic wave in Kerr media is given by

$$n \equiv \frac{ck}{\omega} = n_0(\omega) + n_2|\bar{E}|^2. \tag{11}$$

Thus $k', k'', \partial k/\partial |\bar{E}|^2$ in eq. (10) are given approximately by

$$\left.\begin{aligned} k' &\approx \tfrac{n_0(\omega_0)}{c}, \\ k'' &\approx \tfrac{2}{c}\tfrac{\partial n_0}{\partial \omega} \\ \tfrac{\partial k}{\partial |E|^2} &\approx \tfrac{\omega_0}{c} n_2. \end{aligned}\right\}. \tag{12}$$

We note that to obtain k'' in this expression, we should go back to eq. (7) and take the second derivative of k with respect to ω. It is often convenient to study the evolution of E in the coordinate moving at the group velocity $\tau = t - k'z$. Then the envelope equation becomes

$$i\frac{\partial \bar{E}}{\partial z} - \frac{k''}{2}\frac{\partial^2 \bar{E}}{\partial \tau^2} + \frac{\omega_0 n_2}{c}|\bar{E}|^2\bar{E} = 0. \tag{13}$$

For a lightwave envelope in a fiber, the coefficients of this equation depend on the fiber geometry and modal structure of the guided lightwave. In particular, for a single mode fiber (SMF) $k'' = 0$ occurs at $\lambda = 1.3$ μm which is determined primarily by the glass property itself, while k'' becomes zero at $\lambda = 1.55$ μm for a dispersion shifted fiber (DSF) because of the waveguide property.

For a guided wave in a fiber, k'' is modified by the waveguide dispersion, which depends on the modal structure in the fiber [3]. In particular, for a weakly guided mode, the wave number k is given by the eigen function $\phi(x_\perp)$ for the waveguide mode,

$$k^2 = \frac{(\omega/c)^2 \int |\nabla_\perp \phi|^2 n_0^2 \mathrm{d}S - \int |\nabla_\perp^2 \phi|^2 \mathrm{d}S}{\int |\nabla_\perp \phi|^2 \mathrm{d}S}. \tag{14}$$

Here n_0 is the linear refractive index which is in general a function of the transverse coordinates $x_\perp$ and frequency ω. The integration $\int \mathrm{d}S$ is evaluated across the cross-section of the fiber and ϕ is normalized such that

$$\int |\nabla_\perp \phi|^2 \mathrm{d}S = A_{\mathrm{eff}} E_0^2 \tag{15}$$

where E_0 is the peak intensity of the light electric field, and A_{eff} is the (effective) cross-section of the fiber. In addition, since the light intensity varies across the fiber, n_2 in eq. (13) is reduced by the factor g given by

$$g = \frac{\omega}{kcA_{\mathrm{eff}}E_0^4}\int n_0 |\nabla \phi|^4 \mathrm{d}S \approx \frac{1}{2}. \tag{16}$$

A linear wave packet deforms due to the group velocity dispersion k''. For a lightwave pulse with a scale size of t_0, the deformation takes place at a distance given by the dispersion distance

$$z_0 = t_0^2/|k''|. \tag{17}$$

Thus it is convenient to introduce the distance Z normalized by z_0 and time T normalized by t_0. Equation (13) reduces to

$$\frac{\partial q}{\partial Z} = \frac{i}{2}\frac{\partial^2 q}{\partial T^2} + i|q|^2 q. \tag{18}$$

Here q is the normalized amplitude given by

$$q = \sqrt{\frac{\omega_0 n_2 g z_0}{c}}\bar{E}. \tag{19}$$

Equation (18) is the master equation that describes the evolution of information propagation in fibers and is often called the nonlinear Schrödinger equation. Comparing eq. (19) with eq. (18), we note that $|q|^2$ represents the self-induced phase shift, which is of the order unity for a mW level of lightwave power with dispersion distance of a few hundred kilometers.

In deriving eq. (18), $k'' < 0$ (anomalous dispersion) is assumed. For a normal dispersion $k'' > 0$, the coefficient of the first term on rhs of (18) becomes negative. Equation (18) is the master equation first derived by Hasegawa and Tappert [2] and is now widely used, with proper modification, in the design of lightwave transmission systems. In a practical system, the fiber dispersion k'' often varies in Z. In addition, the fiber has amplifiers with gain $G(Z)$ and loss with loss rate γ. Then eq. (18) should be modified to

$$\frac{\partial q}{\partial Z} = \frac{i}{2} d(Z) \frac{\partial^2 q}{\partial T^2} + i|q|^2 q + [G(Z) - \gamma] q. \tag{20}$$

Here $d(z)$ is the group dispersion normalized by its average value.

4.2 *Linear response of a fiber with dispersion*

We note that in the absence of the nonlinear term, eq. (20) can be easily integrated by means of the Fourier transformation in time,

$$\bar{q}(Z,\Omega) = \int_{-\infty}^{\infty} q(Z,T) \mathrm{e}^{i\Omega T} \, \mathrm{d}T. \tag{21}$$

$\bar{q}(Z,\Omega)$ is then given from eq. (22) by

$$\bar{q}(Z,\Omega) = \bar{q}(0,\Omega) \exp\left\{ -\frac{i\Omega^2}{2} \int_0^z \mathrm{d}(Z')\mathrm{d}Z' + \int_0^z [G(Z') - \gamma] \mathrm{d}Z' \right\} \tag{22}$$

and $q(Z,T)$ is obtained from the inverse transformation,

$$q(Z,T) = \frac{1}{2\pi} \int_{-\infty}^{\infty} \bar{q}(Z,\Omega) \mathrm{e}^{-i\Omega T} \, \mathrm{d}\Omega. \tag{23}$$

In particular we note that if the average (or integrated) dispersion, $\langle \mathrm{d}(Z) \rangle$, is zero and the average (or integrated) gain, $\langle G(Z) \rangle$, balance with the loss $\gamma, q(Z,T)$ can recover the initial modulation $q(0,T)$ exactly. That is, if the fiber response is completely linear and if the amplifier has no noise one can design a fiber transmission line such that the initial information can be transmitted without loss by making $\langle d(Z) \rangle = 0$ and $\langle G(Z) \rangle = \gamma$.

As an example of linear transmission, let us consider a case of a lossless fiber in which the initial pulse shape is given by a Gaussian,

$$q(0,T) = \frac{q_0}{\sqrt{2\pi T_0}} \mathrm{e}^{\frac{-T^2}{2T_0^2}}. \tag{24}$$

The Fourier transform of q is obtained by the formula,

$$\int_{-\infty}^{\infty} e^{-ax^2} e^{-ixy} dx = \sqrt{\frac{\pi}{a}} e^{-y^2/4a} \tag{25}$$

to give

$$\bar{q}(0,\Omega) = \frac{q_0}{\sqrt{2\pi T_0}} \int_{-\infty}^{\infty} e^{\frac{-T^2}{2T_0^2}+\Omega} dT = q_0 e^{-\Omega T_0^2/2}. \tag{26}$$

If we substitute eq. (26) into (22) and assume $d(Z)$ to be a constant, D_0, we have,

$$\bar{q}(Z,\Omega) = q_0 e^{-\Omega T_0^2/2} \cdot e^{-i\Omega D_0 Z/2} = q_0 e^{-\frac{1}{2}(T_0^2 + iD_0 Z)\Omega} \tag{27}$$

If we further substitute this result into eq. (23), we can obtain the wave packet q at a given distance Z,

$$\begin{aligned} q(Z,T) &= \frac{q_0}{2\pi} \int_{-\infty}^{\infty} e^{-\frac{1}{2}(T_0^2 + iD_0 Z)\Omega} \cdot e^{-i\Omega T} d\Omega \\ &= \frac{q_0}{\sqrt{2\pi(T_0^2 + iDZ)}} \exp\left[-\frac{T_0^2 T^2}{2T_0^4 + D_0^2 Z^2}\right] \exp\left[\frac{iDZT^2}{2(T_0^4 + D_0^2 Z^2)}\right]. \end{aligned} \tag{28}$$

Inspecting this result, we can observe various interesting points. First, we note that the pulse width $(T_0^2 + D_0^2 Z^2/T_0^2)$ increases with Z. That is, the fiber dispersion induces the increase of the pulse width approximately in proportion to the distance of propagation. Next we note that the phase varies in proportion to T^2. This indicates that the frequency increases in proportion to T, that is, a chirping is produced. Therefore the dispersion induces increase of the pulse width and frequency chirping.

Unfortunately since an amplifier always adds noise which is proportional to the gain and the gain bandwidth, one needs to provide $q(0,T)$ whose magnitude is sufficiently large in order to maintain the sufficient signal to noise ratio (S/N). The finite level of intensity for the required $q(0,T)$ induces the nonlinear self-frequency shift due to the Kerr effect as shown in eq. (10). We note, however, that if the time dependence of $|E|^2$ in eq. (10) is properly chosen, the self-phase shift can counterbalance the phase shift induced by the (anomalous) dispersion shown in eq. (28). This is the essential feature of the soliton formation. We will discuss ways to overcome this in the next section.

5. Optical soliton

5.1 *Soliton solution in optical fibers*

As we have seen in the lightwave envelope, the information distorts due to fiber dispersion, loss and nonlinearity as it propagates in fibers. In the absence of nonlinearity, the distorted envelope can recover the original shape by making integrated dispersion and loss be identically zero. At present, thanks to advanced optical technologies, optical amplifiers such as EDFA (erbium doped fiber amplifier) and fiber Raman amplifier (FRA) are available in the wavelength range around 1550 nm (EDFA) and in all guided wavelength ranges (FRA).

In addition fibers having practically any value of the group dispersion are available by a proper design of the mode structure and/or fiber grating.

However the fiber nonlinearity is essentially unavoidable because the light intensity should be kept large enough to overcome the deterioration of the signal to noise ratio (S/N). This problem becomes increasingly severe when the distance of propagation becomes large since the accumulated noise by repeated amplification increases and proportionally large intensity is required. In addition the nonlinear problem becomes severe when the bit rate of transmission is increased, since the peak intensity must be increased to have enough number of photons in a light pulse for a shorter pulse width. As was discussed in §4, an optical signal having a peak power of 1 mW faces severe distortion as it propagates a distance of 10^3 km. In practice, this means that the maximum distance of transmission of a signal having a transmission rate of about 10 Gigabits per second (10 Gb/s) is limited to about 10^3 km due to the fiber nonlinearity.

When Hasegawa and Tappert [2] showed that a lightwave pulse described by eq. (20) has a soliton solution given by sech T, and proposed to use this solution as the information carrier, the primary motivation was to bring in fiber nonlinearity to compensate for the fiber (anomalous) dispersion. At present, since the fiber dispersion can be made zero, the argument to use solitons is reversed, i.e., introduce fiber dispersion to compensate for the nonlinearity, since the soliton is useful because it is the only stationary and stable optical pulse shape in the presence of fiber nonlinearity.

If we assign one bit of optical signal to one soliton, the transmission of information can be characterized by only four parameters the amplitude η (also the pulse width), the frequency κ (also the pulse speed), the time position T_0 and the phase σ,

$$q(T,Z) = \eta \operatorname{sech}[\eta(T + \kappa Z - T_0)] \exp[-i\kappa T + \frac{i}{2}(\eta^2 - \kappa^2)Z + i\sigma]. \tag{29}$$

In particular we note that the soliton speed κ is a parameter independent of the amplitude unlike the case of the Korteweg deVries soliton. This fact is very important for the use of an optical soliton as a digital signal. The fact that a soliton transmission can be characterized by only four parameters is an additional important advantage of a soliton system since non-soliton pulses can be characterized only by an infinite dimensional parameter space.

Equation (18) was found to be integrable by Zakharov and Shabbat [4] using the Lax method [5]. The parameters η and κ are then obtained as discrete eigenvalue(s) of the Lax pair operator with the potential given by the initial value, $q(0,T)$. This means that a set of soliton solutions emerges from an arbitrary localized initial pulse at the input of the fiber, if the initial wave intensity $q(0,T)$ exceeds a critical level, which is determined by the amount of dispersion and pulse width.

5.2 *Effect of fiber birefringence*

In a real fiber, the index of refraction depends on the direction of polarization of the electric field. This is called the fiber birefringence. In the presence of the fiber birefringence, the envelope eq. (18) becomes a coupled equation for each orthogonal component of the electric field, *u* and *v* given by

$$\left.\begin{array}{l} i\left(\frac{\partial u}{\partial Z}+\delta\frac{\partial u}{\partial T}\right)+\frac{1}{2}\frac{\partial^2 u}{\partial T^2}+\left(|u|^2+\frac{2}{3}|v|^2\right)u=0 \\ i\left(\frac{\partial v}{\partial Z}-\delta\frac{\partial v}{\partial T}\right)+\frac{1}{2}\frac{\partial^2 v}{\partial T^2}+\left(|v|^2+\frac{2}{3}|u|^2\right)v=0 \end{array}\right\}. \tag{30}$$

Here 2δ is the difference of the group velocity of the two polarizations. Fiber birefringence generally varies randomly in the axial direction. In most fibers, the correlation distance of the variation is shorter than the dispersion distance. Then it was shown by Wai *et al* [6] that the coupled equation may be reduced to the integrable equation called the Manakov equation [7].

$$\left.\begin{array}{l} i\frac{\partial u}{\partial Z}+\frac{1}{2}\frac{\partial^2 u}{\partial T^2}+(|u|^2+|v|^2)u=0 \\ i\frac{\partial v}{\partial Z}+\frac{1}{2}\frac{\partial^2 v}{\partial T^2}+(|v|^2+|u|^2)v=0 \end{array}\right\}. \tag{31}$$

Equation (31) is shown to have a soliton solution which is similar to the nonlinear Schrödinger soliton. This result warrants the validity of the soliton concept in practical fibers. As is seen, the main effect of birefringence is to produce dispersion of optical packets due to the difference of the group velocity of the two polarizations. Such dispersion is called the polarization mode dispersion (PMD) and may present a serious effect in deformation of the pulse shape, particularly for a very short pulse whose dispersion length can become comparable to the random variation of the polarizations. For a soliton the self-trapping effect created by the Kerr nonlinearity helps to confine the pulse even in the presence of PMD [8]. However, even for solitons with the bit rate beyond, say 40 Gb/s, the PMD effect may become a serious problem because the trapping effect becomes weaker for a shorter pulse.

5.3 *Soliton perturbation theory*

Since a soliton can be described by the four parameters η, κ, T_0 and σ, one can study the transmission properties by following the behavior of these limited number of parameters. In an ideal lossless transmission line of a constant dispersion, these four parameters are conserved exactly from the inverse scattering theorem of the nonlinear Schrödinger equation [4]. In the presence of various perturbations these parameters evolve in Z.

If we represent the perturbation by $iR[q \cdot q']$ the perturbed nonlinear Schrödinger equation becomes

$$i\frac{\partial q}{\partial Z}+\frac{1}{2}\frac{\partial^2 q}{\partial T^2}+|q|^2 q=iR[q,q^*]. \tag{32}$$

The evolution equations for the four parameters may be obtained by the perturbation method of conserved quantities of the nonlinear Schrödinger equation, such as energy and momentum, perturbed inverse scattering transform, or the Lagrangean method. For practical cases, all of these methods give the same result. Since the Lagrangean method is also applicable to nonintegrable cases, such as the dispersion-managed system introduced in §6, we present this method here. We first note that the nonlinear Schrödinger equation can be derived from the variation of the Lagrangean density, $\mathscr{L}(q,q^*)$,

$$\delta\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\mathscr{L}\,\mathrm{dZ}\,\mathrm{d}T=0 \tag{33}$$

with

$$\mathscr{L} = \frac{i}{2}\left(q^*\frac{\partial q}{\partial Z} - q\frac{\partial q^*}{\partial Z}\right) + \frac{1}{2}\left(|q|^4 - \left|\frac{\partial q}{\partial T}\right|^2\right). \tag{34}$$

The nonlinear Schrödinger equation (20) can be expressed by means of the functional derivative of $\mathscr{L}$ with respect to q^*,

$$\frac{\delta\mathscr{L}(q,q^*)}{\delta q^*} = 0. \tag{35}$$

Thus the perturbed nonlinear Schrödinger equation (32) may be written as

$$\frac{\delta\mathscr{L}(q,q^*)}{\delta q^*} - iR[q,q^*] = 0. \tag{36}$$

Evolution equations for soliton parameters, η,κ,T_0 and σ can be obtained from the variation of the finite dimensional Lagrangean $L(\eta,\kappa,T_0,\sigma,\ \mathrm{d}\eta/\mathrm{d}Z,\mathrm{d}\kappa/\mathrm{d}Z,\mathrm{d}T_0/\mathrm{d}Z,\ \mathrm{d}\sigma/\mathrm{d}Z)$. In the absence of the perturbation, the evolution equation is obtained from

$$\delta\int_{-\infty}^{\infty} L\mathrm{d}Z = 0, \tag{37}$$

where the Lagrangean $L(r_j,\dot{r}_j)$ is given by

$$L = \int_{-\infty}^{\infty}\mathscr{L}\mathrm{d}T = -2\eta\left(\kappa\frac{\mathrm{d}T_0}{\mathrm{d}Z} + \frac{\mathrm{d}\sigma}{\mathrm{d}Z}\right) + \frac{1}{3}\eta^3 - \eta\kappa^2 \tag{38}$$

where,

$$r_j = \eta,\kappa,T_0,\sigma \tag{39}$$

and

$$\dot{r}_j = \frac{\mathrm{d}\eta}{\mathrm{d}Z},\ \frac{\mathrm{d}\kappa}{\mathrm{d}Z},\ \frac{\mathrm{d}T_0}{\mathrm{d}Z},\ \frac{\mathrm{d}\sigma}{\mathrm{d}Z}. \tag{40}$$

Equation (38) naturally gives the Euler–Lagrange equation of the form,

$$\frac{\mathrm{d}}{\mathrm{d}Z}\left(\frac{\partial L}{\partial\dot{r}_j}\right) - \frac{\partial L}{\partial r} = 0. \tag{41}$$

For example if we take $r_j = \eta$, eq. (41) reads

$$\frac{\mathrm{d}}{\mathrm{d}Z}\left(\frac{\partial L}{\partial\eta}\right) = 0 = \frac{\partial L}{\partial\eta} = -2\left(\kappa\frac{\mathrm{d}T_0}{\mathrm{d}Z} + \frac{\mathrm{d}\sigma}{\mathrm{d}Z}\right) + \eta^2 - \kappa^2, \tag{42}$$

Similarly by taking $r = k$,

$$\frac{\partial L}{\partial\kappa} = -2\eta\left(\kappa + \frac{\mathrm{d}T_0}{\mathrm{d}Z}\right) = 0, \tag{43}$$

We note that in terms of unnormalized time position shift Δt_0, eq. (43) gives,

$$\frac{\Delta t_0}{\mathrm{d}Z} = \Delta\omega_0 k'', \tag{44}$$

which corresponds to eq. (5). In addition by taking $r = T_0$ and σ, we have

$$\frac{\partial L}{\partial T_0} = \frac{\mathrm{d}}{\mathrm{d}Z}\left(\frac{\partial L}{\partial(\mathrm{d}T_0/\mathrm{d}Z)}\right) = -2\left(\kappa\frac{\mathrm{d}\eta}{\mathrm{d}Z} + \eta\frac{\mathrm{d}\kappa}{\mathrm{d}Z}\right) = 0. \tag{45}$$

$$\frac{\partial L}{\partial \delta} = \frac{\mathrm{d}}{\mathrm{d}Z}\left(\frac{\partial L}{\partial(\mathrm{d}\delta/\mathrm{d}Z)}\right) = -2\frac{\mathrm{d}\eta}{\mathrm{d}Z} = 0. \tag{46}$$

In the presence of perturbation the variational equation is modified in accordance with eq. (36). In order to accommodate the modification in the evolution equation (sometimes called the dynamical equation), we write the derivatives of Lagrangean L in eqs (42) to (46) in the form of a chain-rule for example, for r_j,

$$\frac{\partial L}{\partial r_j} = \int_{-\infty}^{\infty}\left(\frac{\delta\mathscr{L}}{\delta q_0(T)}\,\frac{\partial q_0(T)}{\partial r_j} + \frac{\delta\mathscr{L}}{\delta q_0^*(T)}\,\frac{\partial q_0^*(T)}{\partial r_j}\right)\mathrm{d}T. \tag{47}$$

The functional derivation of $L[q_0, q_0^*]$ can be obtained from (35) with

$$q(T,z) = q_0(T,z) + q_1(T,z) \tag{48}$$

and q_0 is the soliton solution

$$q_0(T,Z) = \eta(Z)\mathrm{sech}[\eta(Z)(T - T_0(Z))]\mathrm{e}^{-ik(Z)(T-T_0(Z))+i\sigma(Z)}. \tag{49}$$

Namely, by inserting eqs (48) and (49) into eq. (36), we have

$$\frac{\delta\mathscr{L}}{\delta q_0^*}[q_0, q_0^*] = iR[q_0, q_0^*] - \mathscr{L}_1[q_0, q_0^*; q_1, q_1^*] \tag{50}$$

where $\mathscr{L}_1$ is the linearized portion of $\delta\mathscr{L}_0[q,q^*]/\delta q^*$ around $q = q_0$ and $q^* = q_0^*$. By incorporating the perturbation term in eq. (46) one can rederive the evolution equations for the soliton parameters. The resultant equations read

$$\frac{\partial L}{\partial r_j} - \frac{\mathrm{d}}{\mathrm{d}Z}\left(\frac{\partial L}{\partial \dot{r}}\right) = \int_{-\infty}^{\infty}\left(R^*\frac{\partial q}{\partial r_j} + R\frac{\partial q^*}{\partial r_j}\right)\mathrm{d}T, \tag{51}$$

$$r_j = \eta, \kappa, T_0 \quad \text{and} \quad \sigma. \tag{52}$$

For example, the soliton parameters η and κ are modified according to the following equations.

$$\frac{\mathrm{d}\eta}{\mathrm{d}Z} = \int_{-\infty}^{\infty}\mathrm{Re}\{R[q_0, q_0^*]\mathrm{e}^{-i\varphi}\}\mathrm{sech}\tau\;\mathrm{d}\tau, \tag{53}$$

$$\frac{\mathrm{d}\kappa}{\mathrm{d}Z} = -\int_{-\infty}^{\infty}\mathrm{Im}\{R[q_0, q_0^*]\mathrm{e}^{-i\varphi}\}\mathrm{sech}\tau\tanh\tau\;\mathrm{d}\tau, \tag{54}$$

$$\frac{\mathrm{d}T_0}{\mathrm{d}Z} = -\kappa + \frac{1}{\eta^2}\int_{-\infty}^{\infty} \mathrm{Re}\{R[q_0, q_0^*]\mathrm{e}^{-i\varphi}\}\tau\mathrm{sech}\tau \, \mathrm{d}\tau$$
$$+ \varphi \int_{-\infty}^{\infty} \mathrm{Im}\{q_1 \mathrm{e}^{-i\varphi}\} \tanh\tau \, \mathrm{sech}\tau \, \mathrm{d}\tau, \tag{55}$$

$$\frac{\mathrm{d}\delta}{\mathrm{d}Z} = \frac{1}{2}(\kappa^2 + \eta^2) + \frac{1}{\eta}\int_{-\infty}^{\infty} \mathrm{Im}\{R[q_0, q_0^*]\mathrm{e}^{-i\varphi}\}\mathrm{sech}\tau(1 - \tau\tanh\tau)\mathrm{d}\tau,$$
$$+ \frac{\kappa}{\eta^2}\int_{-\infty}^{\infty} \mathrm{Re}\{R[q_0, q_0^*]\mathrm{e}^{-i\varphi}\}\tau\mathrm{sech}\tau \, \mathrm{d}\tau,$$
$$+ \eta \int_{-\infty}^{\infty} \mathrm{Re}\{q_1 \mathrm{e}^{-i\varphi}\}\mathrm{sech}\tau \, \mathrm{d}\tau. \tag{56}$$

here

$$\phi = -\kappa T + \sigma. \tag{57}$$

5.4 *Problems in soliton transmission systems*

The perturbation theory introduced in §5.2 shows that the soliton parameters evolve in Z due to the perturbation. When one soliton is used to represent one digit, the major cause that leads to the loss of information is the timing jitter which originates from the variation of the soliton position T_0 since the waveform of the soliton is robust. This situation is quite different from a non-soliton pulse where the major cause of information loss is due to the deformation of the waveform itself. The variation of T_0, as can be seen from eq. (55), originates from the frequency shift κ and the finite dispersion (since Z is normalized by the dispersion distance, see eq. (44)). This is because when the soliton frequency shifts, it modifies the group velocity through the group dispersion, and results in the change in soliton position.

For example, the amplifier noise randomly modulates the soliton frequency and induces random variation of the soliton position (random walk). This fact was pointed out first by Gordon and Haus [9] who derived the mean square variation of the soliton position, $\langle T_0^2(Z)\rangle$ at $Z = Z_a$ which is given by

$$\langle T_0^2 \rangle = \frac{(G-1)\eta}{9N_0 Z_a} Z^3, \tag{58}$$

where G is the amplifier gain, Z_a is the normalized amplifier spacing and N_0 is the number of photons per unit energy.

Interactions between adjacent solitons also produces timing jitter caused by the frequency shift, given by [10].

$$\frac{\mathrm{d}\Delta\kappa}{\mathrm{d}Z} = 8\eta^3 \mathrm{e}^{-\eta\Delta T} \cos\phi, \tag{59}$$

where η is the amplitudes of two adjacent solitons and ϕ is the phase difference between the two solitons. Equation (59) indicates that if the phase of the two solitons is the same (opposite), two solitons are attracted to (repelled from) each other, forming a bound (separate) state; i.e., two solitons eventually collide with (separate from) each other. In order to avoid the collision, two adjacent solitons should be separated enough ($\geq$ six times pulse width in time). This requirement is a severe drawback for soliton-based communications because it requires bandwidth at least three times wider than linear systems where the pulse separation can be twice or less than the pulse width.

When solitons are used in a wavelength division multiplexing (WDM) system, imperfect (or asymmetric) collision between solitons at different wavelength channels also induces timing jitter because of reminiscent frequency shifts [11,12]. Such imperfect collisions occur at the input, output, or at amplifiers.

Although solitons are clearly the better choice as the information carrier in optical fibers because of their robust nature, the timing jitter caused by various effects as described above can lead to the loss of information even though the wave shape is intact throughout the transmission. These problems can be solved to some extent by various means of soliton control [13–15].

5.5 *Soliton transmission control*

The control of timing jitter induced by the Gordon-Haus effect and/or interactions with solitons in other WDM channels by means of frequency filters can be demonstrated by the use of perturbed eqs (53) and (54). If we designate the filter strength by β and the excess gain to compensate for the energy loss by the filter by δ, the perturbation term R in eq. (32) reads

$$R = \delta q + \beta \frac{\partial^2 q}{\partial T^2}. \tag{60}$$

The evolution equations for soliton amplitude η and the velocity κ then are given from eq. (53) and (54) as

$$\begin{aligned}\frac{\mathrm{d}\eta}{\mathrm{d}Z} &= \int_{-\infty}^{\infty} \mathrm{Re}\left[\left(\delta q + \beta \frac{\partial^2 q}{\partial T^2}\right) \mathrm{e}^{i\kappa T - i\sigma}\right] \mathrm{sech}\tau \; \mathrm{d}\tau \\ &= 2\delta\eta - 2\beta\left(\frac{1}{3}\eta^3 + \kappa^2\eta\right), \end{aligned} \tag{61}$$

$$\begin{aligned}\frac{\mathrm{d}\kappa}{\mathrm{d}Z} &= -\int_{-\infty}^{\infty} \mathrm{Im}\left[\left(\delta q + \beta \frac{\partial^2 q}{\partial T^2}\right) \mathrm{e}^{i\kappa T - i\sigma}\right] \mathrm{sech}\tau \; \tanh\tau \; \mathrm{d}\tau \\ &= -\frac{4}{3}\beta\eta^2\kappa. \end{aligned} \tag{62}$$

Equations (61) and (62) can be reduced to a gradient flow by introducing new variables, $x = \sqrt{3/2}\kappa, y = \eta$ and $s = 2\beta(z/3)$,

$$\frac{\mathrm{d}x}{\mathrm{d}s} = -2xy^2 \equiv \frac{\partial\phi}{\partial x}, \tag{63}$$

$$\frac{\mathrm{d}y}{\mathrm{d}s} = \frac{3\delta}{\beta}y - y^3 - 2x^2y \equiv \frac{\partial\phi}{\partial y}, \tag{64}$$

with the potential function $\phi(x,y)$ given by

$$\phi = \frac{3\delta}{2\beta}y^2 - \frac{1}{4}y^4 - x^2y^2. \tag{65}$$

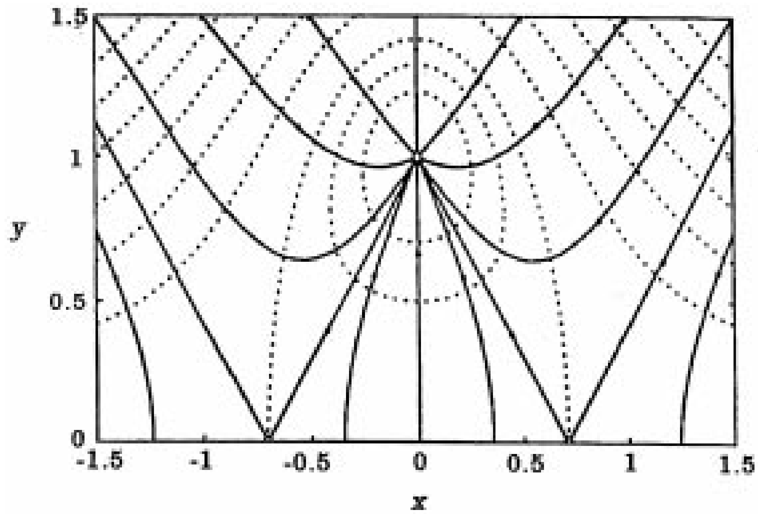

Figure 2. Flow (solid curves) and equi - potential (dashed curves) line in the $x-y$ plane determined by the dynamical equations (4.26) and (4.27). Note that for a given value of β and δ, the soliton amplitude $(\eta = y)$ and velocity $(k = \sqrt{2/3}x)$ acquire a stable fixed value designated by the sink $(x = 0, y = 1)$.

Figure 2 shows the flow lines (solid curves with arrows) and the equipotential lines (dashed curves) for the case with $\beta = 3\delta$. The flow has a sink at $\eta = 1$ and $\kappa = 0$. This indicates solitons having an initial range of amplitudes and velocities (or frequency shifts) emerge as solitons with an identical amplitude and a velocity given by the value at the prescribed sink after repeated amplifications. This process may be interpreted as soliton self-organization. Thus, the frequency-dependent gain makes the prescribed soliton an attractor, and this property of a soliton can be used to overcome the Gordon-Haus effect (figure 2).

This type of adiabatic behavior of a soliton results from the soliton property. Suppose the amplitude increases. From the soliton property it accompanies the decrease of the pulse width thus the increase of the spectral width. The increased spectral width is cut off by the frequency filter, which brings back the amplitude to the original fixed point.

Unfortunately, however, the perturbation given by eqs (61) and (62) does not provide stable behavior on the original infinite dimensional solution itself. For example, the linear wave having $\kappa = 0$ component suffers no effect of the filter and sees the excess gain δ only. Thus the same perturbation leads to unstable growth of the linear wave, which eventually destroys soliton itself.

6. Dispersion management

6.1 *Behavior of an optical pulse*

An optical pulse can propagate free of distortion if the fiber response is completely linear and integrated dispersion and the loss compensated by gain over the entire span is zero even

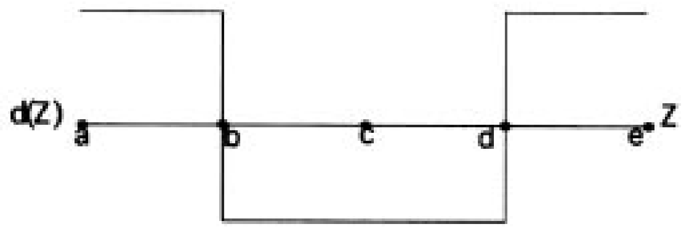

Figure 3. A canonical dispersion map for dispersion managed soliton systems.

if the local value of dispersion is finite and fiber has a finite loss. Furthermore since the timing jitter induced by the frequency perturbation is proportional to the fiber dispersion (see eq. (55)), solitons faces use no timing jitter if the integrated dispersion is zero.

To illustrate these, let us consider the propagation of an optical pulse in a lossless fiber in which the group velocity dispersion varies periodically as shown in figure 3. If the fiber is linear, the pulse that starts at point a can recover the original shape completely at point e if the dispersion in anomalous region d_0 at $a \leq Z \leq b$ and $d \leq Z < e$ is opposite from that in normal dispersion region, $-d_0$ at $b \leq Z < d$, provided $\overline{ab} + \overline{de} = \overline{bd}$. However if the dispersion map is not symmetric, the pulse at $z = e$ cannot recover the original shape.

Let us consider what happens if we take into account the fiber nonlinearity. As was discussed in §3 the major nonlinearity of a fiber originates from the Kerr effect. This produces self-induced phase shift and induces chirp in the pulse. The fiber dispersion also induces a chirp. In an anomalous (normal) dispersion region, the direction of the chirp due to nonlinearity is opposite from (same as) that due to the dispersion.

A soliton is produced if the self-induced chirp due to nonlinearity cancels with that due to the anomalous dispersion. Let us now consider what happens if the local dispersion is much larger than that which allows a soliton solution for the given pulse intensity. Then the pulse acquires certain amount of chirp at the end of the anomalous dispersion section $Z = b$ of figure 3. However this amount of chirp is reversed as the pulse propagates through the normal dispersion section and may be completely reversed at point $Z = d$. Then the amount of the chirp can again become zero at the end of periodicity at $Z = e$. In fact, it can be shown that for a proper choice of the initial pulse width and pulse intensity, one can construct a nonlinear pulse that can recover the initial pulse shape at each end of the periodic dispersion map for a wide range of the value of the average dispersion $\langle d \rangle$. This include $\langle d \rangle = 0$ and even $\langle d \rangle$ is slightly negative (average normal dispersion) [16].

Furthermore it can be shown that even if the pulse does not recover its original shape at the end of one period, it recovers the original shape after several spans of the periodic map with a proper choice of the initial pulse width and the intensity.

Such nonlinear stationary pulses are produced by a balance between properly averaged dispersion, weighted by spectral change, and nonlinearity and behave quite similar to the ideal soliton in terms of their stability and dynamic range of nonlinearity and dispersion. Consequently, they are often called dispersion managed (DM) solitons. The dispersion managed solitons are attractive because, by taking a map having zero or close to zero average dispersion, their timing jitter induced by frequency modulation can be made to zero or close to zero, since the rate of change of the time position T_0 is proportional to the amount of the dispersion eq. (5).

6.2 *Analysis of behavior of dispersion managed soliton*

In this section we present analysis of behavior of dispersion managed solitons based on the Lagrangean method.

The envelope equation for properly normalized electric field of optical field in a fiber with the group velocity dispersion variation, $d(Z)$ in the direction of propagation Z satisfies

$$i\frac{\partial q}{\partial Z} + \frac{d(Z)}{2}\frac{\partial^2 q}{\partial T^2} + \alpha|q|^2 q = -i\Gamma q + iG(Z)q, \tag{66}$$

Here T is the time normalized by the pulse width t_0, Z is the distance normalized by the dispersion distance $(-t_0^2/k"), \Gamma$ is the loss rate per dispersion distance and $G(Z)$ is the amplifier gain. Equation (66) may be reduced to a Hamiltonian structure by introducing a reduced amplitude u

$$u = q/a \tag{67}$$

where

$$\mathrm{d}a/\mathrm{d}Z = [-\Gamma + G(Z)]a \tag{68}$$

and

$$i\frac{\partial u}{\partial Z} + \frac{d(Z)}{2}\frac{\partial^2 u}{\partial T^2} + \alpha a^2(Z)|u|^2 u = 0, \tag{69}$$

Equation (69) is not integrable because of inhomogeneous coefficients $a(Z)$ and $d(Z)$. The dispersion managed soliton normally requires the average $d(Z), \langle d(Z)\rangle$, much smaller than the local $|d(Z)|$;

$$\langle d(Z)\rangle = \frac{1}{L_p}\int_0^{L_p} \mathrm{d(Z)dz} \ll |\mathrm{d(Z)}|, \tag{70}$$

where L_p is a periodic length of the dispersion map. If $\langle d(z)\rangle = 0$ and in the absence of nonlinearity, eq. (70) has an exact periodic solution given by a Gaussian with frequency chirp. Thus, let us apply an additional transformation of u to a new real amplitude $f(T,Z)$ by eliminating the chirp C and frequency shift κ through

$$u(T,Z) = \sqrt{p(Z)}f[p(Z)(T - T_0(Z), Z] \times \exp\{iC(Z)(T - T_0(Z))^2/2 - i\kappa(Z)T + i\theta_0(Z)\} \tag{71}$$

where p, C and κ represent the inverse of the pulse width, chirp coefficient and frequency shift all real functions of Z. T_0 and θ_0 represent the soliton position and phase. f then satisfies

$$i\frac{\partial f}{\partial Z} + \frac{dp^2}{2}\frac{\partial^2 f}{\partial T^2} + \alpha p a^2(Z) f^3 = \frac{\kappa_1 T^2 p}{2} f, \tag{72}$$

Where $\mathrm{d}p/\mathrm{d}Z = -Cpd$ is chosen and

$$\kappa_1 = \frac{\mathrm{d}C/\mathrm{d}Z + C^2 d}{p^2}. \quad (73)$$

We note here that the dispersion coefficient now becomes p^2d. This is because if the pulse width changes in Z, spectrum width changes in proportion to p and the dispersive effect becomes proportional to p^2d. This implies that a nonlinear stationary solution in this case is obtained by balance between the reduced dispersion $\langle p^2 d\rangle$ and the nonlinearity $\langle a(Z)p\rangle$ rather than the dispersion and nonlinearity like in the case of an ideal soliton in a constant dispersion.

Equation (69) can be constructed by the variation of the Lagrangean density $\mathscr{L}(T,Z)$ given by

$$\mathscr{L}(T,Z) = \frac{i}{2}(u_Z u^* - u_Z^* u) + \frac{\alpha a^2(Z)}{2}|u|^4 - \frac{d(Z)}{2}|u_T|^2. \quad (74)$$

If we substitute the ansatz (71) into (74) and assume explicit variation of f in Z is negligible, and integrate the result over T, one can construct the Lagrangean for parameters p,κ and θ_0,

$$L = \frac{\alpha a^2 p I_4}{2} - \frac{\mathrm{d}p^2 I_3}{2} - \frac{\mathrm{d}C^2 I_2}{2p^2} - \frac{\mathrm{d}\kappa^2 I_1}{2} - \frac{I_2}{2p^2}\frac{\mathrm{d}C}{\mathrm{d}Z} + I_1\left(T_0\frac{\mathrm{d}\kappa}{\mathrm{d}Z} - \frac{\mathrm{d}\theta_0}{\mathrm{d}Z}\right), \quad (75)$$

where

$$\left.\begin{array}{ll} I_1 = \int_{-\infty}^{\infty} f^2(\tau)\mathrm{d}\tau, & I_2 = \int_{-\infty}^{\infty} \tau^2 f^2(\tau)\mathrm{d}\tau \\ I_3 = \int_{-\infty}^{\infty} \left(\frac{\mathrm{d}f}{\mathrm{d}\tau}\right)^2 \mathrm{d}\tau, & I_4 = \int_{-\infty}^{\infty} f^4(\tau)\mathrm{d}\tau \end{array}\right\}. \quad (76)$$

Variation of the Lagrangean gives the Lagrangean equation of motion

$$\frac{\partial}{\partial z}\left(\frac{\partial L}{\partial \dot{r}_j}\right) - \frac{\partial L}{\partial r_j} = 0 \quad (77)$$

for $r_j = p, C, \kappa, T_0$ and θ_0, which for a stationary Gaussian pulse of the type (26), can be expressed by a coupled set of evolution equations for these parameters,

$$\frac{\mathrm{d}p}{\mathrm{d}Z} = -pCd \quad (78)$$

$$\frac{\mathrm{d}C}{\mathrm{d}Z} = -C^2 d - \frac{\alpha a^2 p^3}{\sqrt{2}} + 2\mathrm{d}p^4 \quad (79)$$

$$\frac{\mathrm{d}\kappa}{\mathrm{d}Z} = 0 \quad (80)$$

$$\frac{\mathrm{d}T_0}{\mathrm{d}Z} = -\kappa d \quad (81)$$

$$\frac{\mathrm{d}\theta_0}{\mathrm{d}Z} = \frac{\kappa^2 d}{2} - \mathrm{d}p^3 + \frac{5\alpha a^2 A^2}{4\sqrt{2}}. \quad (82)$$

Here we present some properties of dispersion managed solitons by taking a simple example of the dispersion map as shown in figure 3.

Eliminating Z from eqs (78) and (79), we have

$$\frac{\mathrm{d}}{\mathrm{d}p}\left(\frac{C^2}{2}\right) = \frac{2}{p}\left(\frac{C^2}{2}\right) + \frac{\alpha a^2 p^2}{2\sqrt{2}d} - p^3, \tag{83}$$

which can be formally integrated to give

$$\frac{C^2}{2} = C_0 p^2 - \frac{p^4}{2} + p^2 \int \frac{\alpha a^2 p^2}{2\sqrt{2}d}\mathrm{d}p, \tag{84}$$

where C_0 is a constant and plays a role of the Hamiltonian.

If we substitute C from eq. (84) into (78), we can obtain the differential equation for $\mathrm{d}p/\mathrm{d}Z$ involving only $a^2(z)$, $\mathrm{d}(z)$ and p. This can be solved for $p(Z)$.

To demonstrate the nature of the solution $p(Z)$ and $C(Z)$, let us take a simple example of a lossless fiber $a(Z) = a_0$ (=constant) and $d(Z) = \pm d_0$ (piecewise constants) as shown in figure 3.

In this case eq. (84) gives

$$\frac{C^2}{2} = C_0 p^2 - \frac{p^4}{2} - A_0 p^3 \tag{85}$$

where

$$A_0 = \frac{\alpha a_0^2}{2\sqrt{2}d_0}. \tag{86}$$

represents the strength of nonlinearity. One can obtain a periodic solution with the periodicity L_p by a proper choice of initial conditions $p(0)$ and $C(0)$ so that $p(L_p) = p(0)$ and $C(L_p) = C(0)$. For a linear pulse with $A_0 = 0$, the periodic solution for f is a Gaussian with a periodically varying chirp parameter $C(Z)$ and the trajectories in $p-C$ plane at $d = d_0$ and $d = -d_0$ completely overlays themselves. However the nonlinearility produces a gap in these trajectories, because of the frequency chirp produced by the self-induced phase shift.

The peak value of p at point c (normal dispersion regime) is less than that at anomalous dispersion regime because of the nonlinearity induced self phase shift. This makes $\langle p^2 d\rangle > 0$ even if $\langle d\rangle = 0$ and the balance between the weighted average $\langle p^2 d\rangle$ and the nonlinearity produces the stationary solution. This situation is analogous to an ideal soliton solution, which is constructed by a balance of (constant) dispersion and nonlinearity. However, in the dispersion managed case, the above argument indicates that the nonlinear stationary solution is possible even for $\langle d\rangle < 0$ or $\langle d\rangle \geq 0$, provided that $\langle p^2 d\rangle > 0$ and proper nonlinearity exists.

An ideal soliton solution for a fiber with $d = d_0 =$ const, can be constructed for an arbitrary value of d_0 by a proper choice of the amplitude, while the linear stationary solution exists only for $d_0 = 0$. Similarly for a dispersion managed case, a nonlinear stationary periodic pulse can be constructed for an arbitrary value of $\langle d\rangle$ by a choice of initial amplitude and chirp, while the linear stationary is possible only for $\langle d\rangle = 0$. This allows DM solitons to have much larger tolerance in the fiber dispersion. We here note again that, if the system is linear, the trajectory in $p-C$ plane shown in figure 3 returns at the original point d only when the average dispersion $\langle d\rangle$ is exactly zero. We further note that if $p(0)$

and/or $C(0)$ is not chosen so that after one period $p(L)$ and/or $C(L)$ does not return to the original value, it was numerically confirmed $p(nL)$ and/or $C(nL)$ returns to a limit area in $p-C$ plane, where $n = 1, 2, \ldots$. In other words eqs (72) and (73) in general have doubly periodic solution.

6.3 *Quasi-soliton*

Although dynamical equations describe evolution of the pulse parameters, the DM soliton introduced in §6.2 has no exact solution. Therefore, those dynamical equations make sense only under the assumption that there exists a localized stable nonlinear pulse as assumed. However, this conjecture is not always valid. For example, when two DM solitons in two wavelength channels interact, they can be strongly deformed. Kumar and Hasegawa [17] derived a concept of a soliton in a dispersion managed system that allows an exact solution, which is found numerically stable. By inspecting eq. (66), we note that if we choose the dispersion $\mathrm{d}(Z)$ such that $pd =$ constant, coefficients of the resultant equation becomes constant in the new coordinate $Z' = \int p\mathrm{d}Z$. Kumar and Haseagawa have found that a map that allows $pd =$ constant, is in fact feasible and constructed a new stable soliton whose shape is given by eq. (66) having constant coefficients and called it the quasi-soliton. The quasi-soliton has a chirp and is characterized by a solution of nonlinear Schrödinger equation having a combination of linear parabolic potential and nonlinear potential of eq. (66). The pulse shape is near Gaussian at the tail and sec$T$ near the peak. Serkin and Hasegawa [18] extended this concept and obtained exact nonlinear pulses in a periodically varying dispersion and confirmed their stability. The quasi-soliton differs from the DM soliton in that it is a stable nonlinear pulse in specifically programmed dispersion, while the latter is a nonlinear localized pulse in a step-wise periodic dispersion. Constructing a fiber that has a specifically designed continuously varying dispersion map may be more difficult than a simple periodic map with a piece-wise constant dispersions, however, the stable nature of a quasi-soliton may warrant the trouble of constructing such a fiber.

7. Dispersion managed solitons versus quasilinear pulses

7.1 *Dispersion managed solitons*

We saw that one can construct a nonlinearly stationary pulse in a fiber with a periodic variation of group dispersion; Even if the pulse width oscillates the pulse recovers exactly the original shape at the end of each period.

Let us now discuss what is the ideal value for the choice of periodicity Z_p of the map and the variation of the dispersion. There exists a parameter that determines the strength s of the map defined by

$$s = \frac{(k_2'' - \langle k'' \rangle)z_2 - (k_1'' - \langle k'' \rangle)z_1}{t_0^2}. \tag{87}$$

Here k_1'' and k_2'' are the dispersion in ps^2/km in the fiber having distances z_1 and z_2 respectively, and t_0 is the pulse FWHM. The parameter s is sometimes called the power enhancement factor [16] because DM solitons have a power higher than an ideal soliton for the same value of $\langle k'' \rangle$. And the amount of the power enhancement is numerically found to be proportional to s. In addition, the maximum pulse width of a dispersion managed soliton (DMS) is determined by s since the pulse tends to spread during its propagation over a fiber with constant dispersion. Thus it is expected that larger (smaller) the value of s, smaller (larger) the magnitude of the pulse width oscillation becomes. However if we take a limit of infinitely small s, the transmission line becomes effectively equivalent to that having a constant dispersion given by $\langle k'' \rangle$. In this case the breathing of the pulse disappears since the pulse becomes an ideal soliton. In this limit, however, the pulse shape becomes sech$T$ and accompanies a long exponential tail and the interaction between adjacent soliton increases. On the contrary, if we take too large a value of s, adjacent pulses overlap during the transmission and induce undesirable interaction. The interactions induce four wave mixing components among different side bands of the carrier, which contribute to the noise. (Four-wave mixing will be discussed later in this section.) In addition two pulses eventually collide each other.

It was numerically found by Golovchenko *et al* [19] that the collision distance of two adjacent DMS with the duty ratio of 1:4 becomes longest for a choice of $s = 1.65$ for a loss less fiber. That is for a choice of $s = 1.65$ the interaction between two adjacent pulses becomes minimum. From the spirit of soliton concept, the dispersion managed soliton should choose the value of s around this value.

For an extremely high bit-rate transmission over 40 Gb/s, t_0 becomes very small. In order to keep the ideal value of s, the map period becomes very small. Liang *et al* [20] have demonstrated a short period dispersion management is very effective for bit-rate higher than 80 Gb/s and called it dense dispersion management (DDM).

7.2 *Quasi-linear pulse*

Linear transmission groups also have come up with the concept of dispersion management based on different arguments. Linear groups have recognized early in the stage of all optical transmission that nonlinear effects severely distort NRZ pulses. They applied phase modulation at the bit-rate to overcome this problem and recognized the pulses eventually become RZ pulses. They also have recognized that when the local dispersion is made large pulse width increases and nonlinear effects are reduced because of reduced intensity. If the dispersion is reversed such that the integrated dispersion between end to end is zero and if the system is ideally linear, one can recover the original pulse at the receiver side.

However they also recognized that the nonlinear effect which induces pulse distortion even for RZ pulses is unavoidable and designed a map that compensates for the nonlinearly induced chirp and pulse width. An RZ pulse which is generated this way is called a quasi-linear pulse.

7.3 *Four wave mixing and cross phase modulation*

The cubic nonlinearity given by $|q|^2 q$ produces four wave mixing and cross phase modulation in the presence of two waves at different frequencies. Let us consider two waves at frequencies ω_1 and ω_2. In addition to the self induced phase shift terms, $|q_i|^2 q_i (i = 1.2)$

for each frequency component, the cubic nonlinearity produces combinations of frequencies given by $\omega_1 - \omega_1 + \omega_2 (= \omega_2), \omega_2 - \omega_2 + \omega_1 (= \omega_1), 2\omega_1 - \omega_2$ and $2\omega_2 - \omega_1$ in Fourier space, which are given by terms $|q_1|^2 q_2,\ |q_2|^2 q_1,\ q_1^2 q_2^*,\ q_2^2 q_1^*$ respectively. The effect that originates from the former two terms is called the cross phase modulation and that from the latter two the four wave mixing. When two pulses overlap these terms contribute to nonlinear effects in addition to the self-frequency shift.

In an ideal loss-less transmission line with a constant dispersion, two solitons which start from well-separated position suffers no reminiscent effect caused by the cross terms when they collide other than a slight position and phase shifts. This interesting fact originates from the integrability of the nonlinear Schrödinger equation in which the soliton parameters behave as invariant. This fact is a very important merit of the use of solitons as information carrier in fibers in addition to the fact that the pulse itself is nonlinearly stable and stationary.

However when the pulse deviates from an ideal soliton such as in the case of dispersion managed soliton, one should consider effects of these cross terms. When only one wavelength channel is used in which nonlinear interaction is limited only by those induced by neighboring pulses, one may avoid the effect by minimizing overlapping of the two pulses or by reducing nonlinearity. The former choice can be achieved by taking the *s* value around at 1.65, while the latter by taking a very large value of *s*. The latter choice avoids the cross term effects by spreading the pulse very thinly during the transmission allowing very large overlap of the pulses. Although this choice is not ideal in minimizing interactions between neighboring pulses, it provides an interesting merit in wavelength division multiplexed (WDM) systems. In WDM transmission, cross-nonlinear effects among different channels present serious problems because pulses in different wavelength channels inevitably collide many times during the transmission. A DM soliton having a large value of *s* may suffer less cross nonlinear effect than the one having a small *s*, unless the ideal soliton property is utilized by choosing relatively small value of the map strength, *s*. What is the optimum choice of value of *s* in WDM transmission is still unknown.

8. Some recent experimental results of optical solition transmission

Dispersion managed solitons are now experimented both in a single channel ultra-high speed transmissions and in wavelength division multiplexed (WDM) transmissions. Here two representative experimental results are introduced, one, 40 Gb/s single channel DMS transmission over 10,000 km by Morita *et al* [21] and others, 1.1 Tb/s WDM DMS transmission over 3,000 km by Fukuchi *et al* [22].

8.1 *40 Gbps single channel DMS transmission experiment*

The most remarkable single channel experiment that the author is aware of is the one done by Morita *et al* of KDD group [16]. Figure 4 shows the experimental set up. The transmission experiments were conducted in a 140 km recirculating loop. In the 20 Gb/s transmitter, a 20 Gb/s optical soliton data stream was produced by optically time- division-multiplexing (OTDM) 10 Gb/s RZ data pulses, which were generated with a DFB-LD,

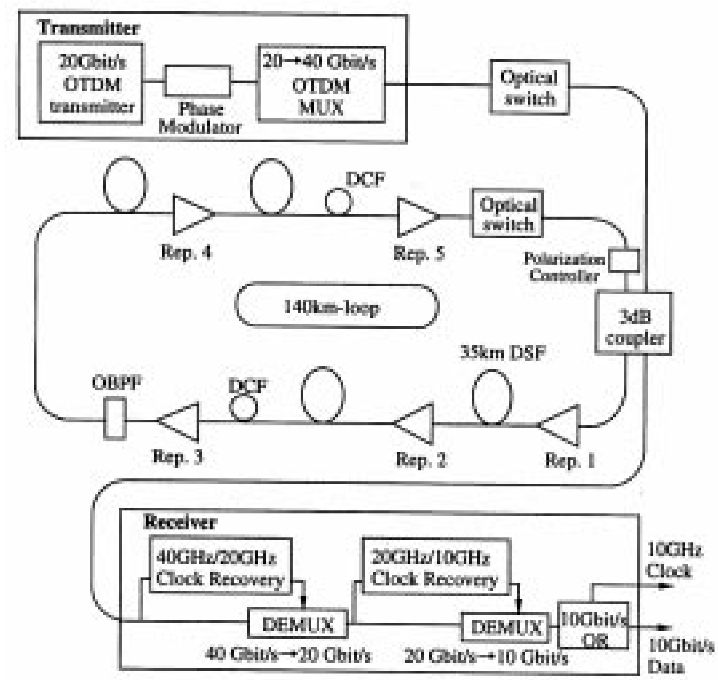

Figure 4. Experimental setup for 40 Gb/s soliton transmission experiment by Morita *et al* [16].

sinusoidally-driven electroabsorption (EA) modulators and two $LiNbo_3$ intensity modulators operated at 10 Gb/s with a $2^{15}-1$ pseudorandom binary sequence. The signal wavelength was 1552.8 nm and obtained pulse width was about 9 ps. To improve the transmission performance, input phase modulation was applied to the output signal from the 20 Gb/s transmitter. The 40 Gb/s signal was generated by OTDM 20 Gb/s signals in 2 ways regarding the state of polarization. The state of polarization of adjacent OTDM channels was set to be parallel or orthogonal. The 140 km recirculating loop consisted of 4 spans of dispersion-shifted fiber (DSF), 5 EDFA repeaters pumped at 980 nm and an optical bandpass filter (OBPF) with 6 nm bandwidth. At the signal wavelength, the average dispersion of DSF was 0.29 ps/nm/km and the system average dispersion was reduced to 0.028 ps/nm/km by compensating for the most of the accumulated dispersion after every two DSF spans. Figure 5 shows this dispersion map schematically. The average span length of the DSF was 35.7 km. The repeater output power was set to about $+4$ dBm.

In the receiver, the transmitted 40 Gb/s signals were optically time-division-demultiplexed with optical gates generated by sinusoidally-driven polarization insensitive EA modulators in two stages; 40 Gb/s to 20 Gb/s and 20 Gb/s to 10 Gb/s. The bit error rate (BER) for the demultiplexed 10 Gb/s signals was measured.

Figure 6 shows the average BER for the four OTDM channels as a function of transmission distance in the cases with the state of polarization of the adjacent OTDM channels parallel $(^\circ)$ and orthogonal $(\cdot)$. In these experiments, the condition of the initial phase modulation was optimized. In addition, in the single- polarization experiment, a low-speed polarization scrambler was used to reduce polarization hole-burning effects of the EDFA repeaters. As shown in figure 6 by setting the state of the polarization orthogonal transmission performance was greatly improved and transmission distance for BER of 10^{-9} was extended from 8,600 km to 10,200 km. The signal waveforms were measured using a high-speed photodetector with and without a polarizer. Figure 7 shows the measured eye diagrams of the 40 Gb/s signals and polarization-division-demultiplexed 20 Gb/s signals before and after 10,000 km transmission. The preserved polarization orthogonality reduced the soliton-soliton interaction effectively through the transmission and made possible to transmit 40 Gb/s data over 10,000 km. This experimental result clearly shows effectiveness of the dispersion management in TDM soliton transmissions.

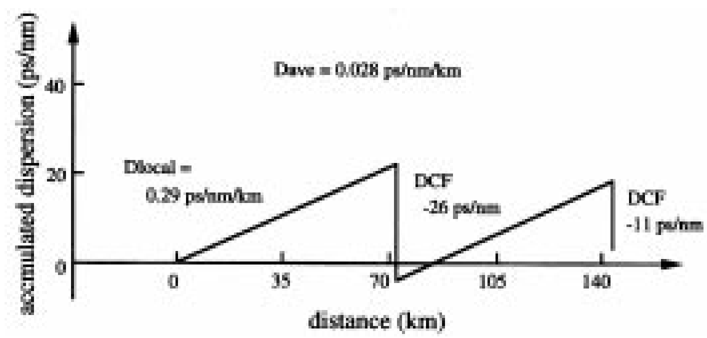

Figure 5. Dispersion map for the 40 Gb/s transmission [16].

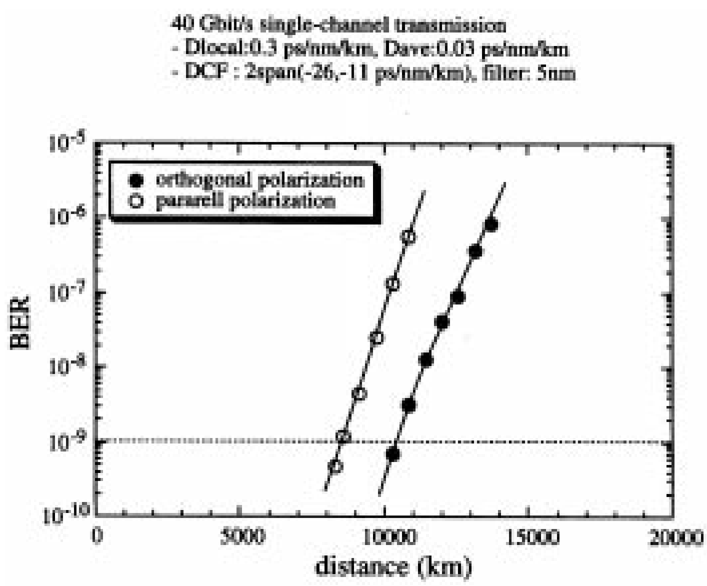

Figure 6. Bit error rate measurement after 10,000 km transmission of the 40 Gb/s soliton signals by Morita *et al* [16]. $(^\circ)$ for parallel polarization and $(\cdot)$ for orthogonal polarization between adjacent pulses.

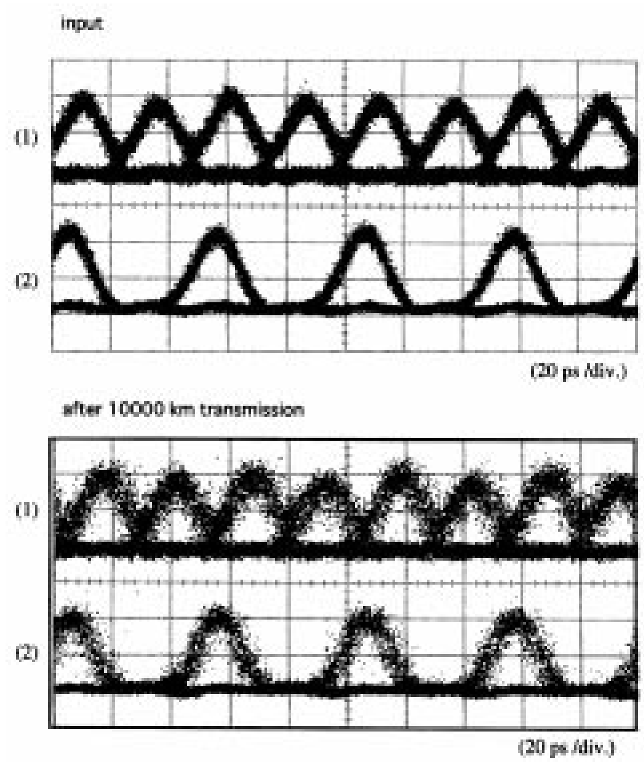

Figure 7. Measured eye diagram of soliton pulses after 10,000 km of transmission. The picture is taking by overlaying a sequence of arrived pulses in the bit frame. The opening of the eye indicates error free transmission [16].

8.2 *1.1 Tb/s (55X20-Gbps) WDM-DMS transmission experiment*

Let us now introduce a recent WDM-DM soliton transmission experiment done by STAR Project Consortium supported by the Japanese Ministry of Post and Telecommunications [17]. Unlike ideal solitons, DM solitons in different wavelength channels collide many times because of the periodic change of dispersion. This could enhance timing jitter and, in a worst case, lead to the destruction of solitons. In order to avoid this problem certain channel separation is required. Fukuchi *et al* [17] have recognized this and used both *C* (1.55 μm) and L (1.58 μm) band of EDFA in parallel so that sufficiently large ($\sim$ 40 mm) wavelength window was available.

The transmission fiber was a combination of a single mode fiber (SMF) and a negative dispersion fiber (NDF). The SMF had a pure-silica core for low loss characteristics. The NDF was designed both to compensate the dispersion and the dispersion slope at 1.55 μm and to minimize the loss in the two bands. The residual dispersion slope in the 1.58 μm band was compensated by a large positive-dispersion-slope fiber placed inside the repeater. Each span consisted of the SMF in the first half and the NDF in the second half. The average span length was 45.7 km. Figure 8 shows the averaged loss and dispersion for 8 fiber spans. The loss increase at 1590 nm was 0.02 dB/km which resulted in an optical-SNR decrease of only 0.9 dB after 3,000 km. The average dispersion was successfully flattened at 0.08 ± 0.025 ps/nm/km in the 1.55 μm band and 0.0 ± 0.05 ps/nm/km in the 1.58 μm band. This 0.1 ps/nm/km difference in dispersion was small enough for soliton transmission at all wavelengths in these bands with the help of prechirp optimization.

Figure 9 shows the experimental setup. 30 DFB-LDs ranging from 1536 to 1559.2 nm in the 1.55 μm band and 25 DFB-Lds ranging from 1572.4 to 1591.6 nm in the 1.58 μm band were used. The wavelength separation was 0.8 nm. For both bands, even and odd channels were multiplexed separately and modulated to form a 20Gb/s RZ signal with

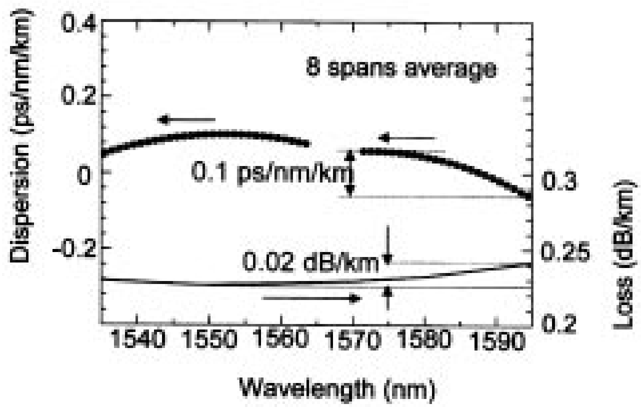

Figure 8. Averaged loss and dispersion of the 1.1 Tb/s WDM soliton transmission experiment by Fukuchi *et al* [17].

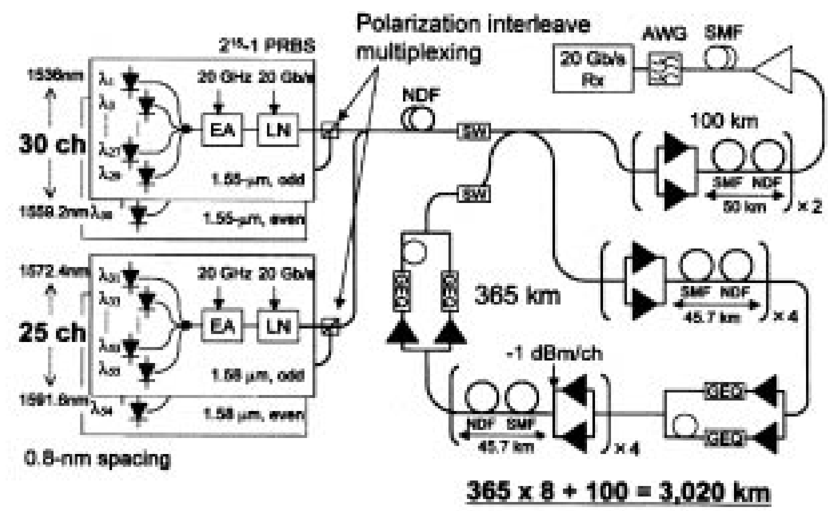

Figure 9. Experimental setup for 1.1 Tb/s WDM soliton transmission by Fukuchi *et al* [17].

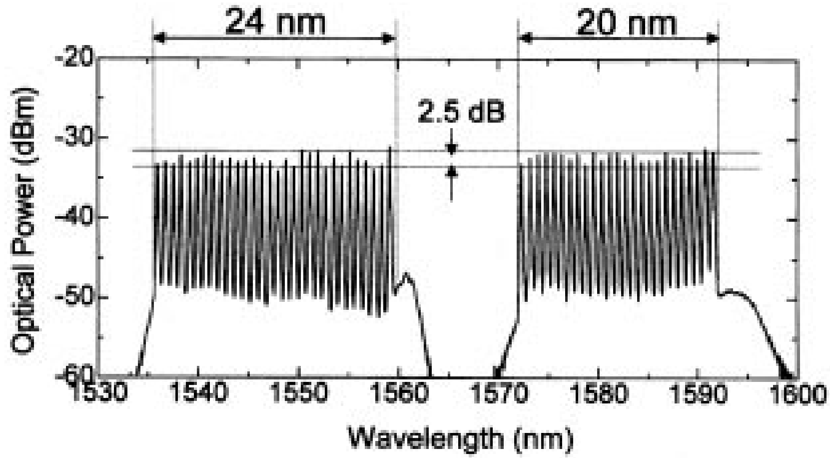

Figure 10. Optical spectrum of 55 channel 20 Gb/s soliton signals after transmission over a distance of 3020 km [17].

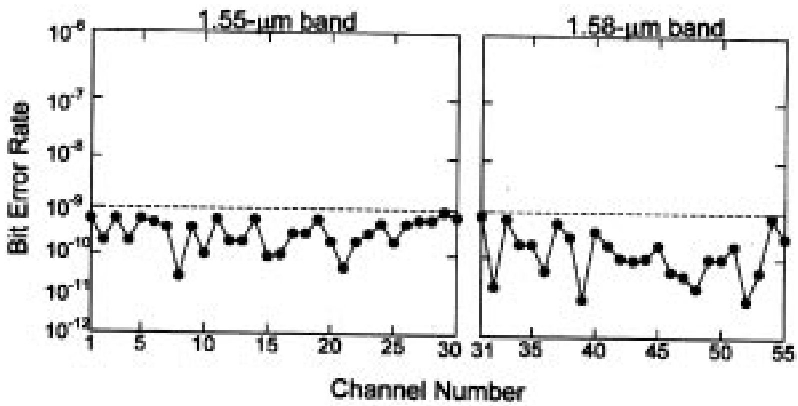

Figure 11. Bit error measurement after 3020 km transmission. Note error rate less than 10^9 has been obtained for all the channels [17].

25ps pulse width. To suppress the cross phase modulation effect, polarization interleave multiplexing was employed by coupling the modulated lights by polarization beam splitters. The prechirp value was optimized for each wavelength. The loop transmission line consisted of 8 spans of the dispersion-flattened fiber, and 2 extra spans were placed at the loop output. Inside the loop, extra repeaters for gain equalization and dispersion slope compensation fiber were placed after every 4 spans. At the receiver end, the measurement channel was extracted by an 0.8 nm-spaced all optical wave guide (AWG) demultiplexer and was received by a 20-Gb/s electrical receiver. Figure 10 shows the optical spectrum after 3,020 km transmission. By adopting the precise gain equalization, a 24 nm bandwidth in the 1.55 μm band and 20 nm in the 1.58 μm band were achieved, and the power difference over the 44 nm window was as small as 2.5 dB. Figure 11 shows the bit error rate measured at 3,020 km transmission. For all 55 channels, the bit error rates were

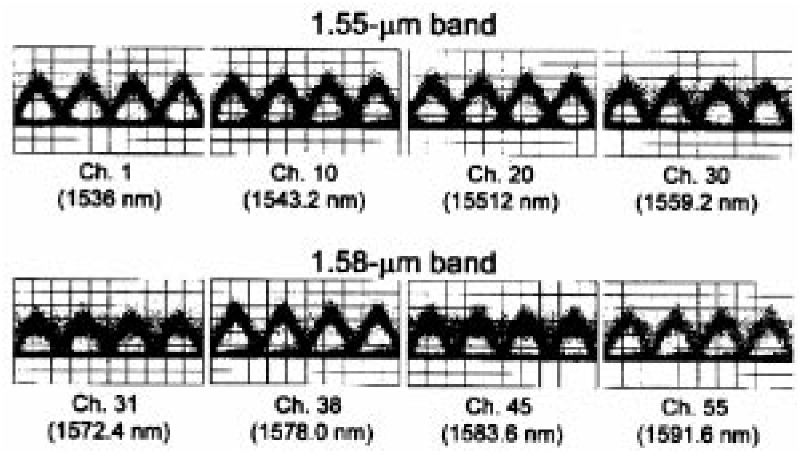

Figure 12. Eye diagram of received soliton signals of selected channels after 3020 km of transmission [17].

less than 10^{-9}. Eye diagrams, for various channels are shown in figure 12. The eye diagrams were similar, indicating the uniformity of the soliton transmission at each wavelength.

This experimental result shows feasibility of WDM DM-soliton transmissions. Although a terabit ($100 \times$ 10-Gbps), 10,000 km WDM-RZ pulse transmission has been announced elsewhere by the use of quasilinear pulses, they used the forward error corrector (FEC) while the present result uses no error correction. Furthermore the result presented here is the only 20 Gb/s per channel WDM experiment beyond 1 Tbps for this distance. Recently experimental result of WDM transmission of 10 Gb/s $\times$ 27 channel DMS is reported by Mollenauer *et al* [23] with a proper soliton control by means of frequency filters and help of Raman amplifications. They succeeded error free transmission over a distance of 10,000 km without the use of FEC. They also claim that limitation in the number of channels is only due to the absence of enough pump power and not essential.

9. Future prospects

The ever-increasing demand for internet traffic will soon exceed today's performance limit of terabits per second per fiber. Although 3.2 Tb/s of capacity has been demonstrated for a distance of 1,500 km [24], the total capacity is limited by the available bandwidth of EDFA. In addition, the error free distance of transmission is limited by cross talk in wavelength channels caused by four wave mixing and cross phase modulation.

Recently new fibers that allow transmission between 1300 nm to 1700 nm have been developed [25] by eliminating water, which had contributed to excessive absorption loss near 1400 nm. The fiber has a maximum loss of about 0.3dB/km over this 400 nm- wavelength range. The only optical amplifier available over this large bandwidth is the Raman amplifier. Since the fiber itself works as a Raman medium, the transmission fiber can be converted to a Raman amplifier by simply pumping at wavelengths approximately 100 nm shorter than the signal wavelength. In fact the first long distance transmission of soliton pulses has been demonstrated by the use of Raman pumps [26] based on the original proposal of the author [1]. Since semiconductor laser diodes with sufficiently large power are now available over this wavelength range, the Raman based ultra-wide band transmission is becoming a reality.

Because Raman amplification has a distributed gain along the fiber, the pump light wave can be inserted into the fiber both in co- and counter-propagation directions with respect to the signal. Since the gain is distributed, a Raman amplifier is equivalent to an EDFA with short amplifier spacing. In this manner a Raman amplifier can provide a significantly better noise figure than an EDFA with larger amplifier spacing. For an example of a hundred-Kilometer amplifier spacing by means of an EDFA, the noise figure reduction could reach about 5dB if a Raman amplifier is used. This allows the use of signal at much reduced power levels, and as a result, an extended distance of massive WDM can be achieved.

The Raman gain is approximately proportional to the difference in wavelength between the signal and the pump until the difference is about 100 nm, beyond which the gain starts to drop. Therefore, in order to obtain a flat gain over sufficient wavelength span, pumps at several wavelengths should be provided. In addition since the Raman energy cascades downward into longer wavelengths, the pump power should be programmed such that the shortest wavelength pump has the highest power, or pump at the shorter wavelengths must be arranged to have denser wavelength separation, or both. Furthermore, care should be taken so that the Rayleigh back-scattered waves do not mix with the signal and contribute to noise. With these measures, it is expected that Raman amplifications will convert a water-free fiber (or equivalent of it) to lossless transmission line over 1300 nm to 1700 nm wavelength. This bandwidth is approximately an order magnitude larger than what an EDFA can cover. Therefore, a fraction of a petabits per second of transmission capability may become a reality once proper dispersion compensation measures are realized.

10. Concluding remarks

Recently it has become increasingly clear that, for a trans-oceanic transmission of 10 Gb/s per channel or beyond in optical fibers, RZ pulse is the only choice. Coherent or NRZ format of transmission has been discarded because of its intolerance to nonlinear effects of fibers. There seem to exist two modes of RZ pulse transmission, however, the dispersion managed soliton and the other so called the quasi-linear or chirped RZ pulse. The dispersion managed soliton is, as shown in this text, a nonlinear stable pulse whose pulse width/pulse height and chirp oscillate periodically or aperiodically. This may be illustrated by a Poincaré map in p, C plane of the set of points obtained at every amplifier position, which is confined within a finite domain. This occurs because of the balance of spectrum weighted average dispersion and nonlinearity similar to an ideal soliton. On the other hand, the quasi-linear pulse has a map which can not be confined within a limited area and significant overlap between adjacent pulses are allowed, albeit the nonlinear effect is taken into account in the analysis of the transmission. The well-separated pulses are recovered only by introduction of dispersion compensating fiber either at the end and/or the beginning of transmission, or at every several hundred kilometers of transmission. Since a large spread of pulse width is allowed, pulse height is significantly reduced and nonlinear interaction between adjacent pulses as well as among different wavelength channels are believed to be reduced. On the other hand, a DM soliton with $s \sim 1.6$ can avoid interaction with adjacent solitons by reducing the pulse width oscillation. The non-overlapped pulse provides a big advantage in the use of transmission within a network since add-drop module can be installed at any point in the line. However, in this case, inter-pulse interaction between other wavelength channels should be avoided either by providing sufficiently large wavelength separation or by soliton control such as by the use of filters. A DM soliton having a large value of s accompanies a large pulse width oscillation, somewhat like a quasi linear pulse, and may be more suitable for the use in WDM transmission because of reduced nonlinear interactions among different wavelength channels.

Acknowledgements

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