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Soliton models in resonant and nonresonant optical fibers

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Abstract. In this review, considering the important linear and nonlinear optical effects like group velocity dispersion, higher order dispersion, Kerr nonlinearity, self-steepening, stimulated Raman scattering, birefringence, self-induced transparency and various inhomogeneous effects in fibers, the completely integrable concept and bright, dark and self-induced transparency soliton models in nonlinear fiber optics are discussed. Considering the above important optical effects, the different completely integrable soliton models in the form of nonlinear Schrödinger (NLS), NLS-Maxwell-Bloch (MB) type equations reported in the literature are discussed. Finally, solitons in stimulated Raman scattering (SRS) system is briefly discussed.

Keywords. Group velocity dispersion; Kerr nonlinearity; optical solitons; self-induced transparency; complete integrability; Lax pair; Bäcklund transformation.

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1. Introduction

We are at the edge of another industrial revolution due to the developments in photonics, namely, the information age. In reality, the amount of information produced by humanity is increasing more than twice every few months. To keep pace with this fast development trend, there is a greater need to connect fiber optic networks undersea and over ultra-long-haul connections. Various technologies have cropped up in the race to move more bits of data faster and farther, with increasing reliability and drastically decreasing costs. Photonics promises to revolutionize the area of information technology and the 21st century, just as electronics revolutionized the past hundred years or so [1].

Solitons are one among the many exciting new technologies emerging in optical networking and they are poised to benefit the commercial ultra-long haul all-optical multi-terabit networks spanning distances up to many millions of kilometers. It could well become one of the fundamental technologies in the current communication revolution [2–7]. Solitons are localized nonlinear waves that have highly stable properties that allow them to propagate very long distances with very little change [3,13–23].

The response of a dielectric to an electromagnetic wave is typically frequency dependent. This results in the dependence of the index of refraction on the frequency, which in turn is dependent on certain natural frequencies at which absorption takes place in the material. Since the speed of a wave depends on the index of refraction, a wave composed of several harmonics will tend to disperse. When a pulse propagates along a fiber, its width increases. The narrower the pulses, the more rapid will be the increase in dispersion. One then can imagine that if we inject too many short pulses into a fiber, they will overlap after propagating over some distance. We then will not be able to distinguish between pulses - and information will be lost. This is known as chromatic dispersion and it tends to distort the incoming signal. The dispersive characteristics of a fiber can be minimized in the operating region of the system by changing the core radius and the difference in the refractive indices of the core and the cladding. The main parameter that characterizes dispersion is the rate of change of the group velocity with respect to the frequency. This is known as group velocity dispersion (GVD). The wavelength at which there is minimum dispersion is the zero-dispersion wavelength (1.33 μm in SiO_2). For wavelengths below this point, the fiber exhibits positive, or normal GVD; while, for larger wavelengths, the dispersion is known as negative, or anomalous GVD [1,6,7].

It is fortunate that there is a counter-effect which shortens the width of a pulse. This effect is called Kerr nonlinearity [24]. In particular, the index of refraction is found to depend on the square of the amplitude of the applied electric field. If the optical fibers are designed appropriately, the dispersive and nonlinear effects can be balanced, thereby allowing for the possibility of a localized optical soliton pulse to propagate through the fiber without distortion due to either effect. At low intensities and for shorter distances, this nonlinear effect is negligible. Even at moderate powers, the nonlinear Kerr effect must be considered when designing wavelength division multiplexing (WDM) communication systems. The lower order nonlinearities manifest themselves in three different ways: self-phase modulation (SPM), cross-phase modulation (XPM) and four wave mixing (FWM). XPM and FWM involve multiple channels interfering with each other and causing inter-channel cross talk [5–7].

In the case of SPM, the power at the rising edge of an optical pulse changes the refractive index of the fiber seen at the middle of the optical pulse. And, similarly the middle of the optical pulse changes the refractive index seen at the falling edge of the optical pulse. As the refractive index governs the speed at which light travels in the fiber, the front, middle and rear portions of the pulse travel at different velocities. Thus, SPM causes compression in the pulse. When this effect is exactly balanced by the dispersion of the fiber, a pulse that travels without any change in shape or energy is formed.

These highly stable, steady and highly localized pulses are called optical solitons [24]. Due to their short pulse duration and high stability, solitons could form the high-speed communication backbone of tomorrow's information super-highway [25]. One of the key technological developments that make use of such soliton pulses for our future cost-effective and repeater less communication is the invention of erbium doped fiber amplifier (EDFA) [1].

Though we have several technological advantages, the major constraints in optical fiber communication (OFC) are the error detection, signal distortion and cross talk [1,2–7]. These are mainly due to the optical losses, dispersion, nonlinearity and amplifier induced noise in the optical fiber. To handle these problems, several new techniques have been proposed. One among them is the use of ultra–short pulses (USP) and to transmit soliton pulses governed by nonlinear Schrödinger (NLS) type equations. The problem of dissipation can be handled with either Raman amplification or by amplification through continuous wave pumping in erbium doped fibers. Both the type of amplification need external pumping source for amplification. But with doping of two-level resonant impurity atoms like erbium (Er), one can achieve self-induced transparency (SIT) phenomenon in fibers [26]. SIT can also compensate the effect of losses in a fiber. Actually SIT effect is also a soliton effect in a two-level resonant medium. So, in Er doped fibers in addition to the optical soliton, SIT soliton can also be achieved which can almost solve the major constraints in the field of OFC.

For high bit rate transmission in OFC, we have to transmit USP. But narrow width pulses will induce higher order effects like higher order dispersion (HOD), self-steepening (SS) and stimulated Raman scattering (SRS). From the experimental point of view, these effects perturb the optical soliton propagation. But theoretically, the nonlinear partial differential equations (NPDE), which govern these effects, can be solved for soliton solutions, both analytically and numerically. In the theoretical analysis, the conditions which are to be satisfied for the soliton propagation can be explicitly obtained. So with such theoretical results, the parametric conditions may be achieved in the fiber so as to propagate solitons in reality. Usually, all these higher order effects are treated as perturbation to the NLS system. Taking into account the higher order effects, the reduced higher order (HNLS) equation for the normalized amplitude reads [27,28],

$$q_z = i(\alpha_1 q_{tt} + \alpha^2|q|^2 q) + \varepsilon[\alpha_3 q_{ttt} + \alpha_4(|q|^2 q)_t + \alpha_5 q(|q|^2)_t], \tag{1}$$

where q is the slowly varying envelope of the electric field, the subscripts z and t are the spatial and temporal partial derivatives, and $\alpha_1, \alpha_2, \alpha_3, \alpha_4$ and α_5 are the parameters related to GVD, SPM, HOD, SS and SRS, respectively. Though eq. (1) was first derived in the year 1985, only for the fast few years, it has attracted much attention among the researchers from both theoretical and experimental points of view.

This paper is organized in the following manner. Section 2 deals with the discussion of some of the completely integrable soliton models in single mode fibers with different linear and nonlinear optical effects. Soliton with two electric fields in a modal birefringent or in multi-channel wavelength division multiplexing (WDM) is dealt in §3. When doping with erbium atoms, the pulse dynamics is governed by the NLS-MB type equations. We discuss some of the integrable models with SIT effects in §4. Finally, we also briefly discuss the integrable SRS problem.

2. Soliton models

Though solitons or soliton-type solutions have been investigated in different areas of science, due to their potential applications, it has attracted a lot of attention only in the area of OFC [14]. Hasegawa and Tappert [24] first proposed the idea of the formation of optical soliton in nonlinear fiber optics in the year 1973. During the same period, Zakharov and Shabbat had solidified the above conjecture of the stable soliton-type nonlinear pulse transmission of the light wave in fiber. Any optical pulse which is transmitted into a lossless fiber forms itself to become solitons like Fourier transmission modes in a linear transmission system.

The model equation in a coordinate moving with a group velocity, has the structure of the NLS equation in which the dispersion terms comes from the group velocity dispersion in a fiber and the potential term which is quadratic in the electric field comes from the fiber nonlinear Kerr effect. Based on this equation, reasonable engineering aspects of its application for soliton transmission in fibers are introduced with a particular emphasis on all-optical transmission systems using EDFA. The aim of this section is to discuss the complete integrability and soliton aspects of the NLS type equations and to concentrate mainly on the integrability aspects of the NLS type equations in nonlinear fiber optics.

2.1 *Bright solitons*

In 1973, Hasegawa and Tappert derived the NLS equation from the Maxwell equations, and they explained that the dynamics of the wave propagation in optical fibers with the effect of Kerr nonlinearity is governed by the NLS equation of the form

$$iq_z + q_{tt} + |q|^2 q = 0 \tag{2}$$

Many mathematical methods are available for obtaining the soliton solution of a given NPDE [4]. Zakharov and Shabbat gave the linear eigenvalue problem for the NLS equation in 1972 [4] and solved the NLS equation using the inverse scattering transform (IST) [4]. The linear eigenvalue problem

$$\psi_t = U\psi, \quad \psi_z = V\psi \quad \text{where} \quad \psi = (\psi_1 \;\; \psi_2)^T \tag{3a}$$

Here, the Lax operators U and V are given in the form:

$$U = \begin{pmatrix} -i\lambda/2 & -\mu q \\ \mu q & i\lambda/2 \end{pmatrix} \quad \text{and}$$

$$V = \lambda^2 \begin{pmatrix} -i\bar{\mu}/2\mu & 0 \\ 0 & i\bar{\mu}/2\mu \end{pmatrix} + \lambda \begin{pmatrix} 0 & -\bar{\mu} q \\ -\bar{\mu} q^* & 0 \end{pmatrix}$$
$$+ \begin{pmatrix} i\mu\bar{\mu}|q|^2 & -i\bar{\mu} q_t \\ -i\bar{\mu} q_t^* & -i\mu\bar{\mu}|q|^2 \end{pmatrix}, \tag{3b}$$

where λ is the eigenvalue parameter and μ and $\bar{\mu}$ are arbitrary constants whose choices make the resultant equation to be either for bright solitons or dark solitons. For bright solitons, we choose $\mu = \bar{\mu} = 1$. In this case, the compatibility condition $U_z - V_t + [U,V] = 0$, gives the NLS equation for bright solitons. One of the remarkable properties of integrable systems is that they possess an infinite number of conserved quantities. Indeed, this is considered to be one of the definitions of complete integrability, as it gives all the important properties about solitons. The first few conserved quantities of NLS equation are

$$W = \int_{-\infty}^{\infty} |q^2| \mathrm{d}t$$
$$M = \frac{i}{2} \int_{-\infty}^{\infty} (qq_t^* - q_t q^*) \mathrm{d}t$$
$$H_0 = \frac{1}{2} \int_{-\infty}^{\infty} \left(|q|^4 - |q_t|^2\right) \mathrm{d}t$$
$$H_l = \frac{1}{2} \int_{-\infty}^{\infty} \left\{ q_{tt} q_t^* - q_t q_{tt}^* + 3|q|^2 (q_t q^* - q q_t^*) \right\} \mathrm{d}t$$

where, W is the total energy, M is the momentum and H_l is the Hamiltonian for the first integrable hierarchy of the NLS equation (Hirota equation (10)). The one-soliton solution of eq. (2) takes the form,

$$q(z,t) = \beta \operatorname{sech}[\beta(t - \alpha z)] \exp[i(\beta^2 - \alpha^2)z/2 - i\alpha t], \quad \lambda : \alpha + i\beta\eta \tag{4}$$

Similarly, one can construct the higher order soliton solutions and N-soliton solutions using a different analytical method known as bilinear method [3]. From the soliton solution, it is clear that the initial pulse is a hyperbolic secant shaped one and we can get some idea about the initial pulse shape, amplitude and the width of the pulse to be propagated inside the fiber. Technically this means that any optical pulse transmitted into a loss-less fiber forms itself to become soliton as it propagates. Thus solitons are natural forms of signal carrier as are the Fourier modes in a linear transmission. On the other hand, the higher order solitons are useful for pulse compression. The two-soliton solution of NLS equation is plotted in figure 1. In 1980, Mollenauer *et al* [25] have succeeded in propagating solitons in the silica fiber experimentally.

In the above NLS model, for exact soliton solutions we assume that the fiber loss is zero. In reality and also for long distance communication, one has to include this effect in the model and analyze the impact of damping on solitons so that we can get real perception about the nature of soliton-type nonlinear pulse transmission inside the fiber. Several papers have reported about this very important investigation and have shown that this effect is responsible for cross talk, error detection and loss of information [5,6]. For example, if we include damping, then the pulse evolution is governed by

$$iq_z + q_{tt} + |q|^2 q + i\Gamma q = 0 \tag{5}$$

where Γ is the damping (> 0) and gain (< 0) parameter. Using the perturbation method and zeroth approximation, one-soliton solution is constructed and the amplification and damping of soliton is explained in figure 2. In addition, by introducing the initial phase

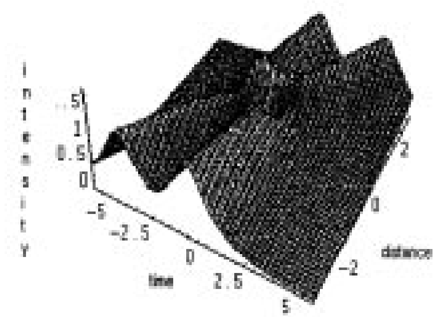

Figure 1. Two soliton solutions of the NLS equation.

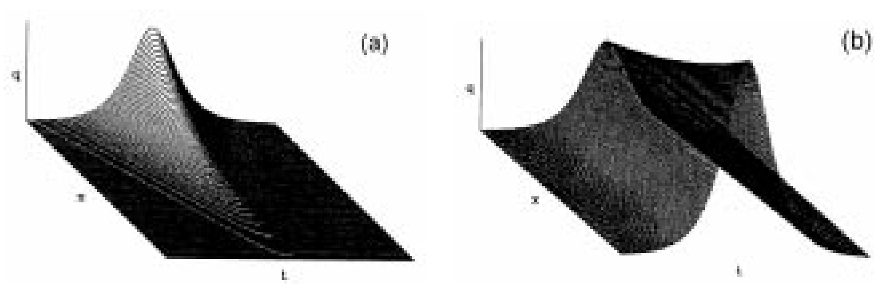

Figure 2. (**a**) Plot of $q = \exp(-\Gamma x)$ sech (t) with $\Gamma = 0.1$. (**b**) Plot of $q = \exp(-\Gamma x)$ sech (t) with $\Gamma = -0.015$.

at $z = 0$, we propose that the effect of damping can be completely compensated and soliton is found to be alive even with the inclusion of damping. In the following section, considering the single channel and zero fiber loss, we shall discuss how one can overcome the interaction between adjacent soliton pulses.

2.2 *Interaction between bright-bright solitons*

Now, we shall discuss the interaction between solitons in the same channel where we assume small frequency differences between solitons. One of the main factors in limiting the full utilization of bandwidth offered by soliton communication system is the interaction between adjacent soliton pulses. In order to tackle this important technical problem, several effective schemes have been proposed and successfully tested by many groups [5,6,35–47]. The interactions occur due to the overlapping of frequency components of either pulse. This is nonlinear in nature and is either attractive or repulsive depending on the phase difference between the pulses. These interactions alter the phases and positions of solitons while their carrier frequencies and amplitudes remain unaffected. For simplicity, we consider the two-soliton solutions of the NLS equation having the following initial launching condition of the form

$$q(0,\tau) = A_1 \mathrm{sech}(\tau - \tau_0) + A_2 \mathrm{sech}(\tau + \tau_0) \mathrm{e}^{(i\theta)}, \tag{6}$$

where A_1 and A_2 are the amplitudes of first and second pulse respectively, τ_0 is the initial pulse separation and θ is the initial phase difference. Considering the above solution, we investigate the following possible cases:

2.2.1 *Equal amplitudes with equal phase*: In this case, we find that the interaction is almost negligible for a short distance after it reaches its maximum. Hence at periodic distances, where the interaction amplitude is minimum, the signals can be recovered without any distortion. Figure 3 shows the variation of the interaction amplitude with distance. Here, we find that the interaction is almost negligible for short distance after it reaches its maximum, but the repetition rate is almost 15 times than that of the system with a separation of 3 times the pulse width. Hence the period of minimum interaction is reduced making

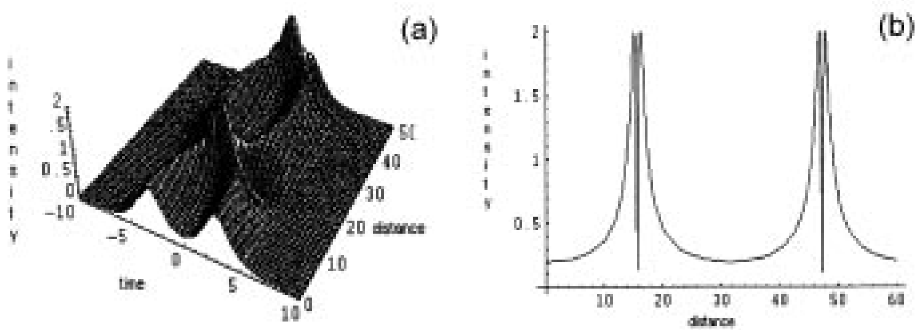

Figure 3. (**a**) Propagation of two solitons for $\tau_0 = 3$. (**b**) Variation of interaction intensity with respect to distance for $\tau_0 = 3$.

the receiver position slightly unstable. But, the important aspect in this case is that the bandwidth of the system is increased and that the signal can be fully recovered at more number of positions, giving us dual advantage.

2.2.2 *Equal amplitudes with unequal phase*: In this case, it has been predicted that the bandwidth of the system can be further increased without major effects of soliton interactions, if there exists an initial phase shift between the two solitons. If the solitons are shifted in phase, then they travel with different velocities and hence interaction is avoided. Unlike solitons traveling with equal phase, which interact at periodic distances, phase shifted solitons experience a repulsive force. If the phase shift is less than 90°, there exists an initial attractive force, but interaction ceases to exist. Due to the repulsive force between the soliton pulses, they travel with different velocities and hence the interaction is prevented. The interactive amplitude increases initially, after which it decays gradually indicating that the interaction decreases as the distance increases. Further, the force of repulsion after the initial attraction is greater than that of the previous case. The repulsion between the solitons is high and hence the gap between them increases more rapidly. The maximum interaction amplitude for $\theta = 120^\circ$ is approximately 2.5 times less than that of $\theta = 30^\circ$ (figure 4). Thus it attains negligible interaction at a shorter distance.

The advantage provided by introducing initial phase difference between the solitons can be utilized to increase the bandwidth of the system by reducing the initial time separation between them. In the extreme case, the time difference is chosen to be zero i.e. the pulses are transmitted simultaneously with initial phase difference. The magnitude of the force between the pulses is found to be more than that of pulses separated by 3 pulse widths, but the pulses start separating faster than that of the previous case. Thus, decreasing the time separation and increasing the initial phase difference between the pulses can reduce the interaction and at the same time increases the bandwidth of the system.

2.2.3 *Unequal amplitudes with equal phase*: When the different amplitude pulses are launched inside the fiber, the larger amplitude soliton will travel with larger phase change than that of the smaller amplitude soliton, and the phase between the pulses changes continuously (figure 5). The soliton interactions can be avoided if the amplitudes of

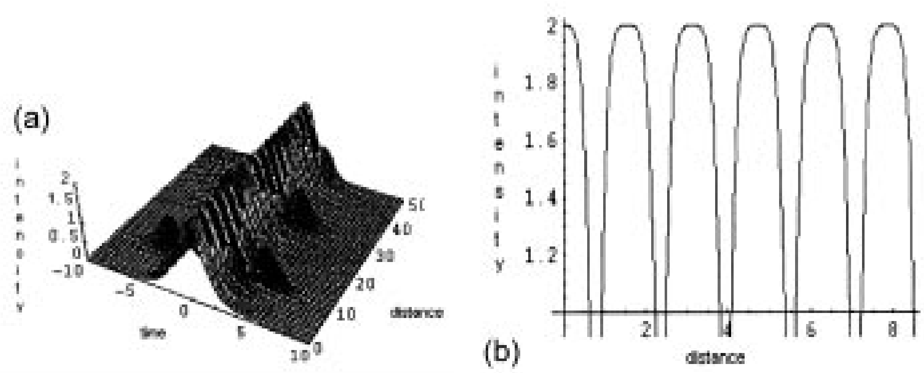

Figure 4. (**a**) Propagation of two solitons for $\tau_0 = 0$. (**b**) Variation of interaction intensity with respect to distance for $\tau_0 = 0$.

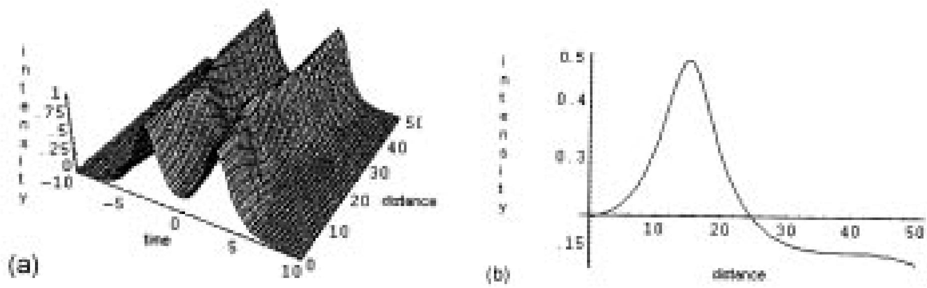

Figure 5. (**a**) Propagation of solitons for $\tau_0 = 3$ and $\theta = 30°$. (**b**) Variation of interaction intensity with distance for $\tau_0 = 3$ and $\theta = 30°$.

alternating solitons are slightly different. The phase difference between the two solitons changes periodically, and therefore the force between them changes periodically inducing small oscillations in the soliton pulses. Thus the points at which the receiver can be placed are increased. The magnitude of interaction is reduced and the frequency of interaction amplitude is increased for $\tau_0 = 0$, giving us the dual advantage of increased bandwidth and increase in the interaction minimum points, where the receiver can be placed to receive signals without interference (figures 6 and 7).

It has been predicted that the force of interaction is almost negligible at approximately 3 times the pulse width separation between adjacent solitons. It is clear that in order to increase the bandwidth utilization, the pulse separation period must be reduced. But this in turn leads to increased interaction between the pulses. It is also observed that the force between the pulses decreases as the separation between them increases and vice versa. The force is found to be attractive for θ varying from $0°$ to $90°$ and beyond that the force is found to be repulsive. It is also observed that the force of interaction increases with decrease in pulse separation. Hence the effect of interaction is more severe if the pulse separation is decreased. Also it is predicted that the force of interaction between the pulses turns out to be repulsive for a phase shift greater than $90°$. We also infer that when the phase difference between the pulses is increased, the interaction is reduced.

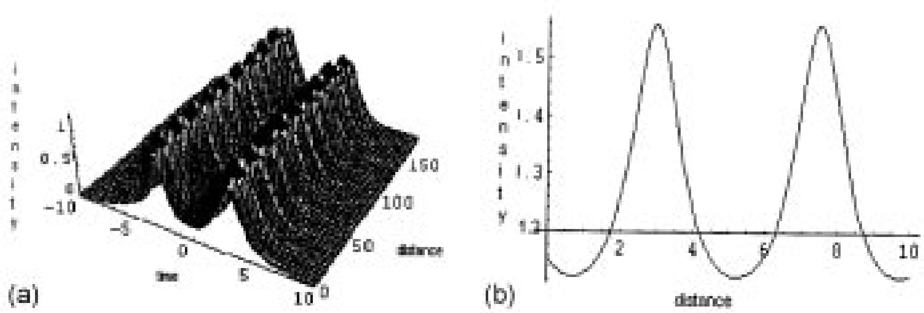

Figure 6. (**a**) Propagation of soliton pulses with different amplitudes for $\tau_0 = 3$, $\theta = 0°$ and amplitude ration $= 1.2$. (**b**) Variation of interaction intensity with respect to distance for $\tau_0 = 0$, $\theta = 0°$ and amplitude $= 1.2$

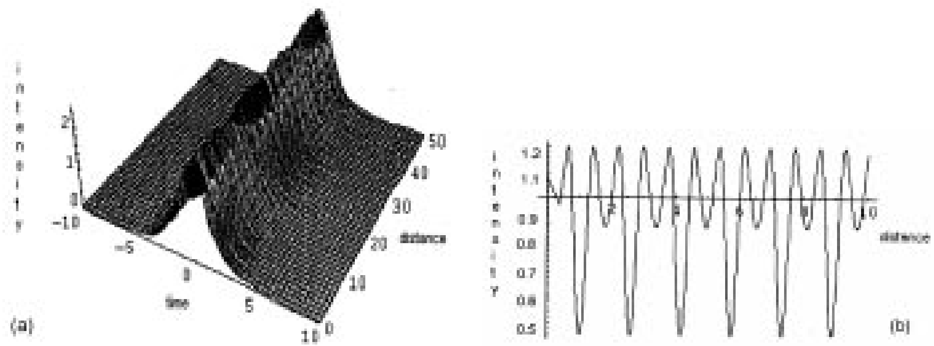

Figure 7. (**a**) Propagation of soliton pulses with different amplitudes for $\tau_0 = 3$, $\theta = 0°$ and amplitude ration = 1.2. (**b**) Variation of interaction intensity with respect to distance for $\tau_0 = 0$, $\theta = 0°$ and amplitude = 1.2

2.3 *Effect of self-steepening on solitons*

The discussion of NLS equation is based on the simplified propagation equation, which includes its lowest order effects of GVD. In 1986, Mitschke *et al* [29] reported an interesting problem regarding the NLS soliton. During their experimental observation on NLS soliton, they witnessed a self-frequency shift to the soliton pulse. This is due to the USP solitons, which is affected by the SRS. If the pulse duration is shorter than one pico second, the higher order dispersion (HOD) of group velocities and the dispersion of the nonlinear susceptibility become important. A theory, describing the propagation of such a short pulses is based on the generalization of the NLS equation. Reduction of the pulse duration results in the necessity to take into account the fourth-order dispersion of the group velocities and even the high orders. In the following sections, we shall restrict our discussions up to the third-order. Actually SS gives asymmetric broadening to the optical soliton pulses. But there are so many interesting results such as the existence of soliton propagation even in the presence of SS. In the presence of SS, the wave propagation in a fiber is governed by the mixed derivative NLS (MDNLS) equation of the form,

$$q_z = i(\alpha_1 q_{tt} \pm \alpha_2 |q|^2 q) \mp \varepsilon \alpha_4 (|q|^2 q)_t, \tag{7}$$

The Lax pair for eq. (7) was given by Wadati *et al* [16]. The one-soliton solution is derived in the form (with $\alpha_2 = k, b = \alpha_4 k$ and $k = \pm 1$)

$$q(z,t) = \frac{4\eta \exp\left[2i\xi t - 4i(\xi^2 - \eta^2)z + 3i\tan^{-1}\left\{\frac{e^{2y} + k(1+b\xi)}{kb\eta}\right\} + i\delta\right]}{\left\{[k(1+b\xi)e^{-y}e^{y}]^2 + (b\eta e - y)^2\right\}^{1/2}} \tag{8}$$

where $y = 2\eta(t - t_0) - 8\eta\xi z \quad \lambda = \xi + i\eta$.

It should be remembered that eq. (7) could be transformed to the derivative NLS (DNLS) equation by change of variables. If we define

$$\tilde{q}(z,t) = q(z,t)\exp[-iat - ibz]$$
$$T = t - \left(\frac{2\alpha_2}{\varepsilon\alpha_4}\right) z$$

where $a = \alpha_2/\alpha_4\varepsilon, b = -(\alpha_4/\varepsilon\alpha_4)^2$, then we get the equation.

$$iq_z + q_{TT} + i\tilde{\varepsilon}(|q|^2 q)_T = 0.$$

The one-soliton solution contains the free real parameters ξ, connected with its velocity, η, determining its amplitude and x_0 and δ, corresponding to the phase of the soliton. From the construction of two-soliton solutions, it can be seen that the solitons with different speeds separate after a long time and the asymptotic solution is two well-separated solitons. As to the interaction of two-solitons, it has been found that the behavior of the absolute values is identical to that of solitons of the NLS equation.

2.4 *Soliton with higher order dispersion and Kerr dispersion*

Even in the anomalous regime, the effect of HOD is considered as a perturbation for USP propagation. Hence, in all practical cases, the effect of HOD on the NLS soliton is considered as a constraint. But, there is also a possibility for a particular parametric choice for which there can be soliton-type pulse propagation in the presence of SS and HOD. The NPDE which governs the wave propagation is called the extended NLS (ENLS) equation of the form,

$$q_z = i(\alpha_1 q_{tt} + \alpha_2|q|^2 q) + \varepsilon[\alpha_3 q_{ttt} + \alpha_4(|q|^2 q)_t]. \tag{9}$$

It was found that the HOD and optic shock effect lead to forming the highly asymmetric power spectra of a propagating pulse. Its bandwidth increases with increasing pulse intensity. In addition, the peak of spectrum shifts towards the short wavelength end. The solitary wave solution to the ENLS equation was derived in [30] which can also be transformed to the well-known completely integrable Hirota equation of the form,

$$q_z = i(\alpha_1 q_{tt} + \alpha_2|q|^2 q) + \varepsilon(q_{ttt} + 6|q|^2 q_t) \tag{10}$$

Equation (1) with $\alpha_1 = 1/2; \alpha_2 = 1$ can be transformed to eq. (10) using the transformation given by Kodama [27] in the form,

$$q' = q + i\varepsilon\left(3\alpha_3 - \frac{1}{2}\alpha_4\right)\frac{\partial q}{\partial t} + i\varepsilon(6\alpha_3 - 2\alpha_4 - \alpha_5)q\int_{-\infty}^{t}|q(t')|^2 \mathrm{d}t' + O(\varepsilon^2) \tag{11}$$

with $q' \to q$, and omitting the higher order terms due to $O(\varepsilon^2)$. Equation (10) is one of the well-studied completely integrable systems and the exact N-soliton solution was given by Hirota [17]. The Lax pair for the Hirota eq. (10) is constructed with the following A and B,

$$A = -4i\lambda^3 - 2i\lambda^2 + 2i\lambda|q|^2 + i|q|^2 + (qq_t^* - q_t q^*),$$
$$B = 4\lambda^2 q + 2\lambda q + 2i\lambda q_t - 2|q|^2 q + iq_t - q_{tt},$$
$$C = -B^*.$$

2.5 *Optical soliton in the presence of higher order dispersion, SS and SRS*

As we discussed in the introduction, with the inclusion of the higher order effects like HOD, SS and SRS, the wave propagation in a nonlinear light guide is described by the HNLS eq. (1). In general, eq. (1) may not be completely integrable. However, in 1990, it was shown that this equation could be solved by means of IST under some alternation of parameters [31].

$$q_z = i(q_{tt} + |q|^2 q) + \varepsilon[q_{ttt} + 6(|q|^2 q)_t + 3q(|q|^2)_t]. \tag{12}$$

Substituting the following transformations

$$q(t,z) = Q(t,z)\exp(-iat + ibz)$$

into eq. (1), we get

$$\begin{aligned} &iQ_z + [+2ia\alpha_1 + 3ia^2\alpha_3\varepsilon)Q_t + (\alpha_1 + 2a\alpha_3\varepsilon)Q_{tt} + (-b - \alpha_1 a^2 - \alpha_3\varepsilon a^3)Q \\ &\quad + (\alpha_2 - a\alpha_4)|Q^2|Q + i[\alpha_4|Q|^2 Q_t + \alpha_5 Q(|Q|^2)_t - \alpha_3\varepsilon Q_{ttt}) = 0 \end{aligned}$$

The parameter a and b are chosen to vanish the terms with Q, Q_{tt} and $|Q^2|Q$. Thus, with suitable parametric choices, we obtain

$$Q_z = \varepsilon[Q_{TTT} + 6(|Q|^2 Q)_T - 3Q(|Q|^2)_T]. \tag{13}$$

So the final form of the variable transformation connecting eqs (12) and (13) is found to be

$$Q(T,Z) = q(t,z)\exp\left\{\frac{-i}{6\varepsilon}\left(T - \frac{Z}{18\varepsilon}\right)\right\}, \quad z = Z, t = T - \frac{Z}{12\varepsilon}.$$

Soliton solution of eq. (12) has been obtained by the IST method and the corresponding Lax pair is 3 × 3. The special case of two-soliton solution of eq. (12) has been discussed in [31]. In the specific case, this solution has two maxima. The distance between these maxima becomes larger under some conditions. It is noteworthy that this type of soliton is unusual for soliton solutions of the NLS equation and its generalization such as the DNLS equation. Recently, using the Painlevé analysis and Hirota bilinear method, same parametric conditions for which the HONLS equation is completely integrable have been reported [32,33].

2.6 *Dark solitons*

Dark soliton in fibers was first observed by transmitting a light wave in the normal dispersion region of a fiber [34], using a Nd:YAG laser output pulse of 100 ps in which 0.3 ps holes were produced by a modulator into a 10 m single-mode fiber. The output power was measured using the autocorrelation techniques for various input power levels and the results were compared with the numerical solution of the NLS equation. Since a dark soliton is a topological soliton, in order to form a single dark soliton, one should construct a dark pulse with an appropriate pulse change. Weiner *et al* recently performed such an experiment by reversing the phase at the middle of a few-picosecond pulse. An excellent agreement between experimental observations and the theoretical calculations has been observed in the narrowing of the dark soliton with increasing power level. Recent studies on the dark solitons have revealed very interesting properties which may allow their stable transmission with much less spacing between solitons when compared with bright solitons. Also, the interaction effects between two dark solitons are less (only a half) than the bright solitons in the presence of fiber loss. The interaction forces between two dark solitons are always repulsive, unlike the case of bright solitons where the interaction forces change according to their relative phase. The self-induced Raman effect is found to be more destructive in the case of dark solitons. The use of dark solitons for high-speed communication systems and other applications will remain an interesting subject for future research [6,35,37]. In the following, some of the models, which admit dark soliton solutions are briefly discussed.

2.6.1 *NLS dark solitons*: In order to derive the dark soliton version of NLS equation, we chose $\mu = i$ and $\bar{\mu} = -i$ in eq. (3). For this case, the compatibility condition gives the NLS equation for dark solitons of the form:

$$iq_z - q_{tt} + 2|q|^2 q = 0. \tag{14}$$

In the case of bright solitons, the Bäcklund transformation and soliton solutions are well analyzed [3]. However, very recently, Park *et al* have generated the dark solitons through BT for the dark soliton equation [48,49].

The effect of third-order dispersion on dark solitons has been discussed by Kivshar and Afanasjev [50] who showed that near the zero point of the group velocity dispersion, dark solitons exist as humps, instead of dips. It was proved that the solitary wave acts as a source generating trailing oscillations, which with the leading front propagates with the group velocity V_g. When third-order dispersion and self-steepening are taken into account together with the group velocity and self-phase modulation terms of the NLS system, the governing equation is known as Hirota equation. On the other hand, if the stimulated inelastic scattering were included together with these two effects, we would get the higher order NLS (HNLS) equation of the form,

$$q_z = \pm iq_{tt} + 2i|q|^2 q + \varepsilon(q_{ttt} + \alpha_1(|q|^2 q)_t + \alpha_2 q(|q|^2)_t) \tag{15}$$

is obtained. $+$ sign corresponds to the anomalous dispersive regime and the $-$ sign corresponds to normal dispersive regime. The parameter ε represents the relative width of the spectrum that arises due to quasi-monochromaticity and it is assumed that $0 < \varepsilon < 1$.

The linear eigenvalue problem for dark solitons of the Hirota system can be constructed with the following U and V and by choosing $\alpha_1 = -\alpha_2$

$$U = \begin{pmatrix} -i\lambda/2 & -iq \\ iq^* & i\lambda/2 \end{pmatrix},$$

$$V = \lambda^3 \begin{pmatrix} i\varepsilon/2 & 0 \\ 0 & -i\varepsilon/2 \end{pmatrix} + \lambda^2 \begin{pmatrix} i/2 & i\varepsilon q \\ -i\varepsilon q^* & -i/2 \end{pmatrix} + \lambda \begin{pmatrix} i\varepsilon|q|^2 & iq - \varepsilon q_t \\ -iq^* - \varepsilon q_t^* & -i\varepsilon|q|^2 \end{pmatrix}$$
$$+ \begin{pmatrix} i|q|^2 + \varepsilon(qq_t^* - q^* q_t) & 2i\varepsilon|q|^2 q - q_t - i\varepsilon q_{tt} \\ -2i\varepsilon|q|^2 q^* - q_t^* + i\varepsilon q_{tt}^* & -i|q|^2 + \varepsilon(-qq_t^* + q^* q_t) \end{pmatrix}. \tag{16}$$

Since, we are able to get the Lax pair for the dark soliton version of Hirota equation, we conclude that it is possible to perform the IST method for this equation to generate dark

N-soliton solutions. In the following, we use Hirota's direct method and construct one- and two-soliton solutions.

Hirota's bilinear method [3,17] is one of the most direct and elegant methods available to generate multi-soliton solutions of nonlinear PDEs. We consider the dark soliton version of the Hirota equation in the form,

$$iq_z - q_{tt} + 2|q|^2 q - i\varepsilon\left\{q_{ttt} - 6|q|^2 q_t\right\} = 0. \tag{17}$$

To avoid mathematical complexities, it is rather convenient to transform this equation to a simpler form, so that we may be able to generate multi-soliton solutions. We make the following transformations to convert the equation [17] to complex modified K-dV (cmK-dV) equation:

$$q(x,t) = Q(Z,T)\exp\left[i\left(\frac{Z}{3\varepsilon} - \frac{T}{27\varepsilon^2}\right)\right], \quad T = t, \quad Z = x + \frac{t}{3\varepsilon}. \tag{18}$$

Using the above transformations in eq. (17), the resultant cmK-dV equation is obtained in the form:

$$Q_T - \varepsilon\{Q_{ZZZ} - 6|Q|^2 Q_Z\} = 0. \tag{19}$$

Next, we use the bilinear transformation

$$Q = G/F, \tag{20}$$

where $G(Z,T)$ is a complex function and $F(Z,T)$ is a real function. Using eq. (20), we obtain the decoupled forms of the bilinearized cmK-dV equation as follows:

$$(D_T - \varepsilon D_Z^3 - 3\varepsilon\lambda D_Z)G\cdot F = 0, \quad (D_Z^2 + \lambda)F\cdot F = -2|G|^2, \tag{21}$$

where λ is a constant to be determined and the Hirota bilinear operators D_x and D_t are defined as

$$D_x^m D_t^n G(x,t)\cdot F(x,t) = \left(\frac{\partial}{\partial x} - \frac{\partial}{\partial x'}\right)^m \left(\frac{\partial}{\partial t} - \frac{\partial}{\partial t'}\right)^n G(x,t)F(x',t')\Big|_{x=x',t=t'}. \tag{22}$$

Further, for the construction of soliton solutions, we assume that

$$G = g_0(1 + \chi g_1) \quad \text{and} \quad F = 1 + \chi f_1, \tag{23}$$

where, g_0 is a complex constant and g_1 and f_1 are real functions. Substituting eq. (23) in eq. (21) and collecting the coefficients of χ^0, and solving we get

$$\lambda = -2|g_0|^2, \quad g_1 = -f_1 = -\exp[\omega_1 T + c_1 Z + \xi_1^{(0)}], \tag{24}$$

where, $\omega_1 = \varepsilon c_1(c_1^2 + 3\lambda)$ and $c_1^2 = -2\lambda = 4|g_0|^2$. Using the above results, we obtain the dark one-soliton solution in the form:

$$Q = g_0 \tanh\left[\frac{1}{2}\left\{c_1\left(Z - \frac{c_1^2\varepsilon T}{2}\right) + \xi_1^{(0)}\right\}\right]. \tag{25}$$

By using the transformations (18), we can easily obtain the dark one-soliton solution of the Hirota equation. It is seen that the HOD and SS effects affect the velocity of the dark soliton, yet they propagate without any change in their shape and intensity. To get the two-soliton solutions, we assume the following forms for G and F:

$$G = g_0(1+\chi g_1+\chi^2 g_2) \quad \text{and} \quad F = 1+\chi f_1+\chi^2 f_2, \tag{26}$$

and proceeding as usual, we get

$$g_1 = P_1 \exp[\xi_1] + P_2 \exp[\xi_2] \quad \text{and} \quad f_1 = \exp[\xi_1] + \exp[\xi_2],$$

where,

$$\xi_1 = \omega_1 T + c_1 Z + \xi_1^{(0)}, \quad \xi_2 = \omega_2 T + c_2 Z + \xi_2^{(0)}$$
$$\omega_1 = \varepsilon c_1^3 + 3\varepsilon\lambda c_1 \quad \text{and} \quad \omega_2 = \varepsilon c_2^3 + 3\varepsilon\lambda c_2.$$

The values of P_1 and P_2 are found to be

$$P_1 = \frac{2|g_0|^2 - c_1^2}{2|g_0|^2} \quad \text{and} \quad P_2 = \frac{2|g_0|^2 - c_2^2}{2|g_0|^2}.$$

It can be shown that the above system of equations can be satisfied if we assume

$$g_2 = A_{12} P_1 P_2 \exp[\xi_1 + \xi_2] \quad \text{and} \quad f_2 = A_{12} \exp[\xi_1 + \xi_2]. \tag{27}$$

The value of A_{12} is found to be:

$$A_{12} = \frac{(P_2 - P_1)\{-(\omega_2 - \omega_1) + \varepsilon(c_2 - c_1)^3 + 3\varepsilon\lambda(c_2 - c_1)\}}{(1 - P_1 P_2)\{-(\omega_2 + \omega_1) + \varepsilon(c_2 + c_1)^3 + 3\varepsilon\lambda(c_2 + c_1)\}}. \tag{28}$$

From the values of g_1, g_2, g_3, f_1 and f_2, one can construct dark two-soliton solutions of the Hirota equation. From the detailed numerical investigations, we find that the dark two-soliton behaves in an elastic manner characteristic of all soliton solutions. They retain their shape after collision only with a slight change in their phase. Also, like all the dark solitons, they appear to repel each other and hence there is no possibility of forming a bound state between them. This important feature is an attractive factor that makes dark solitons a preferred tool, instead of bright solitons, in long-distance communications. Our next aim is to discuss the integrability aspects of the HNLS system.

2.6.2 *Dark soliton solutions of the HNLS equation*: With the inclusion of all the higher order effects, the integrable version of the dark-HNLS equation takes the form:

$$qz = -iq_{tt} + 2i|q|^2 q + \varepsilon(q_{ttt} - 6q_t|q|^2 - 3q(|q|^2)_t). \tag{29}$$

The Lax operators U and V can be given in the form:

$$U = \begin{pmatrix} -i\lambda/2 & -iq & -ir \\ iq^* & i\lambda/2 & 0 \\ ir^* & 0 & i\lambda/2 \end{pmatrix} \quad \text{and}$$

$$V = \lambda^3 \begin{pmatrix} i\varepsilon/2 & 0 & 0 \\ 0 & -i\varepsilon/2 & 0 \\ 0 & 0 & -i\varepsilon/2 \end{pmatrix} + \lambda^2 \begin{pmatrix} i/2 & i\varepsilon q & i\varepsilon r \\ -i\varepsilon q^* & -i/2 & 0 \\ -i\varepsilon r^* & 0 & -i/2 \end{pmatrix}$$
$$+\lambda \begin{pmatrix} i\varepsilon(|q|^2+|r|^2) & -\varepsilon q_t + iq & -\varepsilon r_t + ir \\ -\varepsilon q_t^* - iq^* & -i\varepsilon|q|^2 & -iq^* r \\ -\varepsilon r_t^* - ir^* & -ir^* q & -i\varepsilon|r|^2 \end{pmatrix} + \begin{pmatrix} A_{11} & A_{12} & A_{13} \\ A_{21} & A_{22} & A_{23} \\ A_{31} & A_{32} & A_{33} \end{pmatrix}, \quad (30)$$

where,

$$\begin{aligned}
A_{11} &= i|q|^2 + i|r|^2 + \varepsilon(qq_t^* - q_t q^* + rr_t^* - r_t r^*),\\
A_{12} &= -q_1 - i\varepsilon q_{tt} + 2i\varepsilon(q|q|^2 + r|r|^2),\\
A_{13} &= -r_1 - i\varepsilon r_{tt} + 2i\varepsilon(r|q|^2 + r|r|^2),\\
A_{21} &= -q_t^* + i\varepsilon q_{tt}^* - 2i\varepsilon(q^*|q|^2 + q^*|r|^2),\\
A_{22} &= -i|q|^2 + \varepsilon(q^* q_t - qq_t^*),\\
A_{23} &= -iq^* r + \varepsilon(q^* r_t - rq_t^*),\\
A_{31} &= -r_t^* + i\varepsilon r_{tt}^* - 2i\varepsilon(r^*|q|^2 + r^*|r|^2),\\
A_{32} &= -ir^* q + \varepsilon(r^* q_t - qr_t^*),\\
A_{33} &= -i|r|^2 + \varepsilon(r^* r_t - rr_t^*),
\end{aligned}$$

with $r = \mathrm{e}^{i\Theta} q^*$ and $\Theta(z,t) = 2/3\left(t + \frac{2}{9}z\right)$. We follow the same method used for the Hirota equation in the previous section to obtain dark soliton solutions for HNLS equation. First, we transform eq. (29) to a cmK-dV equation using eq. (18) as follows:

$$Q_T - \varepsilon(Q_{ZZZ} - 6|Q|^2 Q_Z - 3Q(|Q|^2)_Z) = 0. \quad (31)$$

The decoupled bilinear forms of eq. (31) are given as:

$$(D_T - \varepsilon D_Z^3 + 3\varepsilon\lambda D_Z)G \cdot F = 0, \quad (D_Z^2 - \lambda)F \cdot F = -4|G|^2, \quad D_Z G^* \cdot G = 0. \quad (32)$$

To get one-soliton solutions, we use eq. (23) and, after some manipulations, we obtain

$$\lambda = 4|g_0|^2, \quad g_1 = -f_1 = -\exp[\omega_1 T + c_1 z + \xi_1^{(0)}] \quad (33)$$

$$\omega_1 = \varepsilon c_1(c_1^2 - 3\lambda) \quad \text{and} \quad c_1^2 = 2\lambda = 8|g_0|^2.$$

Finally, the dark one-soliton solution of cmK-dV equation is obtained as:

$$Q = g_0 \tanh\left[\frac{1}{2}\left\{c_1\left(Z - \frac{c_1^2 \varepsilon T}{2}\right) + \xi_1^{(0)}\right\}\right]. \quad (34)$$

Proceeding further, we have also generated higher order soliton solutions and analyzed the nature of the solutions through numerical methods and established the properties of solitons.

2.7 *Soliton in nonuniform fibers*

In this subsection, we discuss the impact of nonuniformities, which are always present in real fibers, on soliton pulses and also play an important role in the area of dispersion management soliton communication systems. In real fibers, there will always be some nonuniformity due to many physical situations. Important among them are (i) variation in the lattice parameters of the fiber medium, so that the distance between two neighboring atoms is not constant throughout the fiber and (ii) variation of the fiber geometry (diameter fluctuations, etc.). These nonuniformities influence various effects such as loss (or gain), dispersion, phase modulation, etc. on the soliton pulses [7,10].

In general, in nonuniform fibers, the pulse propagation is governed by the inhomogeneous NLS equation. Solving these inhomogeneous equations, and finding the solutions are very tedious processes and mathematically complicated. In view of the above facts, for the past few years, the soliton aspects of inhomogeneous nonlinear systems have attracted a lot of attention among researchers. Considering the variable dispersion, variable nonlinearity and amplifier gain, the governing equation is in the form [10],

$$iE_{\xi} + 1/2\mu(\xi)E_{tt} + \sigma(\xi)|E^2|E + i\Gamma(\xi)E = 0 \tag{35}$$

where $\Gamma(\xi)$ is the fiber gain coefficient. If $\Gamma(\xi) = 0$, the resulting equation is found to pass the Painlevé test for the condition $\mu(\xi) = \sigma(\xi)[K_1 \int^{\xi} \mathrm{d}s\mu(\xi) + K_2]$, where K_1 and K_2 are arbitrary integration constants. Using the transformations, $z = \int \mu(\xi')\mathrm{d}\xi'$ and $q = E\sqrt{\sigma}(\xi)/\mu(\xi)$, eq. (35) can be transformed into

$$iq_z + 1/2q_{tt} + |q|^2 q + iF(z)q = 0 \tag{36}$$

where $F(z)$ is an inhomogeneous function related to $F(z) = \Gamma(\xi)/\mu(\xi)$. The above equation admits solitons and other related integrability properties only for the condition $F(Z) = 1/(z+z_0)$, where z_0 is the arbitrary integration constant [51].

In addition to the above inhomogeneous eq. (35), the pulse propagation is also investigated in the following equation

$$iq_z + q_{tt} + 2\varepsilon|q|^2 q + \varepsilon M(z,t)q + G(z,t) = 0; \quad \varepsilon = \pm 1 \tag{37}$$

where $M(z,t)$ and $G(z,t)$ are functions which can be related to gain (or loss) and phase modulation with suitable conditions. Recently, Kumar and Hasegawa [52] derived the chirped stationary solutions with $G(z,t) = 0$ and $M(z,t) = t^2$. Clarkson [53] carried out the Painlevé analysis with $\varepsilon = -1$ and reported that it admits the Painlevé property only for the following choices of $M(z,t)$ and $G(z,t)$

$$M(z,t) = t^2\left(\frac{1}{2}\frac{\mathrm{d}\beta}{\mathrm{d}z} - \beta^2(z)\right) + i\beta(z) + t\alpha_1(z) + \alpha_0(z), \quad G(z,t) = 0, \tag{38}$$

where $\alpha_0(z), \alpha_1(z)$ and $\beta(z)$ are arbitrary, real analytic functions of space coordinate. Using suitable transformations, eq. (37) with the conditions eq. (38) can be transformed to NLS eq. (2). With $\varepsilon = +1$, we have also derived very similar conditions as in eq. (38) and also transformed the resulting equation to NLS eq. [51]. On the other hand, if M is a function of t only, then the system is IST solvable for the following choice of

$M(t) = \beta^2 t^2 - \alpha t + i\beta, G(z,t) = 0$ where α and β are real constants or arbitrary functions [51]. Moores [54] also analyzed eq. (37) with $M(z,t) = -i\alpha(z) - \hat{M}(z)t^2$ and investigated the possibility of clean and efficient nonlinear compression of chirped solitary waves with an appropriate tailoring of the gain or dispersion as a function of distance and with optical phase modulation. The soliton solution and the possibility of amplification of soliton pulses using a rapidly increasing distributed amplification with scale lengths comparable to the characteristic dispersion length has also been reported by Quiroga–Teixeiro *et al* [55].

In ref. [51] following transformations have been given for transforming eq. (36) with $F(z) = (1/z + z_0)$ to the NLS equation.

$$q(z,t) = \frac{1}{1 + z/z_0} q'(\theta, \sigma) \exp\left(\frac{it^2}{4(z+z_0)}\right), \tag{39}$$

where $\theta = z/(1+z/z_0), \sigma = t/(1+z/z_0)$ and the variable transformations $q' \to q$, $\theta \to z, \sigma \to t$. Recently, Burtsev *et al* [56], have also studied similar problems and reported the deformation of completely integrable nonlinear optical models with inhomogeneous coefficients. When compared with the integrability and soliton aspects of homogenous equations, in the case of inhomogeneous systems, we observed the following (i) associated spectral parameter is non-isospectral, i.e. functions of z and t (ii) as time increases, the solitons spread but preserve their energy.

3. Optical solitons in birefringent and WDM Systems

In this section we shall discuss the polarized wave propagation in nonlinear fibre optics. Birefringence is the property of a medium by which the refractive index is different in different polarizations of the light wave. A single mode fiber can support two orthogonal directions. Under ideal conditions of perfect cylindrical geometry and isotropic material, a mode excited with its polarization in the x-direction would not couple to the mode with the orthogonal state. However, in practice, small departures from cylindrical geometry or small fluctuations in material anisotropy result in the mixing up of the two polarization states, thereby breaking the mode degeneracy. Then the mode propagation constant β becomes slightly different for the modes polarized in the x and y directions. This property is referred to as model birefringence (B). It is shown that for a given value of B, the power between the two modes is exchanged periodically as they propagate inside the fiber. The axis along which the effective mode index is smaller is called the fast axis, as the group velocity is larger for light polarized along that direction. Similarly the axis with the larger mode index is called the slow axis [57].

In conventional single mode fibers, B is not constant along the fiber but changes randomly because of fluctuations in the core shape and stress-induced anisotropy. As a result, light launched into the fiber with linear polarization quickly reaches a state of arbitrary polarization. For some applications it is desirable that fibers transmit light without changing its state of polarization. Such fibers are called polarization-preserving fibers. The use of polarization-preserving fibers requires an identification of the slow and fast axes before the linearly polarized light is launched into the fiber. If the polarization axis makes an angle with the slow or the fast axis, the polarization remains unchanged during propagation. But if the polarization axis makes an angle with other than these axes, the polarization changes continuously along the fiber in a periodic manner with a period equal to the beat length.

From the above arguments it is clear that when two or more optical waves co-propagate inside a fiber, they interact with each other through the fiber nonlinearity. This provides a coupling between the incident waves through the phenomenon called cross phase modulation (XPM). XPM occurs because of the effective refractive index of a wave depends not only on the intensity of that wave but also on the intensity of the copropagating wave. XPM is always accompanied by SPM. When the two waves have orthogonal polarizations, the XPM caused coupling induces a nonlinear birefringence in the fiber.

Taking the case of an elliptically polarized optical wave propagating through the fiber, $E(r,t) = 1/2(\hat{e}_x E_x + \hat{e}_y E_y)\exp(-i\omega_0 t)$, where E_x and E_y are the complex amplitudes of the polarization components at the central frequency ω_0, it is shown that the nonlinear contributions to the refractive index are,

$$\Delta n_x = n_2 \left[|E_x|^2 + \frac{2}{3}|E_y|^2 \right]$$

and

$$\Delta n_y = n_2 \left[|E_y|^2 + \frac{2}{3}|E_x|^2 \right]$$

where n_2 is the nonlinear refractive index coefficient. The first term inside the bracket represents SPM and the second term XPM. We see that the XPM induced nonlinear coupling between E_x and E_y create nonlinear birefringence. So, an accurate description of the polarization effect in birefringence fibers requires simultaneous consideration of both intrinsic linear birefringence and induced nonlinear birefringence.

Similarly, for handling more channels, it is necessary to achieve wavelength division multiplexing (WDM) using optical solitons [1,5–7]. This is possible by propagation through different channels with different carrier frequencies. In either case, two or more fields are to be propagated in the fiber. So, the dynamics of the fiber system is governed by the coupled system of equations which are not integrable. In addition to the above situations, coupling is also possible in the system of two parallel wave guides coupled through evanescent field overlap, the coupling of two polarization modes in uniform guides, nonlinear optical wave guide arrays and nonlinear distributed feedback structures. Also, nonlinear couplers use solitons as ideal tools for performing all-optical switching operations.

Normally in WDM, the ratio between the coefficient of SPM and XPM will not be 1:1. In silica fiber, the XPM value will be 2/3. However, for the following ideal cases: i) for the elliptical birefringence when $\theta = 35^\circ$, where θ is the angle between the major and minor axis of the birefringence ellipse ii) in a purely electrostrictive nonlinearity, the ratio between SPM and XPM, with 1:1 is allowed. In this section, we consider only the ratio of 1:1 between SPM and XPM. An elliptically birefringent Kerr medium can also be obtained, for example, by twisting an appropriately doped optical fiber during the drawing stage. In this case, Manakov has shown that the coupled NLS (CNLS) equation which governs the wave evolution can be solved using inverse scattering transform methods [18].

3.1 *Coupled nonlinear Schrödinger solitons*

When two fields are propagated in a single-mode fiber, then the Kerr nonlinearity for a field will depend on the intensity of both the fields. The field q can be represented as the sum of right (q_1) and left (q_2) polarized waves governed by CNLS equations of the form

$$q_{1z} = i[c_1 q_{1tt} + (\alpha|q_1|^2 + \beta|q_2|^2)q_1],$$
$$q_{2z} = i[c_2 q_{2tt} + (\beta|q_1|^2 + \gamma|q_2|^2)q_2]. \tag{40}$$

Here α and γ are the SPM and β is the XPM parameter. Zakharov *et al* [58] have proved that the CNLS equations admits complete integrability properties for the following choices of parameters (1) $c_1 = c_2, \alpha = \beta = \gamma$, (ii) $c_1 = -c_2$, $\alpha = -\beta = \gamma$. Sahadevan *et al* [59], have obtained the same integrability conditions through Painlevé analysis. For these choices of parameters, a large number of papers have reported the occurrence of soliton through different analytical methods [18,51,57–76]. The bilinear and the bright and dark N-soliton solutions for the CNLS systems have been constructed using Hirota bilinearization [60]. The Lax pair for the case (i) takes the form,

$$U = \begin{pmatrix} -i\lambda & q_1 & q_2 \\ -q_1^* & i\lambda & 0 \\ -q_2^* & 0 & i\lambda \end{pmatrix}, \tag{41}$$
$$V = 2i\lambda^2 \begin{pmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} + \lambda \begin{pmatrix} 0 & q_1 & q_2 \\ -q_1^* & 0 & 0 \\ -q_2^* & 0 & 0 \end{pmatrix}$$
$$+ \frac{i}{2} \begin{pmatrix} |q_1|^2 + |q_2|^2 & q_{1t} & q_{2t} \\ q_{1t}^* & -|q_1|^2 & -q_2 q_1^* \\ q_{2t}^* & -q_1 q_2^* & -|q_2|^2 \end{pmatrix}.$$

Recently, using Bäcklund transformation approach, we generated soliton solutions for CNLS equation [72]. The one-soliton solution for the CNLS equation is obtained in the form

$$q_1 = 2\beta \exp(2i\beta^2 z)\mathrm{sech}(2\beta t), \quad q_2 = 2\beta \exp(2i\beta^2 z)\mathrm{sech}(2\beta t), \tag{42}$$

In birefringent fibers, it has been predicted that the pulses with small amplitudes in each of the two polarizations tend to split apart and propagate with different group velocities (polarization dispersion). For large amplitude pulses, Menyuk [57] showed numerically that above a certain amplitude threshold, two polarizations strongly interact and nonlinear pulses consisting of both polarizations are formed. Thus, the Kerr nonlinearity compensates not only group dispersion, but also polarization dispersion and forms a steady nonlinear pulse called vector soliton. In the next section, we shall discuss the photonic logic gate operations in terms of soliton-soliton interactions.

3.1.1 *Inelastic Interaction and photonic gate operations*: In order to analyze the inelastic nature of soliton interaction and photonic logic gate operations in terms of solitons, we consider the CNLS equation in the form

$$iq_{tz} + q_{1tt} + \rho q_1 + \gamma q_2 + 2\mu(|q_1|^2 + |q_2|^2)q_1 = 0,$$
$$iq_{2z} + q_{2_{ttt}} - \rho q_2 + \gamma q_1 + 2\mu(|q_1|^2 + |q_2|^2)q_2 = 0. \tag{43}$$

γ and ρ are the normalized linear coupling constant caused by the periodic birefringence axes and the phase-velocity mismatch from resonance respectively. If γ and ρ are assumed to be zero, the coupled system (43) reduces to the integrable Manakov model [18].

Recently, using Hirota's bilinear form and by introducing additional parameters, the N-soliton solutions for CNLS equations have been constructed and inelastic collision of solitons have been observed [62]. As our main aim is to explain the soliton-soliton interaction in terms of photonic logic gate operations, we consider the general two-soliton solution of the Manakov system reported by Radhakrishnan and Lakshmanan [61]. Using the following transformations

$$\begin{aligned} q_1 &= \cos(\theta/2)\mathrm{e}^{i\Gamma z}Q_1 - \sin(\theta/2)\mathrm{e}^{-i\Gamma z}Q_2, \\ q_2 &= \sin(\theta/2)\mathrm{e}^{i\Gamma z}Q_1 + \cos(\theta/2)\mathrm{e}^{-i\Gamma z}Q_2, \end{aligned} \tag{44}$$

where, $\Gamma = (\rho^2+\gamma^2)^{1/2}, \theta = \tan^{-1}(\gamma/\rho)$, eq. (43) can be transformed into the integrable Manakov model of the form [18]

$$\begin{aligned} &iQ_{1z} + Q_{1tt} + 2\mu(|Q_1|^2+|Q_2|^2)Q_1 = 0, \\ &iQ_{2z} + Q_{2tt} + 2\mu(|Q_1|^2+|Q_2|^2)Q_2 = 0 \end{aligned} \tag{45}$$

In order to analyze the photonic logic operations in terms of solitons, we are considering the two-soliton solution of the CNLS equations in the form

$$\begin{aligned} q_1 = \{&[\cos(\theta/2)\mathrm{e}^{i\Gamma z}\alpha_1 - \sin(\theta/2)\mathrm{e}^{-i\Gamma z}\beta_1]\mathrm{e}^{\eta_1} \\ &+[\cos(\theta/2)\mathrm{e}^{i\Gamma z}\alpha_2 - \sin(\theta/2)\mathrm{e}^{-i\Gamma z}\beta_2]\mathrm{e}^{\eta_2} \\ &+[\cos(\theta/2)\mathrm{e}^{i\Gamma z+\delta_1} - \sin(\theta/2)\mathrm{e}^{-i\Gamma z+\delta_1'}]\mathrm{e}^{\eta_1+\eta_1^*+\eta_2} \\ &+[\cos(\theta/2)\mathrm{e}^{i\Gamma z+\delta_2} - \sin(\theta/2)\mathrm{e}^{-i\Gamma z+\delta_2'}]\mathrm{e}^{\eta_1+\eta_2+\eta_2^*}\}/D_n, \end{aligned}$$

and

$$\begin{aligned} q_2 = \{&[\sin(\theta/2)\mathrm{e}^{i\Gamma z}\alpha_1 + \cos(\theta/2)\mathrm{e}^{-i\Gamma z}\beta_1]\mathrm{e}^{\eta_1} \\ &+[\sin(\theta/2)\mathrm{e}^{i\Gamma z}\alpha_2 + \cos(\theta/2)\mathrm{e}^{-i\Gamma z}\beta_2]\mathrm{e}^{\eta_2} \\ &+[\sin(\theta/2)\mathrm{e}^{-i\Gamma z+\delta_1} + \cos(\theta/2)\mathrm{e}^{-i\Gamma z+\delta_1'}]\mathrm{e}^{\eta_1+\eta_2^*+\eta_2} \\ &+[\sin(\theta/2)\mathrm{e}^{i\Gamma z+\delta_2} + \cos(\theta/2)\mathrm{e}^{-i\Gamma z+\delta_2'}]\mathrm{e}^{\eta_1+\eta_2+\eta_2^*}\}/D_n, \end{aligned} \tag{46}$$

Here,

$$\begin{aligned} D_n = 1 &+ \exp(\eta_1+\eta_1^*+R_1) + \exp(\eta_1+\eta_2^*+\delta_0) + \exp(\eta_1^*+\eta_2+\delta_0^*) \\ &+ \exp(\eta_2+\eta_2^*+R_2) + \exp(\eta_1+\eta_1^*+\eta_2^*+R_3) \end{aligned}$$

and $\eta_j = k_j(t+ik_jz), j = 1,2$. The parameters,

$$\begin{aligned} &\exp(\delta_1) = (k_1-k_2)(\alpha_1k_{21}-\alpha_2k_{11})/(k_1+k_1^*)(k_1^*+k_2), \\ &\exp(\delta_2) = (k_2-k_1)(\alpha_2k_{12}-\alpha_1k_{22})/(k_2+k_2^*)(k_1+k_2^*), \\ &\exp(\delta_1') = (k_1-k_2)(\beta_1k_{21}-\beta_2k_{11})/(k_1+k_1^*)(k_1^*+k_2), \\ &\exp(\delta_2') = (k_2-k_1)(\beta_2k_{12}-\beta_1k_{22})/(k_2+k_2^*)(k_1+k_2^*), \\ &\exp(\delta_0) = k_{12}/(k_1+k_2^*), \quad \exp(R_1) = k_{11}/(k_1+k_1^*), \\ &\exp(R_2) = k_{22}/(k_2+k_2^*), \\ &\exp(R_3) = |k_1-k_2|^2(k_{11}k_{22}-k_{12}k_{21})/(k_1+k_1^*)(k_2+k_2^*)|k_1+k_2^*|^2, \end{aligned}$$

and, $k_{ij} = \mu(\alpha_i\alpha_j^*+\beta_i\beta_j^*)(k_i+k_j^*)^{-1}$, $i,j = 1,2$.

Of the six arbitrary complex parameters α_1 and α_2 are amplitudes, β_1 and β_2 are velocities, k_1 and k_2 are phases of the asymptotic soliton in two orthogonally polarized modes. For our analysis, the important parameters like amplitude (α), velocity (β) and phase (k) are changed. When the normalized amplitudes $(\alpha_1 = \alpha_2 = 0.35)$ and normalized velocities are equal $(\beta_1 = \beta_2 = 0.331)$, then the two pulses interact with a slight change in their relative phase. Moreover, we try to point out that interaction can be used as a switch to suppress or to induce soliton switching in the birefringent fiber.

3.1.2 *Switching of solitons*: The switching of solitons can also be realized in terms of all-optical logic gates. The logic gate operations like OR, AND, EX-OR and NAND have been realized for NLS soliton switching [64]. It has been realized that these components will find useful applications in the field of optical information processing, permitting the realization of logic operations at speeds unattainable by conventional electronic systems. The ultimate aim should be the realization of all-optical digital computers interfaced, without the need for opto-electronic converters, of soliton communication systems in which the data rates are so high that conventional electronic logic is too slow.

When a soliton pulse is transmitted through a birefringent fiber, there is a possibility of exchange of energy between the pulses propagating in two orthogonal modes. The two orthogonally polarized solitons can trap one another and move at a common group velocity in spite of their different modal indices (polarization dispersion). This phenomenon is referred to as soliton trapping and is quite important for optical soliton switching [6,7]. This type of switching in fiber is intensity dependent. Depending upon its intensity, the pulse itself induces switching. Self phase modulation (SPM) in an optical fiber due to its nonlinearity is well suited for this purpose. The control of data flow in the fiber optic communication system has been accomplished by switching of optical signals between fibers. Switching depends on the power and phase of the pulse.

In practice, an optical pulse is used for nonlinear switching. However, the switched pulse is severely distorted because only certain parts of the pulse have the right power for switching. In particular, pulse wings are too weak for switching to occur. Solitons can avoid pulse distortion because of their extraordinary stable property that the optical phase remains uniform across the entire pulse in spite of the fiber nonlinearity [8].

In our case, it has been predicted that the soliton interaction itself acts as a switch to suppress or to enhance the switching dynamics. The graphs that we are presenting, together with similar ones obtained with respect to single excitation, give an idea of the possibility of employing the proposed two-layer dielectric structures for realizing logic functions. We mainly observe, AND, OR, EX-OR and NAND operations, by suitable dimensioning of the geometrical and electrical parameters.

AND gate

We have already discussed the two-soliton interaction, after that, by equally feeding the two input ports, we can obtain at a certain distance, strong peak amplitude in the waveguide central region. In other words, setting $\alpha_1 = \alpha_2 = 1$, we obtain $\alpha_3 = 1$, having denoted by α_3, the Boolean variable identifying the presence or absence of power at the device output port. This clearly happens for the present case as shown in figure 7a and figure 7b. Two solitons having different amplitude $(\alpha_1 = 0.71, \alpha_2 = 0.31)$ and same velocity $(\beta_1 = \beta_2 = 0.015)$, propagating in a birefringent fiber, one output soliton is completely exchanged by changing the relative phase $(k_1 = 0.84, k_2 = 0.65)$. Here one output port is excited performing the desired AND operation (figure 8).

We are interested in choosing a device length so as to ensure the maximum amplitude at the output port. A so-designed structure can be used as an AND gate if, in accordance with the excitation at one input port only ($\alpha_1 = 1$ and $\alpha_2 = 0$ or $\alpha_1 = 0$ $\alpha_2 = 1$), at the distance L we have an amplitude low enough to permit the conclusion that $\alpha_3 = 0$. Two solitons having different amplitudes $(\alpha_1 = 0.74, \alpha_2 = 0.31)$ and different velocities $(\beta_1 = 0.023, \beta_2 = 0.54)$, propagating in a birefringent fiber, with the result that the output soliton splits into two modes below the threshold power with the nearly same relative phase $(k_1 = k_2 = 0.68)$ as above. Therefore, no power exists in the central region if the wave-guide is not excited.

OR gate

When the goal is to realize an OR gate, it is necessary that to get $\alpha_3 = 1$, in accordance with the excitation of one input port only. When the two solitons having different amplitudes $(\alpha_1 = 0.74, \alpha_2 = 0.23)$ and different velocities $(\beta_1 = 0.023, \beta_2 = 0.49)$, propagating through the fiber, there exists a single output power by changing the relative phase $(k_1 = 0.46, k_2 = 0.097)$. To obtain a high output, it is necessary that one of the input ports must be high.

EX-OR gate

We have already seen, although for a low power level, that by the antipodal feeding of the two input ports, the amplitude distribution exhibits a null, independent of 'z'. In other

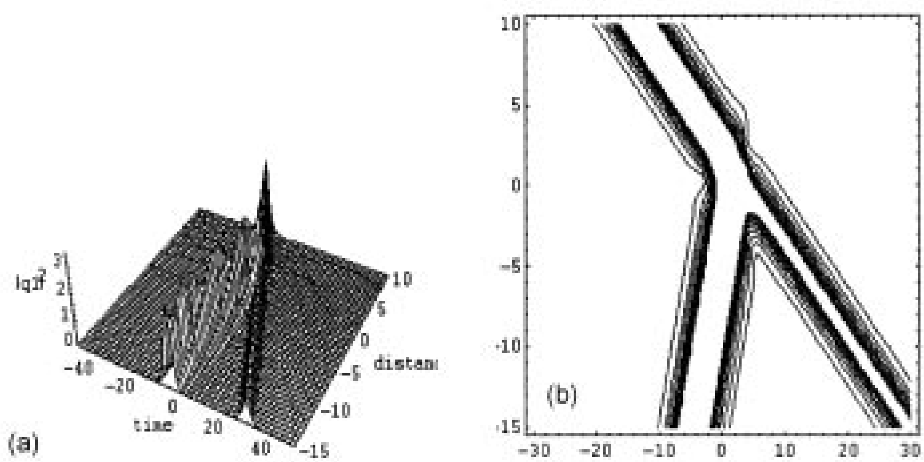

Figure 8. (**a**) AND gate operation when $\alpha_1 = 0.71$, $\alpha_2 = 0.31$, $\beta_1 = \beta_2 = 0.015$, $k_1 = 0.84$ and $k_2 = 0.65$. (**b**) Contour graph for the values as in figure 8a.

words, setting $\alpha_1 = \alpha_2 = 1$, we obtain $\alpha_3 = 0$. This result can be interpreted as a row of the truth table of an EX-OR gate, which must give $\alpha_3 = 1$ only in the case of single excitation of one input port. Then the device length '*L*' has to be designed in such a way as to find an intense peak amplitude at the output port when $\alpha_1 = 1$ and $\alpha_2 = 0$ (or) $\alpha_1 = 0$ and $\alpha_2 = 1$. Figures 9a and 9b shows that when two solitons having same amplitude $(\alpha_1 = \alpha_2 = 0.56)$ and different velocities $(\beta_1 = 0.76, \beta_2 = 0.95)$, propagating in a birefringent fiber, the output power is not excited by changing the relative phase $(k_1 = 0.68, k_2 = 0.46)$.

NAND gate

In order to realize the NAND gate operation, it is also necessary to obtain $\alpha_3 = 0$, when both the input ports are excited. This means that there is no exchange of power. When two solitons having same amplitudes $(\alpha_1 = \alpha_2 = 0.565)$ and different velocities $(\beta_1 = 0.95, \beta_2 = 0.763)$, propagate through the fiber, the output power is not excited by changing the different relative phase $(k_1 = 0.684, k_2 = 0.46)$.

When two-solitons having same amplitudes $(\alpha_1 = \alpha_2 = 0.156)$ and same velocity $(\beta_1 = \beta_2 = 1.209)$, propagate through the fiber, the output power is excited by changing the different relative phase $(k_1 = 0.24, k_2 = 0.86)$ as shown in figure 10.

At the same time we have $\alpha_3 = 1$, in accordance with the excitation of one input port only. This requirement recalls the behavior already discussed in connection with OR gate.

3.2 *CNLS equation with four wave mixing*

In the earlier section, we discussed that the coupling between copropagating optical pulses in a nonlinear medium has led to many important applications in communication, soliton

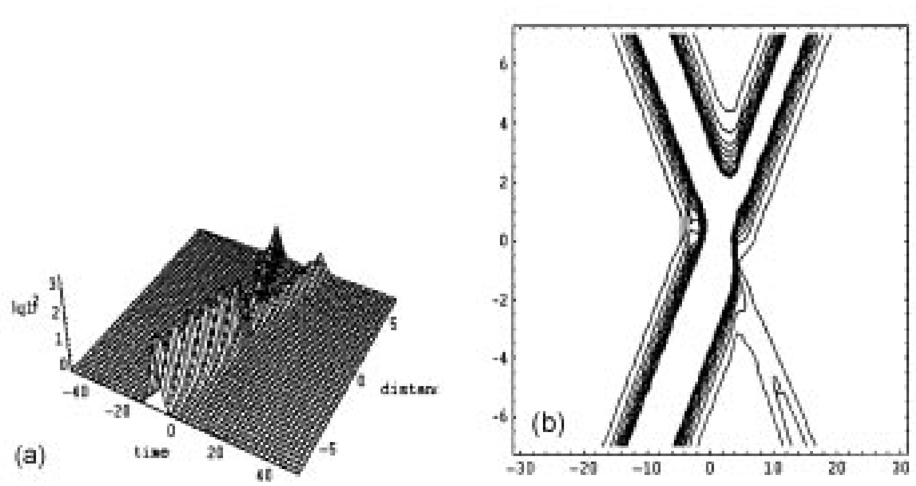

Figure 9. (**a**) EX-OR gate operation when $\alpha_1 = 0.71$, $\alpha_2 = 0.531$, $\beta_1 = 0.830$, $\beta_2 = 0.895$, $k_1 = 0.84$ and $k_2 = 0.65$. (**b**) Contour graph for the values as in figure 9a.

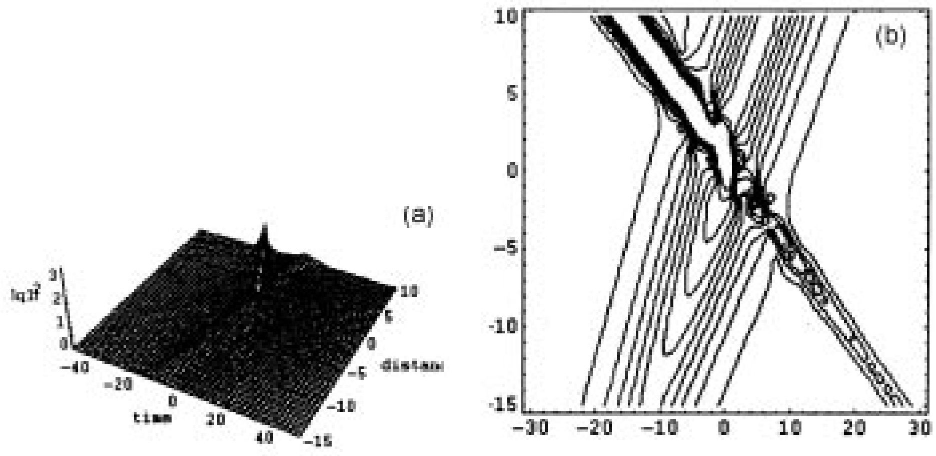

Figure 10. (**a**) NAND gate operation when $\alpha_1 = 0.156$, $\alpha_2 = 0.151$, $\beta_1 = 1.209$, $\beta_2 = 1.208$, $k_1 = 0.246$ and $k_2 = 0.865$. (**b**) Contour graph for the values as in figure 10a.

switching and logic gate operations. Now, generalization of the CNLS equation, with the inclusion of four wave mixing effects, changes to the form [66]

$$iq_{1z} = q_{1tt} + (\alpha|q_1|^2 + \beta|q_2|^2)q_1 + \gamma_1 q_1^* q_2^2 + \gamma_2 q_1^2 q_2^*$$
$$iq_{2t} = \mu q_{2tt} + (\beta|q_1|^2 + \alpha|q_2|^2)q_2 + \gamma_1 q_2^* q_1^2 + \gamma_2 q_2^2 q_1^* \tag{47}$$

where $\mu = \pm 1$ signify the relative sign of the group-velocity dispersion terms and α : SPM, β : XPM, γ_1 and γ_2 : FWM coefficients. The system (47) passes the Painlevé test whenever the parameters belong to one of the following four classes; (i) $\mu = 1, \alpha = \beta, \gamma_1 = \gamma_2 = 0$, (ii) $\mu = 1, \beta = 2\alpha, \gamma_1 = -\alpha, \gamma_2$ arbitrary, (iii) $\mu = 1, \beta = 2\alpha, \gamma_1 = \alpha, \gamma_2 = 0$ and (iv) ($\beta = -1$, $\alpha = -\beta$, $\gamma_1 = \gamma_2 = 0$. The integrability of cases (i) and (ii) have been demonstrated in §3.1. In particular, case (ii) corresponds to the propagation in the isotropic nonlinear medium with the property that $\chi^{(3)}_{xxyy} + \chi^{(3)}_{xyxy} = -2\chi^{(3)}_{xyyx}$. The one-soliton solution is given in the form, $\varepsilon = 1$

$$q_1 = \sqrt{2b}\cos K e^{i(a^2-b^2)t+iaz+\Theta}\operatorname{sec} h(hz + 2abt + \Delta)$$
$$q_2 = \sqrt{2b}\sin K e^{i(a^2-b^2)t+iaz+\Theta}\operatorname{sec} h(hz + 2abt + \Delta) \tag{48}$$

For $\beta = -1$, we obtain the following integrable system

$$q_{1z} = q_{1tt} + 2(|q_1|^2 q_1 - 2|q_2|^2 q_1 - q_2^{*^2} q_1^*)$$
$$q_{2z} = -q_{2tt} + 2(|q_2|^2 q_2 - 2|q_1|^2 q_2 - q_2^{*^2} q_2^{\alpha}) \tag{49}$$

From the above investigations, it is clear that the FWM problem also admits two completely integrable soliton cases like CNLS equations. The explicit Lax pair, bilinear form and soliton solutions have been constructed. From the solutions, it has been concluded that in an isotropic medium, left and right circularly polarized lights do not interact with each other thereby preserving circular polarizations. This case may be compared with a polarization preserving fiber where only one particular polarization direction is preserved.

3.3 *Two orthogonally polarized solitons with self-steepening*

Similar to the single field propagation, SS also plays an important role in the description of solitons in many-field propagation. With the inclusion of SS, the system equation is the integrable-coupled MDNLS equation of the form [6,7],

$$\begin{aligned} &iq_{1z} + q_{1tt} + (|q_1|^2 + |q_2|^2)q_1 \mp i\alpha[(|q_1|^2 + |q_2|^2)q_1]_t = 0 \\ &iq_{2z} + q_{2tt} + (|q_1|^2 + |q_2|^2)q_2 \mp i\alpha[(|q_1|^2 + |q_2|^2)q_2]_t = 0. \end{aligned} \tag{50}$$

Recently, Wadati *et al* [70] have connected the CMDNLS and the CDNLS equations through the Gauge equivalence and hence established the complete integrability properties.

3.4 *Solitons of two fields in presence of higher order dispersion and self-steepening*

In order to increase the bit rate, it is necessary to decrease the pulse width so that the transmission capacity can be increased. As pulse lengths become comparable to the wavelength, however, the above theoretical models become inadequate, as additional terms must now be considered. With the inclusion of the HOD and SS, the two-wave propagation in a nonlinear fiber media can be represented using the coupled Hirota equation [68,69]. The coupled Hirota equation takes the form,

$$\begin{aligned} &iq_{1z} + [c_1 q_{1tt} + 2(\alpha|q_1|^2 + \beta|q_2|^2)q_1] + i\varepsilon[q_{1ttt} + (2\mu_1|q_1|^2 + \nu_1|q_2|^2)q_{1t} + \nu_1 q_1 q_2^* q_{1t}] = 0, \\ &iq_{2z} + [c_2 q_{2tt} + 2(\beta|q_1|^2 + \gamma|q_2|^2)q_2] + i\varepsilon[q_{2ttt} + (\nu_2|q_1|^2 + 2\mu_2|q_2|^2)q_{2t} + \nu_2 q_2 q_1^* q_{2t}] = 0. \end{aligned} \tag{51}$$

From the detailed analysis, we find that eq. (51) admits soliton solutions only for the conditions (i) $c_1 = c_2, \alpha = \beta = \gamma, \mu_1 = \nu_1 = \mu_2 = \nu_2 = 3$ and (ii) $c_1 = -c_2, \alpha = -\beta = \gamma, \mu_1 = -\nu_1 = -\mu_2 = \nu_2 = 3$. For the first condition (i), exact N-soliton solutions have been reported and the second condition is one of the new integrable systems which are not at all studied [69]. Generalizing the 2×2 AKNS method to a 3×3 eigenvalue problem, we construct the linear eigenvalue problem as follows:

$$U = \begin{pmatrix} \frac{-i\lambda}{2} & -k_1 q_1 & -k_1 q_2 \\ k_1 q_1^* & \frac{i\lambda}{2} & 0 \\ k_1 q_2^* & 0 & \frac{i\lambda}{2} \end{pmatrix} \tag{52a}$$

k_1 is an arbitrary constant. Using the integrability condition and choosing the appropriate constants of integration, the V matrix is obtained as

$$\begin{aligned} V = {} & \lambda^3 \begin{pmatrix} \frac{i\varepsilon}{2} & 0 & 0 \\ 0 & \frac{-i\varepsilon}{2} & 0 \\ 0 & 0 & \frac{-i\varepsilon}{2} \end{pmatrix} + \lambda^2 \begin{pmatrix} A_2 & \varepsilon k_1 q_1 & \varepsilon k_1 q_2 \\ -\varepsilon k_1 q_1^* & -A_2 & 0 \\ -\varepsilon k_1 q_2^* & 0 & -A_2 \end{pmatrix} \\ & + \lambda \begin{pmatrix} -i\varepsilon_1^2(|q_1|^2 + |q_2|^2 & i\varepsilon k_1 q_{1t} - 2iA_2 k_1 q_1 & i\varepsilon k_1 q_{2t} - 2iA_2 k_1 q_2 \\ i\varepsilon k_1 q_{1t}^* + 2iA_2 k_1 q_1^* & i\varepsilon k_1^2 |q_1|^2 & i\varepsilon k_1^2 q_1^* q_2 \\ i\varepsilon k_1 q_{2t}^* + 2iA_2 k_1 q_2^* & i\varepsilon k_1^2 q_2^* q_1 & i\varepsilon k_1^2 |q_2|^2 \end{pmatrix} \\ & + \begin{pmatrix} G & H & J \\ K & L & M \\ N & P & Q \end{pmatrix} \end{aligned} \tag{52b}$$

where,

$$
\begin{aligned}
G &= -2A_2k_1^2(|q_1|^2+|q_2|^2) - \varepsilon k_1^2(q_1q_{11t}^* - q_1^*q_{1t} + q_2q_{2t}^* - q_2^*q_{2t}),\\
H &= -\varepsilon k_1 q_{1tt} + 2A_2k_1q_{1t} - 2\varepsilon k_1^3 q_1(|q_1|^2+|q_2|^2),\\
J &= -\varepsilon k_1 q_{2tt} + 2A_2k_1q_{2t} - 2\varepsilon k_1^3 q_2(|q_1|^2+|q_2|^2),\\
K &= \varepsilon k_1 q_{1tt}^* + 2A_2k_1q_{1t}^* + 2\varepsilon k_1^3 q_1^*(|q_1|^2+|q_2|^2),\\
L &= -\varepsilon k_1^2(q_1^*q_{1t} - q_{1t}^*q_1) + 2A_2k_1^2(|q_1|^2),\\
M &= -\varepsilon k_1^2(q_1^*q_{2t} - q_2q_{1t}) + 2A_2k_1^2q_1^*q_2,\\
N &= \varepsilon k_1 q_{2tt}^* + 2A_2k_1q_{2t}^* + 2\varepsilon k_1^3 q_2^*(|q_1|^2+|q_2|^2),\\
P &= -\varepsilon k_1^2(q_2^*q_{1t} - q_1q_{2z}^*) + 2A_2k_1^2q_2^*q_1,\\
Q &= -\varepsilon k_1^2(q_2^*q_{2t} - q_{2t}^*q_2) + 2A_2k_1^2(|q_2|^2).
\end{aligned}
$$

Compatibility condition for the above Lax pair gives the following form of equations:

$$
\begin{aligned}
&k_1q_{1z} + 2A_2k_1q_{1tt} + 4k_1^3A_2(|q_1|^2+|q_2|^2)q_1\\
&\quad -i\varepsilon[-ik_1q_{1ttt} - 3ik_1^3(|q_1|^2+(|q_2|^2))q_{1t} - 3ik_1^3q_1(q_1^*q_{1t}+q_2^*q_{2t})] = 0, \qquad (53a)\\
&k_1q_{2z} + 2A_2k_1q_{2tt} + 4k_1^3A_2(|q_1|^2+|q_2|^2)q_2\\
&\quad -i\varepsilon[-ik_1q_{2ttt} - 3ik_1^3(|q_1|^2+(|q_2|^2))q_{2t} - 3ik_1^3q_2(q_1^*q_{1t}+q_2^*q_{2t})] = 0. \qquad (53b)
\end{aligned}
$$

We have found that the above equations give the bright soliton Hirota equation for the choice :

$$k_1 = 1; \quad A_2 = -i/2$$

Using the BT method, the one-soliton solution can be constructed as,

$$q_1 = 2\beta\exp(2i\beta^2z)\mathrm{sech}(2\beta t + 8\beta^3z), \quad q_2 = 2\beta\exp(2i\beta^2z)\mathrm{sech}(2\beta t + 8\beta^3z), \qquad (54)$$

In [68], Tasgal *et al* have given the IST for the coupled Hirota equation and one-soliton solution is reported. Very recently, we have derived the Hirota bilinear form of the coupled Hirota equation [69] and shown that the above integrable version is the next hierarchy of the CNLS equation and also derived the next hierarchy of the dark CNLS equations [68] i.e., case (i) of eq. (40). We have also generalized eqs. (53) to N-field propagation and established the complete integrability properties [73].

3.5 *WDM solitons with all higher order effects*

In a similar manner, including all the nonlinear effects, the coupled HNLS (CHNLS) equations have been proposed by us and shown that the system admits solitons only for a particular choice of parameters [73–75]. The general form of CHNLS equation is,

$$\begin{aligned} &iq_{1Z}+\alpha_1 q_{1TT}+\alpha_2(|q_1|^2+|q_2|^2)q_1\\ &\qquad +i\varepsilon\left[\alpha_3 q_{1TTT}+\alpha_4(|q_1|^2+|q_2|^2)q_{1T}+\alpha_5 q_1(|q_1|^2+|q_2|^2)_T\right]=0,\\ &iq_{2Z}+\alpha_1 q_{2TT}+\alpha_2(|q_1|^2+|q_2|^2)q_1\\ &\qquad +i\varepsilon\left[\alpha_3 q_{2TTT}+\alpha_4(|q_1|^2+|q_2|^2)q_{2T}+\alpha_5 q_2(|q_1|^2+|q_2|^2)_T\right]=0. \end{aligned} \tag{55}$$

As such eq. (55) is not integrable but using Painlevé analysis, we found that the above system is integrable only for certain parametric conditions. If some restrictions are imposed on the parametric values, one can obtain several integrable, soliton-possessing CNLS -type equations reported in the sections 3.1–3.4. In our recent work, we have shown that the system of two-coupled HNLS equation can be generalized to the integrable form of N-coupled equation and proposed the possibility of the existence of solitons [74–75]. We consider the soliton aspects of the system of N-coupled HNLS equation given in the form,

$$\begin{aligned} &iq_{1Z}+\frac{1}{2}q_{1TT}+q_1\sum_{n=1}^{N}|q_n|^2+i\varepsilon\left[q_{1TTT}+6q_{1T}\sum_{n=1}^{N}|q_n|^2+3q_1\left(\sum_{n=1}^{N}|q_n|^2\right)_T\right]=0,\\ &\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\\ &iq_{NZ}+\frac{1}{2}q_{NTT}+q_N\sum_{n=1}^{N}|q_n|^2+i\varepsilon\left[q_{NTTT}+6q_{NT}\sum_{n=1}^{N}|q_n|^2+3q_N\left(\sum_{n=1}^{N}|q_n|^2\right)_T\right]=0. \end{aligned} \tag{56}$$

Using eq. (18), eq. (56) is reduced to N-coupled cmK-dV equations of the form,

$$\begin{aligned} &E_{1Z}+\varepsilon\left[E_{1TTT}+6E_{1T}\sum_{n=1}^{N}|E_n|^2+3E_1\left(\sum_{n=1}^{N}|E_n|^2\right)_T\right]=0,\\ &\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\\ &E_{NZ}+\varepsilon\left[E_{NTTT}+6E_{NT}\sum_{n=1}^{N}|E_n|^2+3E_N\left(\sum_{n=1}^{N}|E_n|^2\right)_T\right]=0. \end{aligned} \tag{57}$$

In order to construct the explicit Lax pair for eq. (57), we generalize the AKNS method to a $(2N+1)\times(2N+1)$ eigenvalue problem and construct the Lax operators U and V. Applying BT, the one-soliton solution is obtained in the form,

$$E_j(1)=2\beta\,\mathrm{sech}(2\beta t-8\varepsilon\beta^3 z),\quad E_N(1)=2\beta\,\mathrm{sech}(2\beta t-8\varepsilon\beta^3 z). \tag{58}$$

where $j=1,2,3,...,N-1$

3.6 *Optical solitons in inhomogeneous coupled systems*

The wave propagation in nonlinear couplers or multi-field propagation with the inclusion of inhomogeneous effects can also be analyzed along the lines discussed so far. For the propagation of two orthogonally polarized optical fields or N fields in a nonuniform fiber media, we have recently considered the coupled INLS equations and shown that with suitable variable transformation, the system equations can be transformed to the CNLS equations [51]. For N-field propagation the system becomes,

$$iq_{jz} + q_{jtt} + 2q_j \left(\sum_{n=1}^{N} |q_n|^2 \right) + \hat{M}(z,t) q_j + G(z,t) = 0; \quad j = 1, 2, 3, \ldots, N. \tag{59}$$

First, we investigate the soliton solutions of eq. (59) without phase modulation part i.e. $M(z,t) = i\hat{M}(z); G(z,t) = 0$. With these conditions eq. (59) becomes,

$$iq_{jz} + q_{jtt} + 2q_j \left(\sum_{n=1}^{N} |q_n|^2 \right) + i\hat{M}(z) q_j = 0; \quad j = 1, 2, 3, \ldots, N. \tag{60}$$

Using the Painlevé analysis, eq. (60) is found to be integrable only for the condition $\hat{M}(z) = \frac{1}{2(z+z_0)}$ and the integrable version of eq. (60) is,

$$iq_{jz} + q_{jtt} + 2q_j \left(\sum_{n=1}^{N} |q_n|^2 \right) + \frac{i}{2(z+z_0)} q_j = 0. \tag{61}$$

For the first time, we have given Lax pair and soliton solutions to the N-coupled inhomogeneous NLS equation. Further, it is interesting to mention that under the following variable transformations,

$$q_j(z,t) = \frac{\sqrt{2} z_0}{(z+z_0)} Q_j(Z,T) \exp\left(\frac{it^2}{2(z+z_0)} \right), \quad Z = \frac{zz_0}{(z+z_0)}, \ \ T = \frac{\sqrt{2} t z_0}{(z+z_0)}, \tag{62}$$

equation (66) can be transformed to N-coupled NLS equation. We have also constructed similar transformations for inhomogeneous dark soliton equation [51]. Now, we consider the system (60) with $G(z,t) = 0$ and $M(z,t) = i + t^2$ and search for the exact soliton solutions through Lax pair and BT. For example, this system mainly describes pulse propagation in dispersion managed optical fiber with chirp. Now, eq. (59) becomes,

$$iq_{jz} + q_{jtt} + 2q_j \left(\sum_{n=1}^{N} |q_n|^2 \right) + iq_j + t^2 q_j = 0; \quad j = 1, 2, 3, \ldots, N. \tag{63}$$

For eq. (63) also, we have constructed the eigenvalue problem with non-isospectral parameters. Using the BT method, the one-soliton solution of eq. (63) is obtained in the form

$$\begin{aligned} q_j &= 2\alpha_2 \exp\left\{ -2i\alpha_1 t - 4i \int^z (\alpha_1^2 - \alpha_2^2) \mathrm{d}z + i\frac{t^2}{2} \right\} \\ &\quad \times \mathrm{sech}\left(2\alpha_2 t + 8 \int^z \alpha_1 \alpha_2 \mathrm{d}z \right), \quad j = 1, 2, 3, \ldots, N-1. \\ q_N &= 2\alpha_2 \exp\left\{ \frac{i}{(z+z_0)} \left(\frac{t^2}{2} - 2\rho^2 + \delta_N \right) \right\} \mathrm{sech}\left(\frac{2\rho t}{(z+z_0)} + \theta_N \right), \end{aligned} \tag{64}$$

where $\alpha_1 = k_1 \exp(-2z), \alpha_2 = k_2 \exp(-2z)$.

In the literature, it has been reported that some of the integrable inhomogeneous equations with suitable transformations can be transformed into the corresponding homogeneous equations. However, to our knowledge, it should be noted that eq. (63) may not be transformed to the corresponding homogeneous soliton systems.

3.7 *Dark soliton solutions of the coupled Hirota equation*

Recent experimental achievements in nonlinear couplers have increased interest in the potential applications of optical dark solitons such as optical switching. Like our discussions on single-field propagation, in recent years, the integrability of dark solitons in coupled systems has also attracted considerable interest. But, there has not been much work on the dark soliton solutions for the coupled Hirota equations so far. So, the present analysis will uncover the integrability criterion for obtaining dark soliton solutions using various integrability techniques.

The pulse propagation, in the dark soliton case with $k_1 = i$ and $A_2 = i/2$, in eq. (53b) is governed by the following generalized coupled equation of the form:

$$\begin{aligned} & iq_{1z} - q_{1tt} + 2(\alpha|q_1|^2 + \beta|q_2|^2)q_1 \\ & \qquad -i\varepsilon\left\{q_{1ttt} + (2\mu_1|q_1|^2 + v_1|q_2|^2)q_{1t} + v_1 q_1 q_2^* q_{2t}\right\} = 0, \\ & iq_{2t} - q_{2tt} + 2(\alpha|q_1|^2 + \beta|q_2|^2)q_2 \\ & \qquad -i\varepsilon\left\{q_{2ttt} + (2\mu_1|q_1|^2 + v_1|q_2|^2)q_{2t} + v_1 q_1 q_2^* q_{1t}\right\} = 0. \end{aligned} \tag{65}$$

Equation (65) admits the Lax pair, BT and soliton solutions. Similarly, generalising the above results to $(N+1)\times(N+1)$ eigenvalue problem, one can construct a dark soliton model for N-field propagation.

In the following, we generate the dark soliton solutions through Hirota's bilinear method. Before constructing the bilinear form for the coupled dark Hirota system, it would be rather convenient to transform the dark-CH equations into a set of cmK-dV equations of the form

$$\begin{aligned} & Q_{1Z} - \varepsilon[Q_{1TTT} - (6|Q_1|^2 + 3|Q_2|^2)Q_{1T} - 3Q_1 Q_2^* Q_{2T}] = 0, \\ & Q_{2Z} - \varepsilon[Q_{2TTT} - (6|Q_2|^2 + 3|Q_1|^2)Q_{2T} - 3Q_2 Q_1^* Q_{1T}] = 0. \end{aligned} \tag{66}$$

The Hirota bilinear form for the dark-CH equations can be constructed as usual by applying the transformation for the field variables as

$$Q_1(Z,T) = \frac{G(Z,T)}{F(Z,T)}, \quad Q_2(Z,T) = \frac{H(Z,T)}{F(Z,T)}, \tag{67}$$

where $G(Z,T)$ and $H(Z,T)$ are complex functions and $F(Z,T)$ is a real function. Using eq. (67), the bilinear forms of eq. (66), are obtained as:

$$\beta_1(G\cdot F) = 0; \beta_2(H\cdot F) = 0; \beta_2(F\cdot F) = -2(GG^* + HH^*); D_Z(H\cdot F) = 0, \tag{68}$$

where $\beta_1 = D_Z - \varepsilon D_T^3 - 3\varepsilon\lambda D_T$, and $\beta_2 = D_T^2 + \lambda$ and λ is a constant to be determined. Proceeding as we discussed in §2.5, the dark one-soliton solution of cmK-dV equation is obtained as:

$$Q_1 = \tau_1 \tanh\left[\frac{c_1 Z}{2} - \frac{c_1^3 \varepsilon T}{4}\right], \quad Q_2 = \tau_2 \tanh\left[\frac{c_1 Z}{2} - \frac{c_1^3 \varepsilon T}{4}\right], \tag{69}$$

From eqs (69), we can easily obtain the corresponding dark one-soliton solution of the CH equation (65).

In order to construct the two-soliton solutions, substituting the usual expansions, we obtain the following results:

$$g_1 = h_1 = Z_1 \exp[\xi_1] + Z_2 \exp[\xi_2];\; f_1 = \exp[\xi_1] + \exp[\xi_2],$$

and

$$g_2 = h_2 = A_{12} Z_1 Z_2 \exp[\xi_1 + \xi_2];\; f_2 = A_{12} \exp[\xi_1 + \xi_2] \tag{70}$$

where $\xi_j = P_j T - \Omega_j Z \quad j = 1,2$ and $P_j = \varepsilon\Omega_j^3 + 3\varepsilon\lambda\Omega_j$.

The values of Z_j are found to be $Z_j = \frac{2|\tau_1|^2 + |\tau_2|^2 - \Omega_j^2}{2(|\tau_1|^2 + |\tau_2|^2)}$ with $\lambda = -2(|\tau_1|^2 + |\tau_2|^2)$.

The value of A_{12} is found to be:

$$A_{12} = \frac{(Z_2 - Z_1)\left\{-(P_2 - P_1) + \varepsilon(\Omega_2^3 - \Omega_1^3) - 3\varepsilon\lambda(\Omega_2 - \Omega_1)\right\}}{(1 - Z_1 Z_2)\left\{-(P_2 + P_1) + \varepsilon(\Omega_2^3 + \Omega_1^3) - 3\varepsilon\lambda(\Omega_2 + \Omega_1)\right\}}.$$

From the plots of the dark one- and two-soliton solutions, it can be clearly seen that dark solitons exist for the CH system as the femtosecond optical pulses retain their dark solitary wave nature even in the presence of higher order effects like HOD, SS. The presence of higher order terms are felt by their influence on the velocity of dark solitons. But, otherwise they leave the solitons' shape intact. From the plot for two-soliton solution we conclude that the presence of higher order terms certainly influences the phase and velocity of dark solitons. Yet they maintain their inelastic behavior, since after collision, they retain their shape and intensity only with a slight change in their phase.

3.8 *Propagation of dark solitons in the system of coupled HNLS equations*

With the inclusion of all linear nonlinear higher order effects, the CHNLS equation in the normal dispersion region is as:

$$\begin{aligned} q_{1z} &= -iq_{1tt} + 2i(|q_1|^2 + |q_1|^2)q_1 \\ &\quad + \varepsilon\left\{q_{1ttt} + \alpha_1(|q_1|^2 + |q_2|^2)q_{1t} + \alpha_2 q_1(|q_1|^2 + |q_2|^2)_t\right\}, \\ q_{2z} &= -iq_{2tt} + 2i(|q_1|^2 + |q_1|^2)q_2 \\ &\quad + \varepsilon\left\{q_{2ttt} + \alpha_1(|q_1|^2 + |q_2|^2)q_{2t} + \alpha_2 q_2(|q_1|^2 + |q_2|^2)_t\right\}, \end{aligned} \tag{71}$$

where α_1 and α_2 are arbitrary constants. In §3.5, we have discussed in detail the bright soliton version of eq. (71). To our knowledge, the dark soliton solution for eq. (71) has not been reported. In the following, using Hirota's bilinear method, we construct dark soliton solutions. For simplicity, we transform the CHNLS equations into a set of CmK-dV equations in the form:

$$\begin{aligned} &Q_{1T} - \varepsilon[Q_{1ZZZ} - 6(|Q_1|^2 + |Q_2|^2)Q_{1Z} - 3Q_1(|Q_1|^2 + |Q_2|^2)_Z] = 0, \\ &Q_{2T} - \varepsilon[Q_{2ZZZ} - 6(|Q_1|^2 + |Q_2|^2)Q_{2Z} - 3Q_2(|Q_1|^2 + |Q_2|^2)_Z] = 0. \end{aligned} \tag{72}$$

The decoupled bilinear forms of eq. (72) are given as:

$$(D_T - \varepsilon D_Z^3 + 3\varepsilon\lambda D_Z)G \cdot F = 0, \quad (D_T - \varepsilon D_Z^3 + 3\varepsilon\lambda D_Z)H \cdot F = 0,$$
$$(D_Z^2 - \lambda)F \cdot F = -4(|G|^2 + |H|^2), \quad D_Z G^* \cdot G = D_Z H^* \cdot H = 0, \tag{73}$$

where λ is a constant to be determined. Then, the dark one-soliton solution of eq. (72) is obtained as:

$$Q_1 = g_0 \tanh\left[\frac{1}{2}\left\{c_1\left(Z - \frac{c_1^2 \varepsilon T}{2}\right) + \xi_1^{(0)}\right\}\right],$$
$$Q_2 = h_0 \tanh\left[\frac{1}{2}\left\{c_1\left(Z - \frac{c_1^2 \varepsilon T}{2}\right) + \xi_1^{(0)}\right\}\right]. \tag{74}$$

We have also constructed Lax pair and higher order solitons. As eq. (72) admits the required properties for complete integrability, we conclude that systems (72) admit exact N-soliton solutions.

4. Solitons in resonant fibers

In the earlier sections, we have discussed the integrability aspects of single and coupled NLS equations with different linear and nonlinear optical effects. In this section, considering the doping of resonant impurities in the fiber, we shall discuss the concepts of self-induced transparency (SIT) and associated theoretical models. In 1967, McCall and Hahn [26] proposed SIT soliton in two level resonant atoms. Let us consider an ultra short pulse of light, which interacts, with an ensemble of two-level atoms for a time, which is much shorter than any relaxation time of the atoms. Upon its traversal through the medium, the ultra short pulse sees the probed atoms as if they were frozen, which leads to a fully coherent interaction. Under such conditions, and whenever the optical pulse amplitude is large enough, the states of the two-level atoms may continuously evolve along the pulse profile from the fundamental state up to the excited state and back again to the fundamental state. As a result, after the interaction with the ultra short pulse, all the atoms are left back in the initial fundamental state. An optical pulse, which is characterized by such a balance between absorption and stimulated emission, is known as a SIT soliton. Such pulses may propagate indefinitely through the absorbing medium. Extensive investigation of this strongly resonant situation led to the observation of soliton behavior both in experiments and in numerical solutions of the governing equations [26, 42–48]. With these properties, the pulse propagation of this type is governed by the Maxwell-Bloch (MB) equations whose solitons solutions are well studied. In this section, we shall consider several examples of the SIT phenomena.

4.1 *SIT in Kerr-type nonlinear medium*

When USP are used inside the resonant optical fiber, then the dynamics is usually characterized by a high electric field strength, so that nonlinear optical effects take place. Practically speaking, all materials used for fiber fabrication contain impurities that contribute to the absorption of the energy. Breaking the pulse duration shorter than the characteristic relaxation times of the resonant states can minimize these losses. The model of the USP propagation in a Kerr type nonlinear medium doped by resonant impurity atoms incorporates both NLS and MB systems. The evaluation of the USP propagating in a nonlinear multimode fiber in z-direction is described by the NLS-MB equation by the form [43,77].

$$\begin{aligned} q_z &= i\alpha_1 q_{tt} - i\alpha_2 |q|^2 q + \langle p \rangle, \\ p_t &= i\omega p - f q\eta, \\ \eta_t &= f(qp^* + q^* p), \end{aligned} \tag{75}$$

where f is the interaction between light pulse and the medium. To obtain the condition for the existence of non-broadening optical pulse, we must find at what ratio of the parameters in eq. (75), this system admits complete integrability.

The Painlevé analysis of eq. (75) admits solitons only for the parametric choice $-2f^2\alpha_1 = \alpha_2$. Since f is positive, this condition leads to a relation of the Kerr susceptibility and the GVD. Kakei and Satsuma [46] constructed the N-soliton solution for the NLS-MB equations with the following Lax pair:

$$\begin{aligned} U &= \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \lambda + \begin{pmatrix} 0 & q \\ -q^* & 0 \end{pmatrix}, \\ V &= i\left[\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \lambda^2 + \begin{pmatrix} 0 & q \\ -q^* & 0 \end{pmatrix} \lambda + \frac{1}{2} \begin{pmatrix} |q|^2 & q_t \\ q_t^* & -|q|^2 \end{pmatrix} \right] \\ &\quad + \begin{pmatrix} \left\langle \frac{\eta}{\lambda - i\omega} \right\rangle & \left\langle \frac{-p}{\lambda - i\omega} \right\rangle \\ \left\langle \frac{-p^*}{\lambda - i\omega} \right\rangle & \left\langle \frac{-\eta}{\lambda - i\omega} \right\rangle \end{pmatrix} \end{aligned} \tag{76}$$

Choosing the eigenvalue parameter as $\lambda = v + i\rho$ and applying the Bäcklund transformation method, we generated the soliton solutions [77]. For instance, the single soliton solution of NLS-MB equations is obtained as follows,

$$\begin{aligned} q(z,t) &= q(1) = -2\rho \operatorname{sech}(x) \exp(iy - i\theta), \\ p(z,t;\omega) &= \frac{2\rho\{\rho \sinh(x) + i(v-\omega)\cosh(x)\}\exp(iy - i\theta)}{\rho^2 \sinh(x) + (v-\omega)^2 \cosh^2(x) + \rho^2/4}, \\ \eta(z,t;\omega) &= \frac{\rho^2 \sinh^2(x) + (v-\omega)^2 \cosh^2(x) - \rho^2/4}{\rho^2 \sinh^2(x) + (v-\omega)^2 \cosh^2(x) + \rho^2/4}, \end{aligned} \tag{77}$$

where 'x' and 'y' are functions of 'z', 't' and the soliton velocity parameters given by,

$$\begin{aligned} x(z,t) &= 2\rho t + \left[-4\rho v + \int_{-\infty}^{\infty} \frac{2\rho g(\omega)\mathrm{d}\omega}{\rho^2 + (v-\omega)^2} \right] z + x^{(0)}, \\ y(z,t) &= -2vt + \left[2(\rho^2 - v^2) - \int_{-\infty}^{\infty} \frac{2(v-\omega)\mathrm{d}\omega}{\rho^2 + (v-\omega)^2} \right] z + y^{(0)}, \end{aligned}$$

where $x^{(0)}$ and $y^{(0)}$ are independent of both z and t and θ is a real constant. Similarly, two and higher order soliton solutions have been generated. The presence of resonant impurities radically changes the situation (see the phase change in figure 11). 2π pulses of SIT should simultaneously be also a soliton of the NLS equation. i.e., the amplitude and duration of the 2π pulse should preciously be of such values that the corresponding

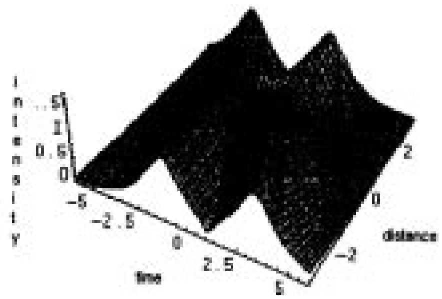

Figure 11. Two soliton solutions of the NLS-MB equation.

self-action would lead to complete compensation of the dispersion broadening of the USP [42–45].

4.2 *Solitons with Kerr dispersion in erbium doped fibers*

Considering the higher order effects, we proposed the coupled system of the Hirota equation and the MB equations (H-MB system), which governs the wave propagation in Er doped fibers [78]. The H-MB system equations are

$$\begin{aligned}
q_z &= i\left(\frac{1}{2}q_{tt} + |q|^2 q\right) + \left(q_{ttt} + 6|q|^2 q_t\right) + \langle p\rangle,\\
p_t &= 2i\omega p + q\eta,\\
\eta_t &= -(qp^* + q^* p).
\end{aligned} \tag{78}$$

In [78], we analysed the possibility of soliton-type pulse propagation in using the Painlevé analysis. With the help of Lax pair, we have also constructed the explicit one-soliton solution using BT. The single soliton solution of H-MB equation can be obtained as

$$q(z,t) = 2\beta\,\mathrm{sech}(\rho_1)\exp(i\sigma_1 - i\theta_1), \tag{79}$$

where ρ_2 and σ_2 are functions of z,t and soliton velocity parameters given by

$$\begin{aligned}
\rho_1(z,t) &= 2\beta t + \left[8\alpha\beta + \int_{-\infty}^{\infty}\frac{2\beta g(\omega)\mathrm{d}\omega}{\beta^2 + (\alpha-\omega)^2} + 8\beta(3\alpha^2 - \beta^2)\right] z + \rho_1^{(0)},\\
\sigma_1(z,t) &= 2\alpha t + \left[4(\beta^2 - \alpha^2) - \int_{-\infty}^{\infty}\frac{2(\alpha-\omega)g(\omega)\mathrm{d}\omega}{\beta^2 + (\alpha-\omega)^2} + 8\alpha(\alpha^2 - 3\beta^2)\right] z + \sigma_1^{(0)},
\end{aligned}$$

($\rho_1^{(0)}$ and $\sigma_1^{(0)}$ are independent of both z and t) and θ_1 is real constant. The impact of these additional effects when compared with NLS solitons is that the speed of the each soliton in a bound-soliton solution becomes different, which leads to splitting of the bound solitons. However, this result can also be used to separate solitons (signal) from linear dispersive waves (noise), even if they occupy the same frequency domain.

4.3 *HNLS-MB solitons*

With all the linear and nonlinear higher order effects, the pulse propagation in the resonant fiber is described by the HNLS-MB equations. We analyzed the possibility of soliton-type pulse propagation in the HNLS-MB equation and presented the Lax pair for the same [79,80]. The HNLS-MB equation reads as

$$\begin{aligned} q_z &= i[1/2q_{tt} + 2|q|^2 q] - \varepsilon\left[\alpha_3 q_{ttt} + \alpha_4 |q|^2 q_t + \alpha_5 q(|q|^2)_t\right] + \langle p\rangle, \\ p_t &= 2i\omega p + q\eta, \\ \eta_t &= -(qp^* + qp^*). \end{aligned} \tag{80}$$

From the careful analysis, eq. (80) admits soliton-type pulse propagation only for the condition, $3\alpha_3 = \alpha_4 = 2\alpha_5$. We have also constructed the soliton solutions and obtained very similar results as in §4.2 [80].

Similarly, we have considered the effect of variable dispersion, variable nonlinearity and gain and obtained the condition for the propagation of solitons [81–82].

5. Solitons in SRS system

In this section, we shall discuss another very important problem, namely stimulated Raman scattering (SRS) in nonlinear optics, which has attracted considerable interest. This effect is inelastic in the sense that there is an energy transfer between the field and the medium. SRS can be explained by the concept of Raman cell, a two-level resonant medium, in which a pump beam A_1 interacts with a Stokes beam A_2. The pump beam is at a higher frequency than the Stokes beam. If the difference between the frequencies of the two beams resonates with the medium, then the electromagnetic wave generates an excited wave E. SRS has both its advantages and disadvantages. On the one hand, it is employed to construct broadband Raman amplifiers and tunable Raman lasers. On the other hand, it is considered to be detrimental in optical communication as it transfers energy from one channel to the other and in the case of ultra short pulses, it transfers energy from the higher frequency components to the lower frequency components leading to the effect of self-frequency shift.

The equations governing SRS, after appropriate normalization, are given by [83–86]

$$\begin{aligned} A_{1x} &= -A_2 E, \\ A_{2x} &= A_1 E^*, \\ E_t + \Gamma E - A_1 A_2^* &= 0, \end{aligned} \tag{81}$$

where A_1 is the complex envelope of the higher frequency wave (pump wave), A_2 is that of the lower frequency wave (Stokes wave) and E is the complex envelope of the probability wave for the material excitation. The quantity Γ is proportional to T_2^{-1}, the collisional de-excitation time. These equations have been intensely studied theoretically and numerically. The nonlinearities in (81) are quadratic nonlinearities and there is no linear dispersion. Thus these equations are in the form of the standard three-wave parameter equations.

The linear eigenvalue problem can be constructed as:

$$\psi_{1x} = -i\lambda\psi_1 - iE\psi_2,$$
$$\psi_{2x} = -iE^*\psi_1 + i\lambda\psi_2$$

and

$$\psi_{1t} = a\psi_1 + b\psi_2,$$
$$\psi_{2t} = c\psi_1 - a\psi_2 \tag{82}$$

where $a = iW/4\lambda - \Gamma/2$; $b = -P/2\lambda$; $c = P^*/2\lambda$ with $W = |A_1|^2 - |A_2|^2$; $P = A_1A_2^*$. By setting $Z = \psi_1/\psi_2$ the BT for the above system is found to be

$$Y' - Y = 4\lambda_i Z/(1+|Z|^2), \tag{83}$$

where $Y = iE$ are the $(N-1)$-soliton solutions and y' is the N-soliton solution. Here λ_i is the imaginary part of the spectral parameter. The one-soliton solution is given as

$$Y = (-2\lambda_i Z_0/|Z_0|)\exp[i(\omega_i t - 2\lambda_i x)]\mathrm{sech}(2\lambda_i x + \omega_r t + \ln|Z_0|) \tag{84}$$

with $\omega_r = (\lambda_i/2)[\lambda_i^2 + \lambda_r^2]$ and $\omega_i = (\lambda_r/2)[\lambda_i^2 + \lambda_r^2]$. The equivalence between SRS and SIT equation with sharp line limit has been established.

6. Summary and conclusions

In this article, we have discussed some of the completely integrable soliton models useful for the field of optical communication. We mainly considered the effects like GVD, HOD, self steepening, stimulated inelastic scattering, self induced transparency, birefringence (WDM) and inhomogeneous effects, and the possibility of soliton-type pulse propagation and explained the existence of solitons through Lax pair, bilinear method and Bäcklund transformation method. The effect of birefringence and WDM in terms of soliton-soliton interaction was explained in terms of photonic logic gate operations. The coexistence of the SIT soliton and the optical fiber soliton with all the higher order effects and inhomogeneities were explained in detail.

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