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Nonlinear compression of optical solitons

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Abstract. In this paper, we consider nonlinear Schrödinger (NLS) equations, both in the anomalous and normal dispersive regimes, which govern the propagation of a single field in a fiber medium with phase modulation and fibre gain (or loss). The integrability conditions are arrived from linear eigen value problem. The variable transformations which connect the integrable form of modified NLS equations are presented. We succeed in Hirota bilinearzing the equations and on solving, exact bright and dark soliton solutions are obtained. From the results, we show that the soliton is alive, i.e. pulse area can be conserved by the inclusion of gain (or loss) and phase modulation effects.

Keywords. Optical solitons; bright and dark solitons; nonlinear compression; phase modulation; fibre amplification; loss.

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1. Introduction

The term soliton refers to special kinds of waves that can propagate undistorted over long distances and remain unaffected after collision with each other. Solitons are very useful for improving the performance of high-speed dispersion-limited fibre optics communication systems since they can maintain their width over long distances by balancing the effect of group velocity dispersion (GVD) through the nonlinear phenomenon of self-phase modulation (SPM). Bright (dark) solitons are manifestations of the fibre nonlinearity in the anomalous (normal) dispersion regime, where the fibre can support them through a balance between the dispersive and nonlinear effects. The simplest possible model of nonlinear pulse propagation is the nonlinear Schrödinger (NLS) equation [1].

There are many nonlinear Schrödinger-type equations which have been studied from the soliton point of view and are shown to be completely integrable by various analytical methods [2,3]. However, there are a number of other factors which can affect the dynamics of optical solitons and the conditions for the generation of optical solitons in real fibres. For instance, the dissipative loss leading to damping of soliton amplitude without changing its velocity [4,5], higher-order dispersion effects [4,6], various inhomogeneities of fibre [7], alternating conditions of exploitation of optical lines [8,9], etc. are important factors which can affect wave propagation through fibres.

Optical pulse compression finds important applications in optical fibres. The pulse compressors based on the nonlinear effects in optical fibres can be classified into two broad categories, referred to as fibre-grating or prism compressors and soliton-effect compressors [1]. In a fibre-grating compressor, the pulse is propagated in the normal dispersion regime of the fibre and then compressed externally using a grating pair. The role of fibre is to impose a linear, positive chirp on the pulse through a combined effect of SPM and GVD. The grating pair provides anomalous GVD required for compression of positively chirped pulses. The soliton-effect compressor consists of only a piece of fibre whose length is suitably chosen. The input pulse propagates in the anomalous dispersion regime of the fibre and is compressed through an interplay between SPM and GVD. The compression is due to an initial narrowing phase through which all higher-order solitons go before the initial shape is restored after one soliton period. The compression factor depends on the peak power of the pulse that determines the soliton order N. The two types of compressors are complimentary, and generally operate at different regions of the pulse spectrum. Thus, the fibre-grating compressor is useful to compress pulses in the visible and near-infrared regimes while the soliton-effect compressor is useful in the range 1.3–1.6 μm. These methods are convenient with sources that inherently produce chirped pulses [10]. Unchirped pulses can be chirped for compression in numerous ways. Nonlinear chirping can be achieved by the use of self-phase modulation in optical fibres, possibly with positive dispersion for linear chirping [11]. Unchirped pulses can also be chirped during amplification, or by phase modulation [12].

An alternative approach to chirping is to combine SPM and dispersion in a distributed manner: mainly, to utilize soliton effects. Early work focused on using the compression of higher-order solitons [13]. This can provide rapid conversion but suffers from residual pedestals. Nonlinear intensity discrimination technique can help to reduce the pedestals [14,15], but energy is wasted. A less rapid technique that provides better pulse quality is adiabatic amplification of fundamental solitons. Solitons have fixed area, so the increased energy from amplification is accommodated by an increase in power and a decrease in width. To avoid distortion, the amplification per soliton period should not be too great. Recently, it has become practical to draw optical fibres with dispersion that decreases along the length of the fibre. This method can be used to achieve the same effect as adiabatic amplification, but the effect can be achieved in a passive fiber [16,17].

Moores in his numerical work, has introduced a novel pulse compression technique [18] which is similar to adiabatic soliton amplification. The technique implements the area conservation property of the soliton and this involves gain and phase modulation whose coefficients are functions of normalized distance. Here Moores has considered various gain profiles, and has explictly shown that chirped solitary waves can be nonlinearly compressed throughout cleanly and efficiently with appropriate tailored gain or dispersion profile. In this way, the pulses exhibit no radiative loss, conserve their area and maintain a constant chirp relative to the pulse width.

In nonlinear fibre optics, nonlinear compression of chirped solitary waves is found to have wide applications in optical telecommunication and switching purposes. In this paper, we are analytically discussing the pulses which are chirped throughout the fibre, and the chirp and amplification are proportional. Nonlinear compression of pulse in a fibre can be described by a modified NLS equation of the form

$$iq_z + q_{tt} + 2|q|^2 q = F(z)q, \tag{1}$$

where $F(z)$ is related to gain for $\beta(z) > 0$, and loss for $\beta(z) < 0$, and $\alpha(z)$ is the coefficient of phase modulation, which is, in general, a function of z [19] and is given by

$$F(z) = \alpha(z)t^2 + i\beta(z). \tag{2}$$

Then eq. (1) becomes

$$iq_z + q_{tt} + 2|q|^2 q = \alpha(z)t^2 q + i\beta(z)q. \tag{3}$$

We identify the integrability condition of the above equation through linear eigenvalue problem and construct soliton solutions. The above equation is numerically studied by Moores [18]. An intersting feature to be noted in the present study is the conservation of pulse area which implies the conservation of energy [18].

2. Linear eigenvalue problem

The Lax pair assures the complete integrability of a nonlinear system, and is especially used to obtain integrability condition and N-soliton solutions by means of inverse scattering transform method. In this article, we follow the AKNS formalism to obtain the Lax pair. In order to construct the Lax pair for eq. (3), it is convenient to introduce a variable transformation

$$q(z,t) = Q(z,t)\exp\left(\frac{-i\beta t^2}{2}\right). \tag{4}$$

Using the above transformation in eq. (3), the following equation is obtained

$$iQ_z + \frac{\beta_z t^2 Q}{2} + Q_{tt} + 2|Q|^2 Q - 2i\beta t Q_t - 2i\beta Q - \left(\beta^2 + \alpha\right)t^2 Q = 0. \tag{5}$$

The linear eigenvalue problem associated with eq. (5) is [20,21]

$$\psi_t = U\psi, \tag{6}$$

$$\psi_z = V\psi, \tag{7}$$

$$\psi = (\psi_1 \psi_2)^T, \tag{8}$$

where we choose

$$U = \begin{pmatrix} -i\lambda_1 & Q \\ -Q^* & i\lambda_1 \end{pmatrix}, \tag{9}$$

$$V = 2i\lambda_1^2 \begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix} + 2\lambda_1 \begin{pmatrix} -i\beta t & Q \\ -Q^* & i\beta t \end{pmatrix} + i\begin{pmatrix} |Q|^2 & Q_t - 2i\beta t Q \\ Q_t^* + 2i\beta t Q^* & -|Q|^2 \end{pmatrix}, \tag{10}$$

where λ_1 is the non-isospectral parameter given by

$$\lambda_1 = \lambda_1(0)\exp(2\int \beta \mathrm{d}z). \tag{11}$$

Now the relation $U_z - V_t + [U,V] = 0$, gives rise to the following equation

$$iQ_z + Q_{tt} + 2|Q|^2 Q - 2i\beta t Q_t - 2i\beta Q = 0. \tag{12}$$

On comparing eqs (12) and (5), it is clear that eq. (5) admits complete integrability when the condition

$$\beta_z = 2\left(\beta^2 + \alpha\right) \tag{13}$$

is satisfied. Thus the integrability condition of eq. (3) is derived by using Lax pair. Integrability condition for constant β has been discussed earlier [22,23].

3. Hirota bilinearization and soliton solution

Hirota's bilinear method [24] is one of the most direct and elegant methods available to generate multi-soliton solutions of nonlinear partial differential equations. To construct Hirota's bilinear form, we consider Hirota bilinear transformation in the form

$$Q = \frac{G}{F}, \tag{14}$$

where $G(z,t)$ is a complex function and $F(z,t)$ is a real function. Now using the transformation (14) in (12) and after decoupling, we get

$$\begin{aligned} &\left[iD_z + D_t^2 - 2i\beta t D_t - 2i\beta\right](G.F) = 0 \\ &D_t^2(F.F) = 2|G|^2, \end{aligned} \tag{15}$$

where the Hirota bilinear operators D_z and D_t are defined as

$$\begin{aligned} D_z^m D_t^n G(z,t) F(z',t') = &\left(\frac{\partial}{\partial z} - \frac{\partial}{\partial z'}\right)^m \left(\frac{\partial}{\partial t} - \frac{\partial}{\partial t'}\right)^n \\ &\times G(z,t) F(z',t')\,|_{z=z',t=t'}\,. \end{aligned} \tag{16}$$

In order to obtain soliton solutions, we are applying a perturbative technique by writing the variables F, G as a series in an arbitrary parameter ε

$$F = 1 + \varepsilon^2 f_2 + \varepsilon^4 f_4 + \cdots, \qquad G = \varepsilon g_1 + \varepsilon^3 g_3 + \varepsilon^5 g_5 + \cdots. \tag{17}$$

For one-soliton solution (1SS), we assume

$$F = 1 + \varepsilon^2 f_2, \qquad G = \varepsilon g_1. \tag{18}$$

Substituting eq. (18) in eq. (15) and then collecting coefficents of ε and ε^2, we get

ε

$$\left[iD_z + D_t^2 - 2i\beta t D_t - 2i\beta\right](g_1.1) = 0, \tag{19}$$

ε^2

$$D_t^2\left(1.f_2+f_2.1\right)=2\left|g_1\right|^2, \tag{20}$$

ε^3

$$\left[iD_z+D_t^2-2i\beta tD_t-2i\beta\right]\left(g_1\cdot f_2\right)=0, \tag{21}$$

ε^4

$$D_t^2\left(f_2\cdot f_2\right)=0. \tag{22}$$

For solving (19)–(22), we assume

$$\begin{aligned} g_1 &= \exp(\eta), \\ f_2 &= \exp(\eta+\eta^*), \end{aligned} \tag{23}$$

where

$$\eta=-2i\alpha_1 t-4i\int\left(\alpha_1^2-\alpha_2^2\right)\mathrm{d}z+2\alpha_2 t+8\int\alpha_1\alpha_2\mathrm{d}z+\eta_0 \tag{24}$$

with $\alpha_1(z)=\alpha_1(0)\ \exp(2\int\beta\mathrm{d}z)$ and $\alpha_2(z)=\alpha_2(0)\ \exp(2\int\beta\mathrm{d}z)$. Using eqs (23), (24), (18) and (14), after absorbing ε, the 1SS of eq. (12) can easily be worked out to be as

$$Q=2\alpha_2\exp\left[-2i\alpha_1 t-4i\int\left(\alpha_1^2-\alpha_2^2\right)\mathrm{d}z\right]\operatorname{sech}\left[2\alpha_2 t+8\int\alpha_1\alpha_2\mathrm{d}z\right]. \tag{25}$$

Using the transformation (4), we can construct 1SS for eq. (3) as

$$\begin{aligned} q=2\alpha_2\exp&\left[-2i\alpha_1 t-4i\int\left(\alpha_1^2-\alpha_2^2\right)\mathrm{d}z-\frac{i\beta t^2}{2}\right] \\ &\times\operatorname{sech}\left(2\alpha_2 t+8\int\alpha_1\alpha_2\mathrm{d}z+\eta_0\right). \end{aligned} \tag{26}$$

Thus we have obtained exact bright soliton solution using Hirota technique. The appearance of the term α_2 in the amplitude shows that the soliton amplitude grows at the same rate as the respective power amplitudes, i.e: $\frac{\mathrm{d}W}{\mathrm{d}z}=2W$ with $W=\int|q|^2\,\mathrm{d}\tau$. As a result of chirping, the pulse broadens as it propagates along the fibre. At each stage of propagation, the product of pulse width and pulse amplitude is found to be conserved due to which the area occupied by the pulse envelope remains preserved with the inclusion of gain and phase modulation. This is clearly depicted in figures 1a and b. The contour plot in figure 1c further strengthens our result. The above results are in agreement with the results obtained numerically by Moores [18].

For NLS equation with phase modulation and damping, eq. (3) becomes

$$iq_z+q_{tt}+2\left|q\right|^2q=\alpha(z)t^2q-i\beta(z)q. \tag{27}$$

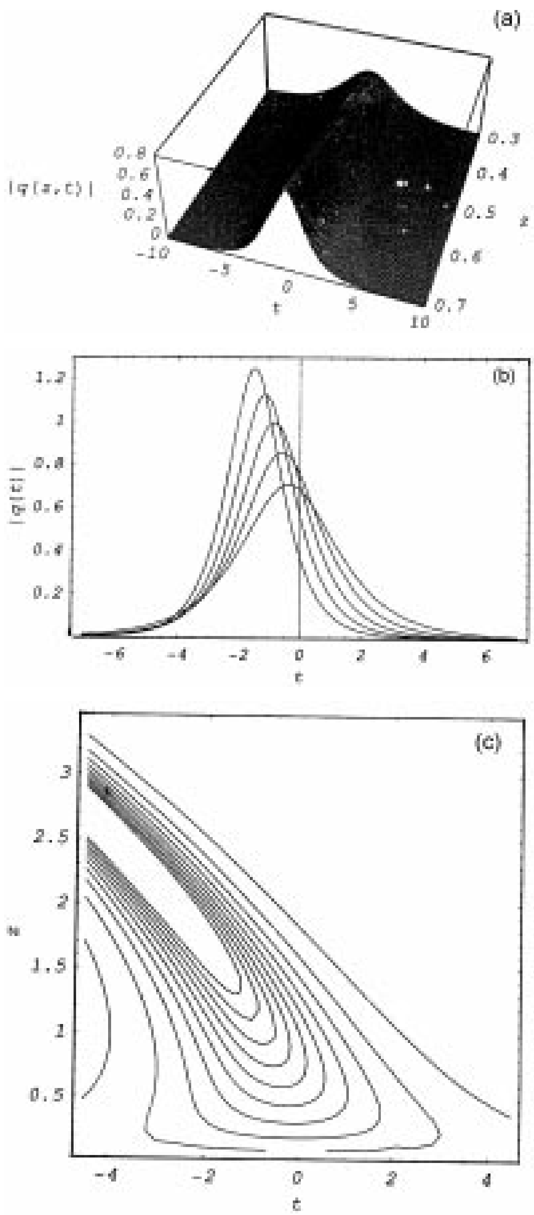

Figure 1. (**a**) Amplitude profile of $|q(z,t)|$ for the one-soliton solution of eq.(26) with the parameter values $\alpha_1(0) = 1$, $\alpha_2(0) = 0.5$ and $\beta(z) = \frac{1}{3z}$. (**b**) Variation of $|Q(t)|$ for the one-soliton solution of eq. (40) with the parameter values $\alpha_1(0) = 1$, $\alpha_2(0) = 0.5$ and $\beta(z) = \frac{1}{3z}$. This figure shows the variation of $|q(t)|$ with t for different values of z ($z = 0.6, 0.8, 1, 1.2, 1.4$). Figure shows that the amplitude increases as z increases. (**c**) Contour plot of $|q(z,t)|$ with respect to z and t for the parameter values $\alpha_1(0) = 1$, $\alpha_2(0) = 0.5$ and $\beta(z) = \frac{1}{3z}$.

The linear eigenvalue problem associated with eq. (27) is

$$U = \begin{pmatrix} -i\lambda_2 & Q \\ -Q^* & i\lambda_2 \end{pmatrix}, \tag{28}$$

$$V = 2i\lambda_2^2 \begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix} + 2\lambda_2 \begin{pmatrix} i\beta t & Q \\ -Q^* & -i\beta t \end{pmatrix} + i \begin{pmatrix} |Q|^2 & Q_t + 2i\beta t Q \\ Q_t^* - 2i\beta t Q^* & -|Q|^2 \end{pmatrix}, \tag{29}$$

where

$$\lambda_2 = \lambda_2(0) \exp\left(-2\int \beta \mathrm{d}z\right) \tag{30}$$

and

$$Q(z,t) = q(z,t) \exp\left(\frac{i\beta t^2}{2}\right).$$

Here the integrability condition is

$$\beta_z = -2\left(\beta^2 + \alpha\right) \tag{31}$$

and proceeding as before, we get the solution of the eq. (27) as

$$q = 2\alpha_4 \exp\left[-2i\alpha_3 t - 4i\int \left(\alpha_3^2 - \alpha_4^2\right) \mathrm{d}z + \frac{i\beta t^2}{2}\right] \times \mathrm{sech}\left(2\alpha_4 t + 8\int \alpha_3\alpha_4 \mathrm{d}z + \eta_0\right) \tag{32}$$

with $\alpha_3(z) = \alpha_3(0)\,\exp(-2\int \beta \mathrm{d}z)$ and $\alpha_4(z) = \alpha_4(0)\,\exp(-2\int \beta \mathrm{d}z)$.

Here the appearance of the term α_3 in the amplitude shows that the soliton amplitude decays at the same rate as the respective power amplitudes. i.e; $\frac{\mathrm{d}W}{\mathrm{d}z} = -2W$ with $W = \int |q|^2\, \mathrm{d}\tau$. In this case also, the product of pulse width and pulse amplitude is found to be conserved due to which the area occupied by the pulse envelope remains preserved when loss and phase modulation are included. This is clearly depicted in figures 2a and b. The contour plot in figure 2c further strengthens our result.

4. Dark solitons

Optical dark solitons have been investigated in many theoretical and experimental papers. Recently there is an increased interest in dark spatial solitons because of their possible applications in optical logic devices [25] and waveguide optics as dynamic switches and junctions [26]. They are also considered for signal processing and communication applications because of their inherent stability [27]. In fact, the influence of noise and fibre loss on dark soliton is much lesser than that on bright solitons [28].

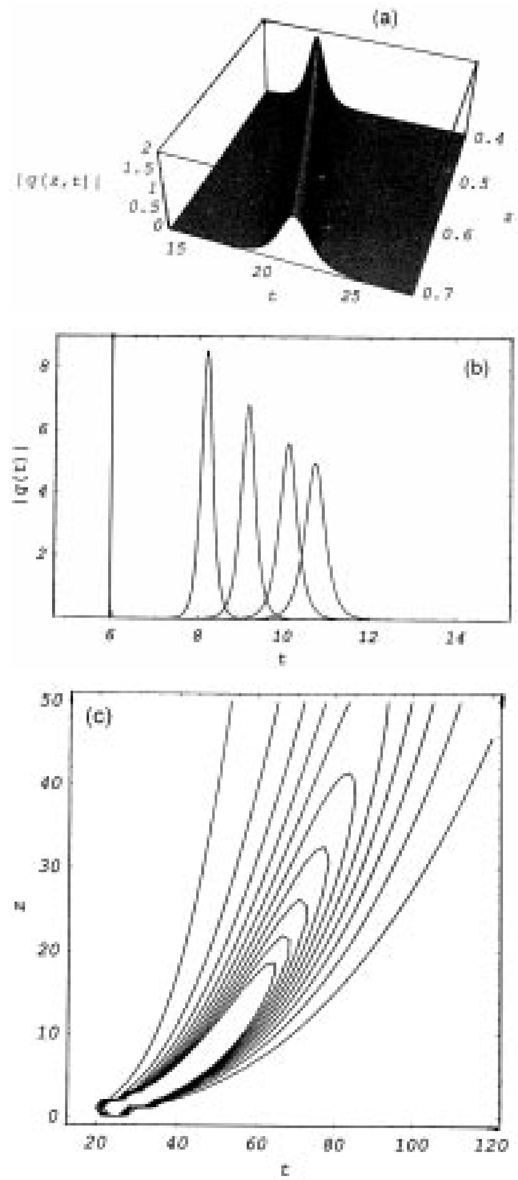

Figure 2. (**a**) Amplitude profile of $|q(z,t)|$ for the dark one-soliton solution of eq. (32) with the parameter values $\alpha_3(0) = 1$, $\alpha_4(0) = 0.5$ and $\beta(z) = \frac{1}{3z}$. (**b**) Variation of $|q(t)|$ for the one-soliton solution of eq. (32) with the parameter values $\alpha_3(0) = 1$, $\alpha_4(0) = 0.5$ and $\beta(z) = \frac{1}{3z}$. This figure shows the variation of $|q(t)|$ with t for different values of z ($z = 0.04, 0.056, 0.0753, 0.09$). Here amplitude decreases as z increases. (**c**) Contour plot of $|q(z,t)|$ with respect to z and t for the parameter values $\alpha_3(0) = 1$, $\alpha_4(0) = 0.5$ and $\beta(z) = \frac{1}{3z}$.

Pulse compression can also be achieved in the normal dispersive regime by reversing the GVD with the inclusion of amplification and phase modulation. Now proceeding to the case of the normal GVD regime, eq. (3) can be written as

$$iq_z - q_{tt} + 2|q|^2 q = \alpha(z)t^2 q + i\beta(z)q. \tag{33}$$

Before proceeding to arrive the integrability conditions from AKNS formalism, we introduce a variable transformation

$$q(z,t) = Q(z,t)\exp\left(\frac{i\beta t^2}{2}\right) \tag{34}$$

in eq. (33), resulting in

$$iQ_z - \frac{\beta_z t^2 Q}{2} - Q_{tt} + 2|Q|^2 Q - 2i\beta t Q_t - 2i\beta Q - \left(\alpha - \beta^2\right)t^2 Q = 0. \tag{35}$$

The linear eigenvalue problem associated with eq. (35) is

$$U = \begin{pmatrix} -\frac{i\lambda}{2} & -iQ \\ iQ^* & \frac{i\lambda}{2} \end{pmatrix}, \tag{36}$$

$$V = \lambda_1^2 \begin{pmatrix} \frac{i}{2} & 0 \\ 0 & -\frac{i}{2} \end{pmatrix} + \lambda_1 \begin{pmatrix} -i\beta t & iQ \\ -iQ^* & i\beta t \end{pmatrix} + \begin{pmatrix} i|Q|^2 & -Q_t - 2i\beta t Q \\ -Q_t^* + 2i\beta t Q^* & -i|Q|^2 \end{pmatrix}, \tag{37}$$

where

$$\lambda_1 = \lambda_1(0)\exp\left(2\int \beta \mathrm{d}z\right). \tag{38}$$

Now the relation $U_z - V_t + [U,V] = 0$, leads to the equation

$$iQ_z - Q_{tt} + 2|Q|^2 Q - 2i\beta t Q_t - 2i\beta Q = 0. \tag{39}$$

On comparing eqs (39) and (35), it is clear that eq. (33) admits complete integrability when the condition

$$\beta_z = -2\left(\alpha - \beta^2\right) \tag{40}$$

is satisfied. Thus the integrability condition of eq. (33) is derived by using Lax pair. On decoupling eq. (39), we get

$$\begin{aligned} \left[iD_z - D_t^2 - 2i\beta t D_t - 2i\beta + \chi\right](G.F) &= 0 \\ (D_t^2 - \mu)(F.F) &= -2|G|^2 \end{aligned} \tag{41}$$

in which χ is a function to be determined.

Next, in order to construct dark one-soliton solution of the eq. (39), we assume

$$G = g_0\left(1+\varepsilon g_1\right), \qquad F = 1+\varepsilon f_1. \tag{42}$$

Substituting eq. (42) in eq. (41) and collecting the coefficients of ε^0, we get

$$\begin{gathered}\left[iD_z - D_t^2 - 2i\beta t D_t - 2i\beta + \chi\right]\left(g_0.1\right) = 0 \\ \chi = 2\left|g_0\right|^2.\end{gathered} \tag{43}$$

To solve eq. (43), we assume

$$g_0 = 2\alpha_2(0)\exp\left[\int(2\beta + i\chi)\,\mathrm{dz}\right], \tag{44}$$

where $\alpha_2(0)$ is a constant. Then

$$\chi = 2\left|g_0\right|^2 = 8\alpha_2^2 = 8\left[\alpha_2(0)\right]^2\exp\left[4\int\beta\,\mathrm{dz}\right]. \tag{45}$$

Using eqs (41) and (42) and the usual Hirota identities [24], and then collecting the coefficents of ε and ε^2, we obtain

ε

$$\begin{gathered}\left[iD_z + D_t^2 - 2i\beta t D_t\right]\left(1.f_1 + g_1.1\right) = 0 \\ \left(D_t^2 - \mu\right)\left(1.f_1 + f_1.1\right) = -4g_1\left|g_0\right|^2,\end{gathered} \tag{46}$$

ε^2

$$\begin{gathered}\left[iD_z + D_t^2 - 2i\beta t D_t\right]\left(g_1.f_1\right) = 0 \\ \left(D_t^2 - \mu\right)\left(1.f_1 + f_1.1\right) = -2g_1\left|g_0\right|^2.\end{gathered} \tag{47}$$

For solving eqs (46) and (47), we assume

$$g_1 = -\exp\left(4\alpha_2 t + 2\eta_0\right), \qquad f_1 = -g_1, \tag{48}$$

where η_0 is a constant. Using eqs (48), (44), (42) and (14), after absorbing ε, dark one-soliton solution can be obtained as

$$Q = 2\alpha_2\exp\left[i\left(\int 8\alpha_2^2\mathrm{dz} \pm \pi\right)\right]\tanh\left(2\alpha_2 t + \eta_0\right). \tag{49}$$

Using eq. (34), we can construct 1SS of eq. (33) as

$$q = 2\alpha_2\exp\left[i\left(\int 8\alpha_2^2\mathrm{dz} + \frac{\beta t^2}{2} \pm \pi\right)\right]\tanh\left(2\alpha_2 t + \eta_0\right). \tag{50}$$

Thus, we obtain exact dark 1SS by following the Hirota technique. Here also we can observe that the product of the pulse width and pulse amplitude is found to be conserved due to which the area occupied by the pulse envelope remains preserved when gain and phase modulation are introduced. This is clearly depicted in figures 3a–c.

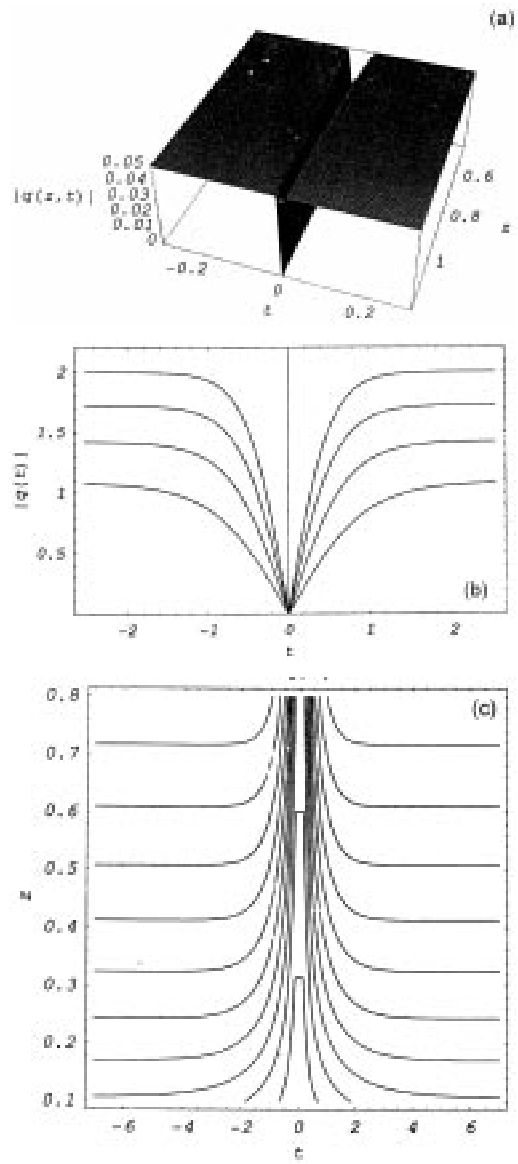

Figure 3. (**a**) Amplitude profile of $|q(z,t)|$ for the one-soliton solution of eq. (50) with the parameter values $\alpha_2(0) = 1$ and $\beta(z) = \frac{1}{3z}$. (**b**) Variation of $|q(t)|$ for the one-soliton solution of eq. (50) with the parameter values $\alpha_2(0) = 1$ and $\beta(z) = \frac{1}{3z}$. This figure shows the variation of $|q(t)|$ with t for different values of z ($z = 0.4, 0.6, 0.8, 1$). Figure shows that the amplitude increases as z increases. (**c**) Contour plot of $|q(z,t)|$ with respect to z and t for the parameter values $\alpha_2(0) = 1$ and $\beta(z) = \frac{1}{3z}$.

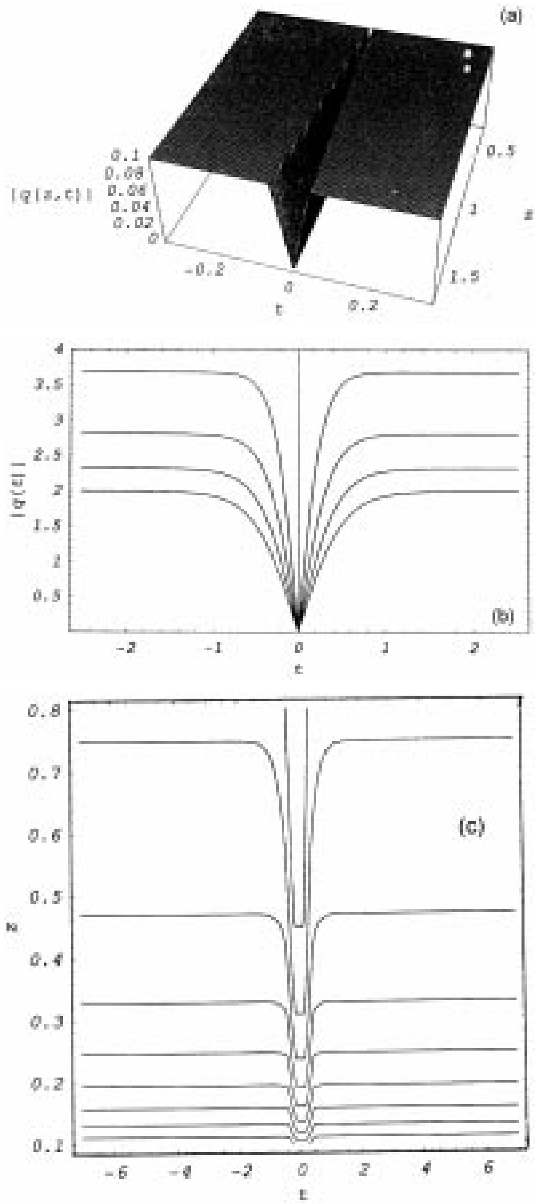

Figure 4. (**a**) Amplitude profile of $|q(z,t)|$ for the dark one-soliton solution of eq. (56) with the parameter values $\alpha_3(0) = 1$ and $\beta(z) = \frac{1}{3z}$. (**b**) Variation of $|q(t)|$ for the one-soliton solution of eq. (56) with the parameter values $\alpha_3(0) = 1$ and $\beta(z) = \frac{1}{3z}$. This figure shows the variation of $|q(t)|$ with t for different values of z ($z = 0.4, 0.6, 0.8, 1$). Figure shows that amplitude decreases as z increases. (**c**) Contour plot of $|q(z,t)|$ with respect to z and t for the parameter values $\alpha_3(0) = 1$ and $\beta(z) = \frac{1}{3z}$.

For phase modulation and damping, eq. (33) becomes

$$iq_z - q_{tt} + 2\left|q\right|^2 q = \alpha(z)t^2 q - i\beta(z)q. \tag{51}$$

The linear eigen value problem associated with eq. (51) is

$$U = \begin{pmatrix} \frac{-i\lambda_4}{2} & -iQ \\ iQ^* & \frac{i\lambda_4}{2} \end{pmatrix}, \tag{52}$$

$$\begin{aligned} V = \lambda_4^2 \begin{pmatrix} \frac{i}{2} & 0 \\ 0 & \frac{-i}{2} \end{pmatrix} + \lambda \begin{pmatrix} i\beta t & iQ \\ -iQ^* & -i\beta t \end{pmatrix} \\ + \begin{pmatrix} i\left|Q\right|^2 & -Q_t + 2i\beta t Q \\ -Q_t^* - 2i\beta t Q^* & -i\left|Q\right|^2 \end{pmatrix}, \end{aligned} \tag{53}$$

where

$$\lambda_2 = \lambda_2(0)\exp\left(-2\int \beta \mathrm{d}z\right) \tag{54}$$

and

$$Q(z,t) = q(z,t)\exp\left(\frac{-i\beta t^2}{2}\right).$$

Here the integrability condition is

$$\beta_z = 2\left(\alpha - \beta^2\right) \tag{55}$$

and solution of the eq. (51) is

$$q = 2\alpha_3 \exp\left[i\left(\int 8\alpha_3^2 \mathrm{d}z + \frac{\beta t^2}{2} \pm \pi\right)\right]\tanh\left(2\alpha_3 t + \eta_0\right) \tag{56}$$

with $\alpha_3(z) = \alpha_3(0)\,\exp(-2\int\beta \mathrm{d}z)$. Thus exact dark 1SS for NLS with damping and phase modulation is obtained. From the figures 4a–c we can observe that the product of the pulse width and pulse amplitude is found to be conserved due to which the area occupied by the pulse envelope remains preserved with the inclusion of damping and phase modulation and the contour plot strengthens our result. To our knowledge, explicit construction of bright and dark soliton solution of modified NLS equations with variable coefficients using Hirota's method have not been reported earlier.

5. Conclusion

Modified NLS equations in anomalous and normal dispersive regimes with phase modulation and gain (or loss) are considered. We have arrived at a general integrability condition from AKNS method for both cases. Finally we have constructed bright and dark soliton solutions. Solutions of eqs (3), (27), (33) and (51) explain how we can achieve pulse compression keeping soliton property when it is transmitted through the fibre with the addition of fibre gain (or loss) and phase modulation. The amplitude of the pulse is found to decrease in an exponential way for damping and increase in an exponential way for gain with the pulse width broadening at each stage of propagation such that area of the pulse envelope remains preserved. From the figures, it is very interesting to note that the soliton area is conserved and hence the soliton nature of the pulse is maintained and the pulse can be said to experience a new type of compression. If we are able to achieve this experimentally, then this method offers a new way of generating compressed pulses.

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