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Bistable soliton states and switching in doubly inhomogeneously doped fiber couplers

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Abstract. Switching between the bistable soliton states in a doubly and inhomogeneously doped fiber system is studied numerically. Both the cases of lossless as well as lossy couplers are considered. It is shown that both up-switching (from the low state to the high state) and down-switching (from the high state to the low state) of solitons between bistable states are realizable, if the amplification of the input soliton for up-switching and the extraction of energy from it for down-switching are suitably adjusted.

Keywords. Nonlinear optics; optical soliton; bistability; switching; directional coupler.

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1. Introduction

In nonlinear optics, bistable solitons were introduced by Kaplan [1] in 1985. Since then, there has been a considerable interest in bistable solitons in glass fibers (with non-Kerr properties), in connection with optical bistability and other possible applications leading to switching and logic-gate devices.

In literature one distinguishes between two kinds of bistable solitons: one for which the nonlinear propagation constant shift is a multivalued function of the soliton energy and the other for which the soliton pulsewidth is a double-valued function of the soliton energy. The former correspond to the original work of Kaplan, while the latter were obtained by Gatz and Herrmann [2] while studying the bright soliton solutions in fibers with saturating nonlinearity. Studies have shown that fibers, doped with one or two semiconductor or organic crystallites, do not support solitons of the first kind. They support the formation and propagation of solitons of the second type. However, it has been demostrated, theoretically, that Kaplan-type bistable solitons can be achieved in [3,4] triply doped fibers with appropriate properties.

Both types of solitons are useful for switching and logic-gate devices [5–9]. However Kaplan-type solitons are preferable since the two stable branches of solitons are separated by a branch with unstable solitons by a range of input pulse peak amplitude for which no soliton solution results. The above mentioned separation between the stable branches ideally suits bistable switching devices in which one likes to have a clear distinction between low and high states.

Unfortunately, bistable solitons have not yet been observed experimentally, except for the case of gaseous SF_6 in which one had an evidence of their existence [12]. The main difficulty for the experimental realisation of bistable solitons is the lack of materials with required nonlinear optical properties. If such materials, bulk or in waveguide/fiber cofiguration, with appropriate intensity dependent nonlinear optical properties for supporting bistable solitons, could be made available it would definitely be very useful for practical applications from the viewpoint of optical switching [10] with solitons and all-optical logic operations [13,14].

Motivated by these developments, recently we proposed a novel fiber system and showed that such a system did support bistable soliton propagation [10]. The main feature of the proposed doped silica fiber is that it is doubly and inhomogeneously doped with appropriate dopants. One of the dopants is uniformly distributed over the entire fiber core while the other is inhomogeneously distributed over a restricted cross-sectional area of the core. Like in Kaplan's model, in our model too, the curve (see figure 1) representing the soliton energy, P, as a function of the nonlinear propagation constant shift, β, has three distinct branches: one with $\frac{\partial P}{\partial \beta} > 0$ (for the lower values of β), the second with $\frac{\partial P}{\partial \beta} < 0$ (for the intermediate values of β), and the third again with $\frac{\partial P}{\partial \beta} > 0$ (for the higher values of β). Since the stability of solitons [1] requires $\frac{\partial P}{\partial \beta} > 0$, we call the first and the third branches as the lower positive-slope branch (LPSB) and the upper positive-slope branch (UPSB), respectively. The soliton states belonging to the LPSB are called low states while those belonging to the UPSB are called high states. These states are separated by the unstable soliton branch which is the second one.

In the given theoretical paper, we numerically study the switching between the bistable states of solitons of our model in a directional coupler configuration. The main objective of the paper is to demonstrate that both the up-switching, from the low state to the high state, and the down switching, from the high state to the low state, are possible, if the amplification of the input soliton (for up-switching) and the extraction of energy from it

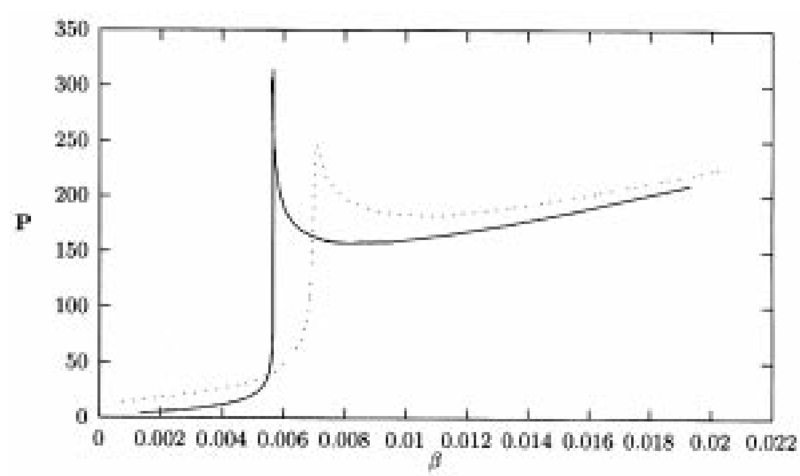

Figure 1. Plot of the soliton energy vs the nonlinear propagation constant shift β for $\alpha = 0.08, \delta = 5.0$ and $\sigma = 1.175$ (solid curve), $\sigma = 1.17$ (dotted curve).

(for down-switching) are suitably adjusted. In our numerical computation we use the well-known split-step fast Fourier transform method with 512 grid points. We have also studied the effect of fiber loss on the switching process.

2. Basic system of coupled equations with fiber loss

Consider a dual core isotropic directional coupler made of doubly inhomogeneously doped fibers. The coupler is assumed to have identical cores with circular cross-section. The system of coupled nonlinear partial differential equations, governing pulse dynamics in such a coupler is given by

$$i\frac{\partial u}{\partial \xi} + \frac{1}{2}\frac{\partial^2 u}{\partial \tau^2} + f\left(|u|^2\right)u = -\kappa v - i\Gamma u \tag{1}$$

$$i\frac{\partial v}{\partial \xi} + \frac{1}{2}\frac{\partial^2 v}{\partial \tau^2} + f\left(|v|^2\right)v = -\kappa u - i\Gamma v \tag{2}$$

where $u(\xi,\tau)$ and $v(\xi,\tau)$ are the normalized (see ref. [7]) complex envelope amplitudes of the pulses in core 1 and core 2 respectively, ξ is the normalized length along the coupler and τ is the delayed time in a reference frame that travels with the common group velocity. κ is the normalized coupling coefficient between the cores and is given by [15]

$$\kappa = \frac{c\,\kappa_l}{\omega\, n_2\, I_S} \tag{3}$$

where the linear coupling coefficient between the cores of the coupler $\kappa_l = \frac{\pi}{2L_c}$. Here L_c is the well-known half-beat length of the linear coupler [11]. Γ is the normalized loss coefficient. The dimensionless function $f(|u|^2)$ describes the intensity dependent refractive index and is given by

$$f(|u|^2) = 1 - \frac{\ln(1+|u|^2)}{|u|^2} - \alpha\delta\left[\sigma|u|^2 - |u|^2\ln(1+|u|^2) + |u|^2\ln(1+\mathrm{e}^{-\sigma}|u|^2)\right] \tag{4}$$

where, α, δ and σ have the same meaning as in ref. [10]. $f(|v|^2)$ has the same functional form as $f(|u|^2)$.

For a given set of values of α, δ and σ each of the uncoupled equations admits bright bistable soliton solutions of the type

$$q(\xi,\tau) = \sqrt{\psi(\tau)}\,\exp[i\beta\xi] \tag{5}$$

where, ψ satisfies the boundary conditions

$$\lim_{\tau\to\pm\infty}\psi(\tau) = \lim_{\tau\to\pm\infty}\frac{\mathrm{d}\psi(\tau)}{\mathrm{d}\tau} = 0 \tag{6}$$

and β has the meaning of the nonlinear propagation constant shift. The bistability manifests itself in the multivaluedness of β as a function of the soliton energy P. In figure 1 we have shown a typical plot of β versus P for two sets of parameters : $\alpha = 0.08$, $\delta = 5.0$ and $\sigma = 1.175$ (solid curve) and $\alpha = 0.08$, $\delta = 5.0$ and $\sigma = 1.170$ (dotted curve). The soliton solutions belonging to the LPSB and the UPSB of the curve $P(\beta)$ are stable while the solitons corresponding to the negative slope branch of $P(\beta)$ are unstable. Also, the solitons of LPSB are smaller and wider while those of the UPSB are taller and narrower and, hence, can be distinguished easily. In what follows we present the results of our study on switching between the bistable states of solitons for the second set of the parameters given above and corresponding to figure 1.

3. Numerical analysis and results

A. *Coupler without loss*

In this case the damping coefficient Γ is zero and the coupled set of differential equations is given by

$$i\frac{\partial u}{\partial \xi} + \frac{1}{2}\frac{\partial^2 u}{\partial \tau^2} + f\left(|u|^2\right) u = -\kappa v \tag{7}$$

$$i\frac{\partial v}{\partial \xi} + \frac{1}{2}\frac{\partial^2 v}{\partial \tau^2} + f\left(|v|^2\right) v = -\kappa u \tag{8}$$

In order to study the switching dynamics of solitons, we launch $u(\xi = 0, \tau) = Au_{\mathrm{sol}}$, where u_{sol} is a soliton solution of the uncoupled system ($\kappa = 0$) and A is a real positive number, into the input end of one of the cores, say core 1, and keep the other core empty, i.e., $v(\xi = 0, \tau) = 0$. Note that $A > 1$ represents an amplification of the soliton while $A < 1$ corresponds to the case when energy is extracted from the soliton [8]. The coupling coefficient κ was varied from $\kappa = 0.1$ (weak coupling) to $\kappa = 1$ (strong coupling). In all our runs we have used the exact (upto the numerical accuracy) numerical soliton solutions.

A.1 *Sliding up and down along the soliton states in the LPSB*: One can slide along the LPSB or UPSB by appropriately choosing the coupling length and the amplification factor, or energy extraction factor, A.

For achieving up-switching in the LPSB, we take an input low-state soliton of peak intensity $u_0 = 1.0$, corresponding to $\beta_{\mathrm{in}} = 6.297538 \times 10^{-3}$ and energy equal to $P_{\mathrm{in}} = 55.834690$, and launch it into the input core, say core 1, of the coupler. The second core is kept empty. The system of differential equations (7) and (8) is then solved by the split-step FFT method with $\kappa = 0.1$ (which corresponds to the weak coupling regime and $L_c = 5.0 \times \pi$) and the initial conditions $u(0, \tau) = Au_{\mathrm{sol}}$ and $v(0, \tau) = 0$. The numerical solution shows that for $A = 1.2$ and at $L = 0.0318 \times L_c$ the input soliton switches to a soliton, with $\beta_{\mathrm{out}} = 6.762296 \times 10^{-3}$ and energy $P_{\mathrm{out}} = 91.819080$, which also belongs to the LPSB. The results are depicted in figures 2a and 2b. Figure 2a shows the up-switching of the input soliton in the first core while figure 2b shows the evolution of the energy which leaks

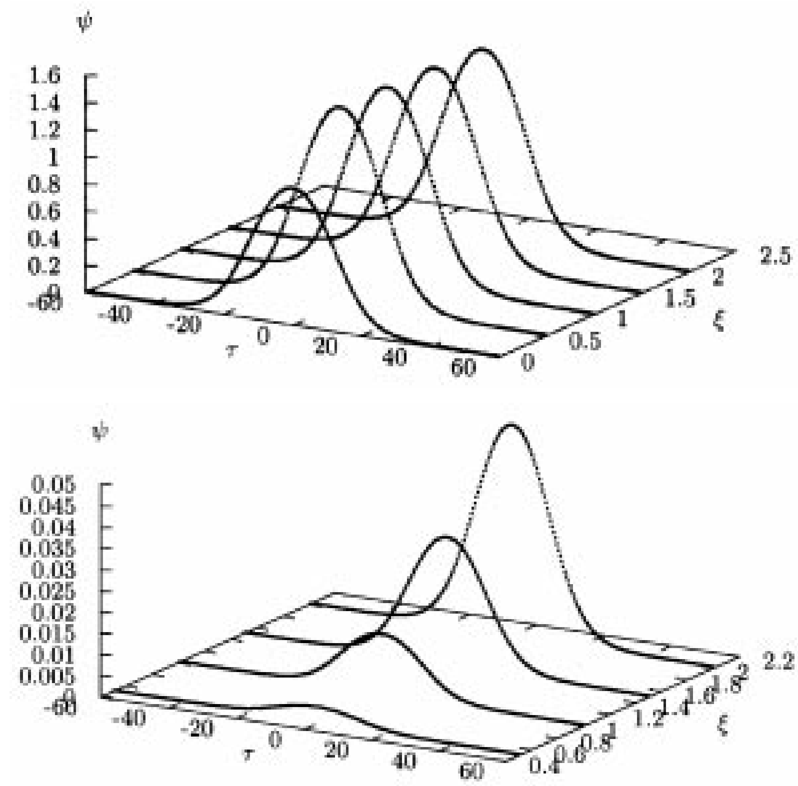

Figure 2. (**a**) Up-switching of the soliton with peak intensity $u_0 = 1.0$, $\beta = 6.297538 \times 10^{-3}$ and $P_{\text{in}} = 55.834690$, belonging to the LPSB, to a soliton of the LPSB with $\beta = 6.762296 \times 10^{-3}$ and $P_{\text{out}} = 91.819080$ for $\kappa = 0.1$, $A = 1.2$ and $L = 0.0318L_c$. (**b**) Plot of the localized radiation that leaks into the second core during switching depicted in figure 2a.

into the second core during the switching process depicted in figure 2a. The figures contain larger propagation distances than the switching length. It is because of the fact that after switching took place we further propagated the pulses to ensure that they propagate without any distortion.

Before proceeding further, we would like to note that for typical values of the parameters, $\lambda = 1.55\mu$m, $n_2^1 = 4.0 \times 10^{-12}$ cm^2/W and $I_s^1 = 200$ MW/cm^2 [16], where the superscript 1 stands for the first dopant (see ref. [10]), The coupler length $L = 0.318L_c$ comes out to be 1.54 mm for the above mentioned switching process.

Similar results are obtained for down-switching in the LPSB for which the results are depicted in figures 3a and 3b. As above, figure 3a shows the down-switching of the input soliton, at the coupler length $L = 0.032 \times L_c$, to a soliton with $\beta_{\text{out}} = 5.891349 \times 10^{-3}$ and energy $P_{\text{out}} = 44.803810$ which also belongs to the LPSB. This has been achieved with $A = 0.9$. Figure 3b shows the evolution of the energy which leaks into the second core.

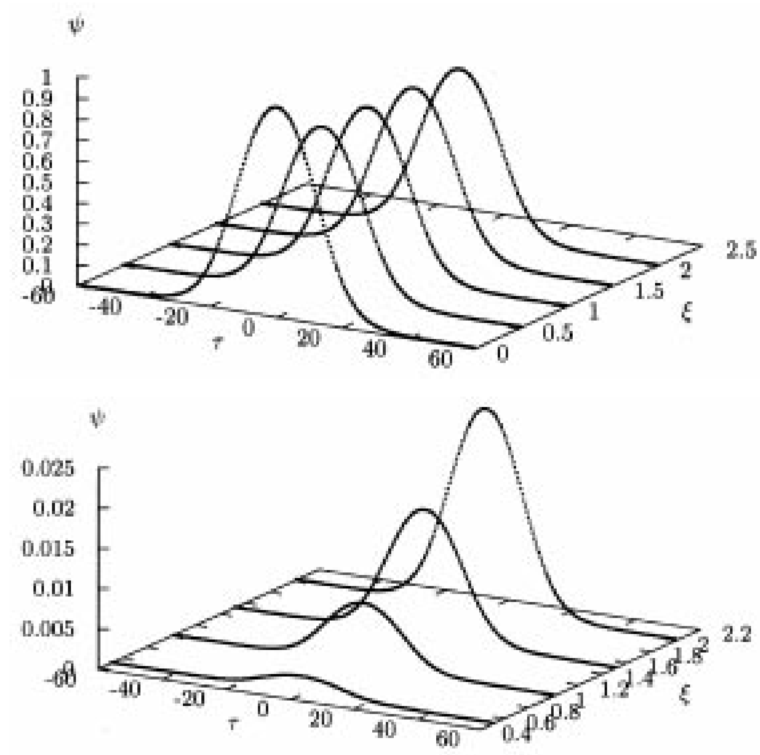

Figure 3. (**a**) Down-switching of the soliton with peak intensity $u_0 = 1.0$, $\beta = 6.297538 \times 10^{-3}$, and $P_{in} = 55.834690$, belonging to the LPSB, to a soliton of the LPSB with $\beta = 5.891349 \times 10^{-3}$ and $P_{out} = 44.803810$ for $\kappa = 0.1$, $A = 0.9$ and $L = 0.0318L_c$. (**b**) Plot of the localized radiation that leaks into the second core during switching depicted in figure 3a.

A.2 *Soliton up-switching from the LPSB to the UPSB*: Keeping the input low-state soliton, the coupler length L, and the coupling coefficient κ the same we take the amplification factor $A = 3.0$ and carry out the numerical solution of the coupled system of differential equations with the second core kept empty. The numerical solutions yield that the input soliton, switches to a soliton with peak intensity $u_0 = 8.98$ (corresponding to $\beta = 1.184462 \times 10^{-2}$ and energy $P_{out} = 186.367200$), belonging to the UPSB, The unused energy leaks into the second core. The results are depicted in figures 4a and 4b, respectively. The longer propagation distances contained in these figures are because of the same reason as explained in the previous case.

In order to see the effect of the coupling coefficient κ on the coupler length, we varied the former while keeping the amplification factor and the input soliton unchanged (i.e., $A = 3.0$). For $\kappa = 0.5$ and 1.0 (corresponding to the moderate coupling and the strong coupling regimes, respectively) up-switching, to the same high-state, was obtained for $L = 0.053 \times L_c$ and $L = 0.01326 \times L_c$ respectively. Thus we see that by increasing the coupling coefficient we can reduce the coupler length for up-switching from the same low state to the same final high state.

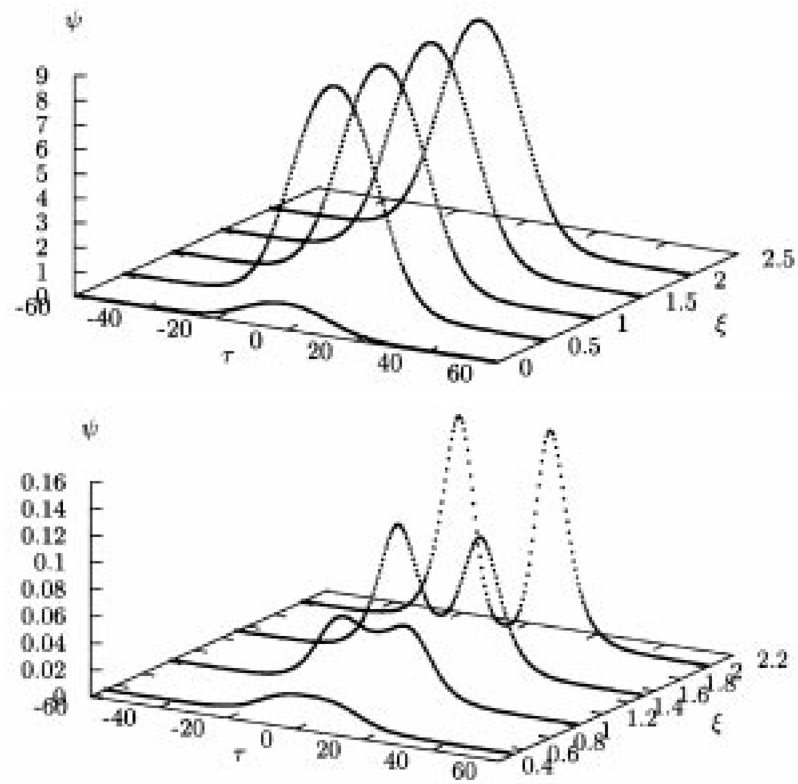

Figure 4. (**a**) Up-switching of the input soliton of the LPSB with peak intensity $u_0 = 1.0$ ($\beta = 6.297538 \times 10^{-3}$, and $P_{\mathrm{in}} = 55.834690$) to a soliton of peak intensity $u_0 = 8.98$ ($\beta = 1.184462 \times 10^{-2}$ and $P_{\mathrm{in}} = 186.367200$), belonging to the UPSB, for $\kappa = 0.1$, $A = 3.0$ and $L = 0.0318 L_c$. (**b**) Evolution of the radiation acquired by the second core during the down-switching depicted in figure 4a.

A.3 *Soliton down-switching from the UPSB to the LPSB*: Our calculations show that, in the moderate coupling regime as well as in the strong coupling regime, it is possible to achieve down-switching from a high state to a low state in one step. We take as an input a soliton with peak intensity $u_0 = 10.0$ ($\beta = 1.298524 \times 10^{-2}$ and energy $P_{\mathrm{in}} = 187.8732$). For $\kappa = 0.5$ we down-switched it to a low state soliton of peak intensity $u_0 = 1.15$ ($\beta = 6.512702 \times 10^{-3}$ and energy $P = 66.299780$) with $A = 0.5$ and $L = 0.6366 \times L_c$. The result is depicted in figure 5.

In the weak coupling regime it is not possible to get down-switching of solitons from the UPSB to the LPSB in one go. One needs at least two steps. In the first step one down-switches from a high state to another high state with a smaller peak intensity, i.e., slides down in the UPSB itself and then down-switches to a low state by further extracting energy from the intermediate high state. This result is the same as the one obtained by Enns *et al* [8]. As an example, we carried out calculations for an input high-state soliton with peak intensity $u_0 = 10.0$ ($\beta = 1.298524 \times 10^{-2}$ and energy $P_{\mathrm{in}} = 187.8732$) with

$\kappa = 0.1$. With $A = 0.9$ and $L = 0.032 \times L_c$, we down-switched it to another high-state soliton of peak intensity $u_0 = 6.384570$ ($\beta = 9.169573 \times 10^{-3}$ and energy $P = 186.92$). The results are shown in figure 6. Then, with $A = 0.5$ and $L = 0.32 \times \pi$, we down-switched the latter to a low-state soliton with peak intensity $u_0 = 1.592637$ ($\beta = 6.840451 \times 10^{-3}$ and energy $P_{out} = 109.7433$). The final step switching results are shown in figure 7. The longer propagation distances contained in these figures are because of the same reason as explained above.

B. *Coupler with loss*

In this case the switching dynamics is governed by the system of differential equations (1) and (2). For demonstrating the general features of the effect of fiber loss on soliton

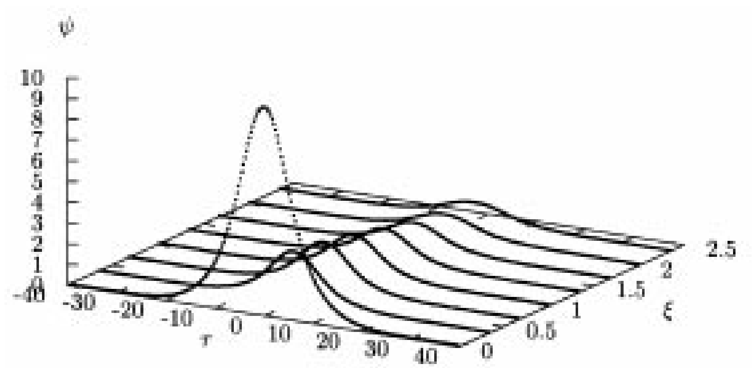

Figure 5. Down-switching of the soliton with peak intensity $u_0 = 10.0$, $\beta = 1.298524 \times 10^{-2}$, and $P_{in} = 187.8732$, belonging to the UPSB, to a soliton of the LPSB with $\beta = 6.512702 \times 10^{-3}$ and $P_{out} = 66.29978$ for $\kappa = 0.5$, $A = 0.5$ and $L = 0.6366 L_c$.

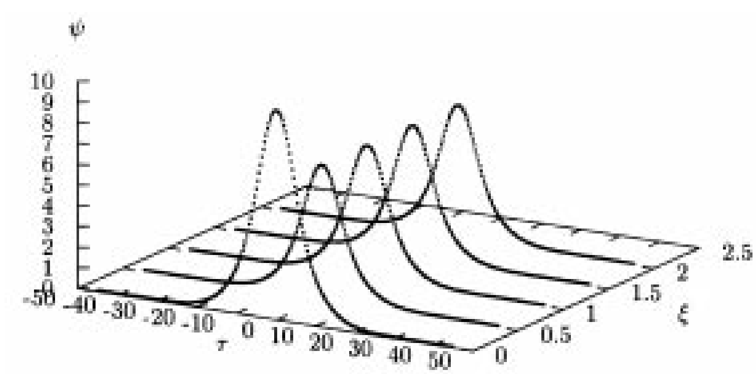

Figure 6. Intermediate down-switching of the soliton with peak intensity $u_0 = 10.0$, $\beta = 1.298524 \times 10^{-2}$, and $P_{in} = 187.8732$, belonging to the UPSB,to a soliton of peak intensity $u_0 = 6.384570$ ($\beta = 9.169573 \times 10^{-3}$) and $P_{out} = 186.9200$), also belonging to the UPSB.

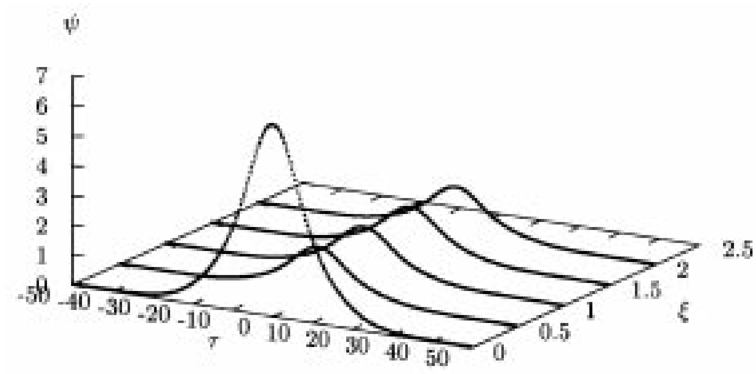

Figure 7. Final down-switching of the soliton with peak intensity $u_0 = 6.384570$ ($\beta = 9.169573 \times 10^{-3}$) and $P_{\text{in}} = 186.9200$), to a soliton of peak intensity $u_0 = 1.592637$ ($\beta = 6.840451 \times 10^{-3}$) and $P_{\text{out}} = 109.743300$), belonging to the LPSB.

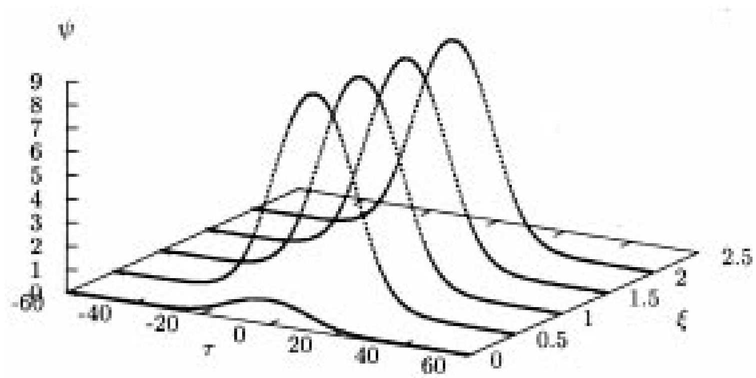

Figure 8. Up-switching of the soliton with peak intensity $u_0 = 1.0$, $\beta = 6.297538 \times 10^{-3}$ and $P_{\text{in}} = 55.834690$, belonging to the LPSB, to a soliton of the UPSB with $\beta = 1.170951 \times 10^{-2}$ and $P_{\text{out}} = 186.0941$ for $\kappa = 0.1$, $A = 1.2$ and $L = 0.0318L_c$ and $\Gamma = 0.0138$.

switching, we have selected two values of the dimensionless loss coefficient: $\Gamma = 0.0138$, corresponding to a low-loss fiber with a loss of 0.12 dB/Km, and $\Gamma = 0.138$, corresponding to a relatively high-loss fiber with a loss of 1.2 dB/Km.

Figure 8 shows the effect of loss on soliton up-switching from the LPSB to the UPSB considered in the subsection A.2 for $\Gamma = 0.0138$. The input condition, the coupling length and the weak coupling regime ($\kappa = 0.1$) were kept unchanged.The input soliton of peak intensity $u_0 = 1.0$, corresponding to $\beta_{\text{in}} = 6.297538 \times 10^{-3}$ and energy equal to $P_{\text{in}} = 55.834690$, switches to a high-state soliton with energy $P_{\text{out}} = 186.0941$ (corresponding to $\beta = 1.170951 \times 10^{-2}$) in place of a soliton with energy $P_{\text{out}} = 186.3672$ and $\beta = 1.184462 \times 10^{-2}$, to which it switches in the absence of loss. Thus, the output state energy is reduced by 0.15 per cent approximately. Hence, in the case of low-loss fiber coupler the switching process is negligibly affected by fiber loss.

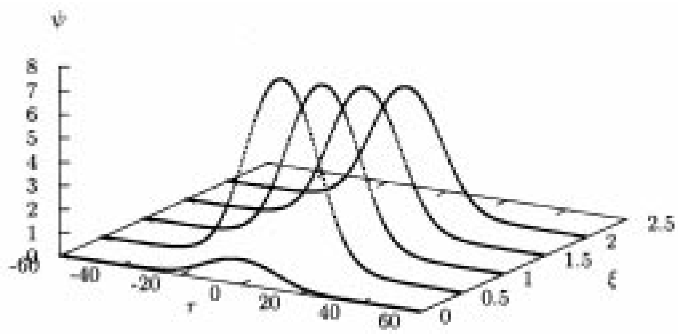

Figure 9. Up-switching of the soliton with peak intensity $u_0 = 1.0$, $\beta = 6.297538 \times 10^{-3}$ and $P_{\rm in} = 55.834690$, belonging to the LPSB, to a soliton of the UPSB with $\beta = 1.058832 \times 10^{-2}$ and $P_{\rm out} = 184.7462$ for $\kappa = 0.1$, $A = 1.2$ and $L = 0.0318L_c$ and $\Gamma = 0.138$.

Figure 9 shows the results for $\Gamma = 0.138$.The same input soliton of peak intensity $u_0 = 1.0$, switches to a high-state soliton with energy $P_{\rm out} = 184.7462$ (corresponding to $\beta = 1.058832 \times 10^{-2}$) in place of a soliton with energy $P_{\rm out} = 186.3672$ and $\beta = 1.184462 \times 10^{-2}$, to which it switches in the absence of loss. Thus, the output state energy is reduced by 0.87 per cent. Hence as the loss factor increases the same low state soliton switches to a high-state soliton of less energy compared to the previous case. The further propagation of the soliton is governed by the well known exponential decay law for the soliton peak intensity.

4. Conclusion

In the given work we have demonstrated that, with a nonlinear directional coupler made of doubly but inhomogeneously doped optical fibers, one can achieve up-switching as well as down-switching operations between the bistable states of the solitons. We have also analyzed the effect of fiber loss on the soliton switching dynamics and shown that the fiber loss does not pose any relevant problem except decreasing the energy of the output soliton slightly. The latter can be taken care of by amplification, as it is done for normal soliton propagation. We hope that our study will be useful for researchers working on the theoretical as well as practical aspects of bistable solitons.

Acknowledgements

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