PRAMANA
— journal of
physics

Vol. 57, No. 4
October 2001
pp. 755–761

Neutrino beam plasma instability

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MS received 10 April 2000; revised 7 March 2001

Abstract. We derive relativistic fluid set of equations for neutrinos and electrons from relativistic Vlasov equations with Fermi weak interaction force. Using these fluid equations, we obtain a dispersion relation describing neutrino beam plasma instability, which is little different from normal dispersion relation of streaming instability. It contains new, nonelectromagnetic, neutrino-plasma (or electroweak) stable and unstable modes also. The growth of the instability is weak for the highly relativistic neutrino flux, but becomes stronger for weakly relativistic neutrino flux in the case of parameters appropriate to the early universe and supernova explosions. However, this mode is dominant only for the beam velocity greater than $0.25c$ and in the other limit electroweak unstable mode takes over.

Keywords. Neutrino; beam plasma instability; neutrino plasma wave.

PACS Nos 13.15.+g; 52.35.-g; 52.35.Qz

Recently, there were a series of work [1–4] on neutrino flux passing through electron–positron or electron–proton plasma in the context of early universe, stars and supernova explosion. Neutrinos, produced in these astrophysical objects, carry lot of energy and escape from the system without much interactions. They can undergo only Fermi weak interaction. Still, it is argued that their collective interactions with background plasma, through weak interactions, can cause loss of energy of neutrino to plasma. One mechanism of loss of energy is via excitation of unstable modes in the plasma, which eventually damps and deposit energy in the plasma. Hence plasma gets heated. It is suggested in the literature that this process may lead to supernova explosions. Similarly, in the early universe and in stars flux of neutrinos streams through plasma and neutrino driven instabilities are important. In this letter, we examine the possibility of neutrino beam plasma instability which can also deposit the neutrino beam energy into the plasma. Here we derive the dispersion relation and conditions for the excitation of electron–neutrino streaming instability due to weak interactions.

We consider a plasma of electron and proton with neutrino beam streaming through it. It is a three species electroweak plasma. However, we neglect the proton dynamics because of its larger mass compared to electron and neutrino. Protons just serve as a constant background of positive charge in our problem. We first derive the fluid set of equations for electrons and neutrinos including the Fermi weak interaction term in addition to electromagnetic interactions starting from single particle relativistic dynamics. Then we perturb the system about the equilibrium, which consists of electrons and protons at rest and neutrinos streaming with velocity $\vec{V}_0$, and derive the dispersion relation and look for the condition for instability. We assume a homogeneous neutrino fluid streaming into a homogeneous electron fluid. This is a valid assumption as long as the scale length of inhomogeneity is large compared to that of perturbation.

It should be noted that our work is purely classical and earlier work in [1–4] were somewhat unconventional with mixture of classical and quantum theory, which is strongly questionable. They treat Klein–Gordon equation for neutrino as fluid equation which is not proper. Of course, in their later work on kinetic theory (KT) [5] of neutrino plasma interaction they followed purely classical approach without mentioning the error they made in their earlier work. Even though they obtained dispersion relation for streaming instability from KT, they have not made any detailed quantitative estimate of growth rates, which we do here using relativistic fluid theory.

Let us first consider the single particle relativistic dynamics of neutrino and electron in electroweak potential. Each neutrino feels a potential $V_N = G\sqrt{2}n_e$ due to the medium of electrons with density n_e. Subscript N in V stands for neutrino. Earlier this form of potential was used by Bethe [6] to explain the solar neutrino puzzle. It follows from electron–neutrino Fermi weak interaction term [7]

$$H_{\rm int} = \frac{G}{\sqrt{2}}\left[\bar{\nu}_e\gamma_\lambda(1+\gamma_5)\nu_e\right]\left[\bar{e}\gamma^\lambda(1+\gamma_5)e\right], \tag{1}$$

where G is the Fermi constant of weak interaction. ν_e and e are neutrino and electron fields. γ_μ and γ_5 are Dirac matrices. The interaction Hamiltonian $H_{\rm int}$ also suggests that electron will also experience a force due to weak interaction potential energy $V_e = G\sqrt{2}n_N$, in addition to usual electromagnetic interactions, due to the medium of neutrinos with density n_N. This term gives the ponderomotive force term due to neutrino field discussed in ref. [1]. Note that, since we are developing a theory (here it is fluid theory) to describe a medium of neutrinos or electrons from a single particle dynamics, we consider the potential, each kind of species feels, due to the medium of other species. Medium effects due to same species is contained in the theory describing the medium. Thus the total relativistic energy of neutrino is given by

$$(E_N - V_N)^2 = \vec{p}_N^2 + m_N^2 \tag{2}$$

and that of electron is

$$(E_e - V_e + e\phi)^2 = (\vec{p}_e + e\vec{A})^2 + m_e^2, \tag{3}$$

where ϕ and $\vec{A}$ are electromagnetic scalar and vector potentials. Here we are interested in only electrostatic mode and hence $\vec{A} = 0$. Thus the Hamiltonian describing the motion of neutrino through plasma is given by

$$H_N = \sqrt{\vec{p}_N^2 + m_N^2} + V_N, \tag{4}$$

and that of electron is

$$H_e = \sqrt{\vec{p}_e^2 + m_e^2} + V_e - e\phi. \tag{5}$$

Equations of motion of neutrino, which follows from Hamilton equations of motion, is

$$\dot{\vec{p}}_N = -G\sqrt{2}\nabla n_e. \tag{6}$$

Similarly, equations of motion for electron is

$$\dot{\vec{p}}_e = -G\sqrt{2}\nabla n_N + e\nabla\phi. \tag{7}$$

Let us now derive relativistic fluid equations without collision starting from Vlasov equation. The relativistic Vlasov equation is [8]

$$\frac{p^\mu}{m}\frac{\partial f}{\partial x^\mu} + \frac{\mathrm{d}p^\mu}{\mathrm{d}\tau}\frac{\partial f}{\partial p^\mu} = 0, \tag{8}$$

where $p^\mu \equiv (p^0, \vec{p})$, the 4-momentum and τ is the proper time. Proper time τ and time t are related by the relation $\mathrm{d}\tau = \mathrm{d}t/\gamma$, where $\gamma = 1/\sqrt{1-\vec{v}^2}$. $\vec{v}$ is the particle velocity. Using this relation, it is easy to see from the equation of motion for neutrino, eq. (6), that

$$\frac{\mathrm{d}\vec{p}_N}{\mathrm{d}\tau} = -G\sqrt{2}\gamma\nabla n_e \ \text{ and } \ \frac{\mathrm{d}p_N^0}{\mathrm{d}\tau} = 0. \tag{9}$$

Hence the Vlasov equation for neutrino becomes

$$p_N^0\frac{\partial f}{\partial t} + \vec{p}_N\cdot\nabla f - G\sqrt{2}\,p_N^0\nabla n_e\cdot\nabla_p f = 0. \tag{10}$$

Now taking different moments of the above equation with respect to momentum (p^μ) we get the continuity equation

$$\partial_\mu(P_N^\mu n_N) = 0, \tag{11}$$

from the zeroth moment and the equation of motion for fluid element

$$n_N P_N^\mu \partial_\mu P_N^\sigma = -G\sqrt{2}\,n_N P_N^0 \delta_{\sigma i}\partial_i n_e - \partial_\mu \Pi^{\mu\sigma}\,, \tag{12}$$

from the first moment, using the well-known procedure given in the text books [8]. P_N^μ is the fluid 4-momentum and $\Pi^{\mu\sigma}$ is the pressure tensor which we will replace by usual pressure gradient term $-\nabla p \approx -\gamma kT\nabla n_N \propto -S_N^2\nabla n_N$, appropriate to isotropic medium. S_N is the speed of sound in neutrino fluid. $\delta_{\sigma i}$ is a Kronecker delta. Similar equation exists for electron also with additional electric potential term.

Linearizing the equation of motion for neutrino fluid, eq. (12), about the equilibrium flux of neutrino with energy E_{N0} and momentum $\vec{P}_{N0}$ we get, $E_N' = 0$ and

$$\left(\omega - \frac{\vec{k}\cdot\vec{P}_{N0}}{E_{N0}}\right)(\vec{k}\cdot\vec{P}_N') = G\sqrt{2}\vec{k}^2 n_e' + S_N^2 k^2 n_N', \tag{13}$$

where prime refers to perturbed quantities and '0' here refers to the unperturbed quantities. ω and $\vec{k}$ are frequency and wave vector of the perturbation respectively. Similarly, continuity equation for neutrino gives

$$\vec{k}\cdot\vec{P}_N' = \left(\omega - \frac{\vec{k}\cdot\vec{P}_{N0}}{E_{N0}}\right)\frac{n_N'}{n_{N0}}E_{N0}. \tag{14}$$

Combining the above two equations we get

$$\left(\omega - \frac{\vec{k}\cdot\vec{P}_{N0}}{E_{N0}}\right)^2 n'_N - S_N^2 k^2 n'_N = \frac{G\sqrt{2}\vec{k}^2 n_{N0}}{E_{N0}} n'_e. \tag{15}$$

For simplicity, let us take the cold plasma limit by neglecting the term involving S_N. It is valid as long as the beam speed is greater than sound speed which we can see by comparing terms.

Similar analysis for electrons, which is assumed to be at rest at equilibrium, gives the set of fluid equations

$$\partial_\mu (P_e^\mu n_e) = 0, \tag{16}$$

and

$$n_e P_e^\mu \partial_\mu P_e^\sigma = -G\sqrt{2}\, n_e P_e^0\, \delta_{\sigma i}\, \partial_i n_N + e\, n_e P_e^0\, \partial_i \phi\, \delta_{\sigma i}, \tag{17}$$

gives

$$\vec{k}\cdot\vec{P}'_e = \omega \frac{n'_e}{n_{e0}} E_{e0} \tag{18}$$

and

$$E'_e = 0; \quad \omega\vec{k}\cdot\vec{P}'_e = G\sqrt{2}\vec{k}^2 n'_N - e\vec{k}^2\phi. \tag{19}$$

The potential generated by charge perturbation is given by Poisson equation

$$-\vec{k}^2\phi = \frac{e}{\varepsilon_0} n'_e, \tag{20}$$

where we assumed a uniform positive background of protons at rest. Combining all the above equations related to electrons we get

$$\left(\omega^2 - \frac{e^2 n_{e0}}{\varepsilon_0 E_{e0}}\right)\frac{E_{e0}}{n_{e0}} n'_e = G\sqrt{2}\vec{k}^2 n'_N. \tag{21}$$

Finally substituting for n'_N from eq. (15) we get the dispersion relation

$$(\omega^2 - \omega_p^2)(\omega - \vec{k}\cdot\vec{V}_0)^2 = \kappa^2 \vec{k}^4, \tag{22}$$

where $\omega_p^2 \equiv \frac{e^2 n_{e0}}{\varepsilon_0 E_{e0}}$, plasma frequency, $\vec{V}_0 \equiv \frac{\vec{P}_{N0}}{E_{N0}}$ and $\kappa^2 = \frac{2G^2 n_{N0} n_{e0}}{E_{N0} E_{e0}}$. Normalizing ω and k by ω_p, the dispersion relation may be expressed as

$$1 = \frac{1}{\omega^2} + \frac{\kappa^2 k^4}{\omega^2 (\omega - kV_d)^2}\ , \tag{23}$$

where $V_d \equiv V_0 \cos\theta$, with θ the angle between $\vec{V}_0$ and $\vec{k}$. Note that it differs from the standard dispersion relation of beam plasma instability by an extra ω^2 in the third term, which gives rise to new effects.

Let us now consider different limits of the dispersion relation eq. (23). For $\kappa^2 = 0$ we get back the usual plasma oscillations due to electric field interactions and no weak interactions. For $\kappa \neq 0$, it may be solved to obtain the roots of ω. Imaginary roots of ω give the growth rate of the instability. The condition for the instability may be obtained using the familiar method given in text books. The exact expression is complicated, however, we can look at various limits of eq. (23). For $\omega \gg 1$, we get

$$\omega = \frac{kV_d}{2} \pm \frac{1}{2}\sqrt{k^2V_d^2 \pm 4\kappa k^2}, \tag{24}$$

which has 4 roots. Two of them are complex for $V_d < 2\sqrt{\kappa}$. One of the complex roots represents the growth of the instability and hence the condition for the instability is $V_d < 2\sqrt{\kappa}$. In astrophysical problems, generally, $n_{e0} \approx n_{N0} \approx 10^{38}/\mathrm{cm}^3$ and E_{e0} and E_{N0} are of the order of MeV. Putting these values along with $G = 1.2 \times 10^{-5}\ \mathrm{GeV}^{-2}$ we get the value of $\kappa \approx 10^{-4}$. For this value of κ and $\omega \gg 1$ we get the growth rate of instability, from eq. (24),

$$\omega_i \approx \sqrt{\kappa}k\sqrt{1 - \frac{V_d^2}{4\kappa}}. \tag{25}$$

Other real roots in this limit are

$$\omega \approx kV_d + \kappa\frac{k}{V_d} \ \text{ and } \ \omega \approx \frac{\kappa k}{V_d}\ . \tag{26}$$

These are new nonelectromagnetic neutrino plasma waves due to weak interactions and modified by the neutrino beam. It is easy to see that for $\omega \ll 1$ there is always an unstable mode. Of course, one more limit discussed in text books, is to get the expression for the maximum growth rate, $\omega_i \approx (\kappa^2k^4)^{1/3}$, for $\omega \approx kV_d \approx 1$. This is what we see in the plot of growth rate given in figure 1. It is purely due to beam mode resonating with plasma mode. However, in the other limits, discussed earlier, the instability exists even when $V_d = 0$, which is not the case in usual beam plasma instability. These are new electroweak unstable mode, excited by electroweak free energy and modified by neutrino beam. In fact, for $V_d = 0$, the dispersion relation, eq. (23), can be easily solved to get

$$\omega^2 = \frac{1}{2} \pm \frac{1}{2}\sqrt{1 + 4\kappa^2k^4}, \tag{27}$$

which has 4 roots consisting of stable and unstable modes. It can be seen from the Hamiltonian, eq. (4), streaming instability is due to free energy in kinetic energy term and electroweak energy term gives electroweak unstable modes as well as stable modes. The new electroweak unstable mode may be explained as follows. From the Hamiltonian of neutrino and electron, we can see that any neutrino density fluctuation causes electron to have higher potential energy in the region where neutrino density is higher. Hence electrons flow to lower neutrino density region and increasing its density there. This increase in electron density causes neutrino potential energy to be large. So neutrino flows away from there, means towards higher neutrino density region. The density fluctuation grows and hence the instability.

We have also solved exactly the dispersion relation, eq. (23) for $\kappa = 10^{-4}$ and plotted in figure 1. The maximum growth rates of the instability are 0.0015 and 0.0322 in units

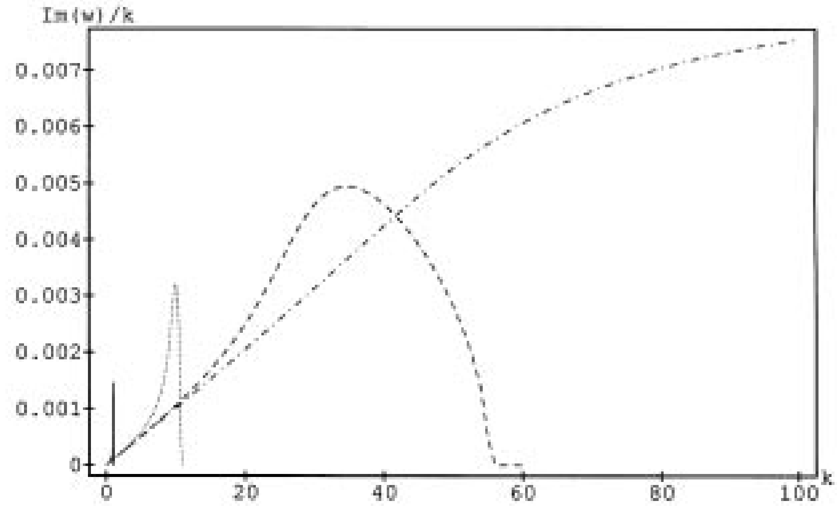

Figure 1. Plots of ω_i/k as a function of k for $\kappa = 0.0001$ and $V_d = 1.0$ (continuous curve), 0.1 (dotted curve), 0.03 (dash-dashed curve) and 0.01 (dash-dotted curve).

of ω_p for $V_d = 1$ and 0.1 in units of the velocity of light (c) respectively. For $V_d < 0.02c$, the growth rate is proportional to k for large k. The growth rate is weak for the mode propagating in the direction of the flow with the velocity of light and becomes stronger for $V_d < 10^{-2}c$. The dependence of the growth rate (ω_i) on the neutrino beam velocity V_d is shown in figure 1. For $V_d > 0.25c$ only long wavelength modes are unstable and the growth rate is maximum for $kV_d \approx 1$. This is mainly due to streaming instability. Whereas in the other limits, the growth rate increases with k and finally it becomes proportional to k. This is due to electroweak unstable mode. Both of the above observations are consistent with our earlier discussions of dispersion relation at various limits.

In conclusion, we have derived relativistic fluid equations for neutrino and electron fluids from relativistic Vlasov equations with electric and weak interactions. We found that for typical value of various parameters of electroweak plasma of early universe and supernova, the neutrino beam electron instability may be weak for the electrostatic-weak mode propagating in the direction of the flow of neutrino with the speed of light, but becomes stronger for weakly relativistic and $V_d > 0.25c$. For $V_d < 0.25c$, electroweak unstable mode dominate over streaming instability. We also find that there exists new nonelectromagnetic waves, neutrino plasma wave, in this electroweak plasma.

Acknowledgements

The author is thankful to the organizing committee of Plasma 99 where this article was presented. The author would like to thank Institute for Plasma Research for the facilities provided to initiate this work there.

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