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A simple coordinate space approach to three-body problems – Examples: Halo nucleus and double-λ hypernucleus

S MAHAPATRA, J NAG, D P SURAL and S N MUKHERJEE*
Department of Physics, Jadavpur University, Kolkata 700 032, India
*Banaras Hindu University, Varanasi 221 005, India

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Abstract. We show how to treat the dynamics of an asymmetric three-body system consisting of one heavy and two identical light particles in a simple coordinate space variational approach. The method is constructive and gives an efficient way of resolving a three-body system to an effective two-body system. It is illustrated by explaining the structural properties of some nuclei of current interest, namely halo nuclei and double-λ hypernuclei. The ansatz used here may be of value in a number of three particle problems of similar nature.

Keywords. Three-body model; halo nuclei; double λ hypernuclei; scaling parameter; folding potential.

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1. Introduction

In the present work we made use of a simple coordinate transformation that reduces a three-body problem to an effective two-body equation including a long range attractive potential generated by the coordinate transformation. The method has been implemented in a computer program designed to solve variationally a three-body system consisting of a heavy core and two equal mass light particles – for example, Borromean halo nuclei and double-λ hypernuclei. Our method is tested against the structural properties of these nuclei and a good agreement is found between theory and experiment. Some preliminary studies based on the present method have been reported earlier [1].

2. Theory

To simplify the model we assume that the three-body system representing the core and two extra core particles is composed of three spinless particles. The particles are labelled as 1(core), 2, 3(neutrons or lambdas). The mass of the core is M_1 and the mass of each equal mass extra core particle is M_2. r_1, r_2 and r_3 are the distances between the particle pairs (1-2), (1-3) and (2-3) respectively.

We first remove the centre of mass motion from the Hamiltonian. Then the variational principle for the Schrödinger equation for the internal motion of the three-body system is given by

$$\delta\langle\psi|H-E|\psi\rangle = 0, \tag{2.1}$$

where H, E, ψ refer to the Hamiltonian, energy and wave function for the internal motion. At this stage we make some simplifications. We start with a ψ that depends on r_1, r_2 and r_3. Then from eq. (2.1) we obtain after some calculations

$$\begin{aligned} 0 = 8\pi^2 \int_0^\infty \mathrm{d}r_1 \int_0^\infty \mathrm{d}r_3 \int_{|r_1-r_3|}^{r_1+r_3} \mathrm{d}r_2 r_1 r_2 r_3 \\ \times \left\{ \frac{\hbar^2(M_1+M_2)}{2M_1M_2} \left[\sum_{i=1}^{3} \left(\frac{\partial\psi}{\partial r_i} \right)^2 + \lambda(1,2,3) + \lambda(2,3,1) + \lambda(3,1,2) \right] \right. \\ + \frac{\hbar^2(M_1-M_2)}{2M_1M_2} \left[\left(\frac{\partial\psi}{\partial r_3} \right)^2 - \lambda(1,2,3) + \lambda(2,3,1) + \lambda(3,1,2) \right] \\ \left. + [U(r_1)+U(r_2)+V(r_3)]\psi^2 - E\psi^2 \right\}, \end{aligned} \tag{2.2}$$

where $\lambda(i,j,k) \equiv \{(r_i^2 + r_j^2 - r_k^2)/2r_ir_j\}(\partial\psi/\partial r_i)\ (\partial\psi/\partial r_j)$ and $U(r_1), U(r_2), V(r_3)$ are the two-body interactions.

Following Feshbach and Rubinow [2] and Ren [3], we assume that the wave functions for the three-body system are described simply by

$$\psi = \Phi(R). \tag{2.3}$$

The new space coordinate defined by

$$R(\eta) \equiv \frac{r_1 + r_2 + \eta r_3}{2},$$

contains a scaling parameter η. η controls the way the wave functions depend on r_1, r_2 and r_3. In addition to R we define two new coordinate variables R_2, R_3:

$$R_2 = r_2 \quad \text{and} \quad R_3 = \frac{1}{2}(1+\eta)r_3. \tag{2.4}$$

The assumption that the wave function depends on a single coordinate space variable R, allows the integrations over R_2 and R_3 in (2.2) and this simplifies the eigenvalue problem initially contained in (2.1). After R_2 and R_3 integrations have been performed in (2.2), eq. (2.2) takes the form

$$\delta \int_0^\infty \mathrm{d}R R^5 \left\{ \left(\frac{\mathrm{d}\phi}{\mathrm{d}R} \right)^2 D + \left[V_{\mathrm{eff}}(R) - \frac{(\eta^2+5\eta+8)}{15(\eta+1)^3} E \right] \phi^2 \right\} = 0, \tag{2.5}$$

where

$$D \equiv \frac{1}{60(1+\eta)^3}\{[2\mu+(\mu+\mu')\eta^2](\eta^2+5\eta+8)+\eta(\eta+5)(3\mu+\mu')\}$$
$$\equiv \frac{D'}{60(1+\eta)^3},$$

where $D' = \{[2\mu+(\mu+\mu')\eta^2](\eta^2+5\eta+8)+\eta(\eta+5)(3\mu+\mu')\}$,

$$\mu = \frac{\hbar^2(M_1+M_2)}{2M_1M_2} \quad \text{and} \quad \mu' = \frac{\hbar^2(M_1-M_2)}{2M_1M_2}.$$

The $V_{\text{eff}}(R)$ appearing in (2.5) is a long range attractive potential which emerges through the R_2 and R_3 integrations of the actual potentials $U(r_1), U(r_2)$ and $V(r_3)$. It is defined as follows:

$$V_{\text{eff}}(R) = \frac{1}{R^5}\int_0^R \mathrm{d}R_3 \int_{R-R_3}^{R-\beta R_3} \mathrm{d}R_2 (2R - R_2 - \nu R_3) R_2 R_3$$
$$\times \{U(r_1)+U(r_2)+V(r_3)\}, \tag{2.6}$$

where $\beta = (\eta-1)/(\eta+1)$ and $\nu = 2\eta/(\eta+1)$.

Applying the variation to $\phi(R)$ in (2.5), we finally obtain an eigenvalue equation which is

$$\frac{\mathrm{d}^2F(R)}{\mathrm{d}R^2} + \left[\frac{4(\eta^2+5\eta+8)}{D'}E - \frac{V_{\text{eff}}(R)}{D} - \frac{\frac{3}{2}\left(\frac{3}{2}+1\right)}{R^2}\right]F(R) = 0, \tag{2.7}$$

where $F(R) = R^{5/2}\Phi(R)$.

Thus the three-body problem reduces finally to an 'effective two-body equation' (2.7) when a wave function $F(R)$ is used. Energy E and the wave function $\phi(R)$ depend on the scale parameter η which may be adjusted for the best results.

3. Applications

A. *Halo nuclei*

The above formulation allows us to establish a framework to study three-body problems within the variational method. First we will apply this method to study structural properties of neutron rich nuclei ($N/Z \sim 2$–2.5) near the neutron drip line such as $^{11}_{3}$Li, which is a well-studied neutron halo nucleus that contains two loosely bound valence neutrons and shows anomalously large size not governed by the $r_0A^{1/3}$ rule. In literature such nuclei are called Borromean halo nuclei because they are bound with only one bound state but have no bound state in the binary subsystems in a three-body model. Some recent reviews on neutron halo nuclei by its discoverer, Tanihata [4] and also by Hansen *et al* [5] describe how these nuclei opened up a rich new vein of nuclear physics dealing with nuclear interaction in a low density asymmetric nucleus [6].

Before we describe the method of solving eq. (2.7), we first specify the two-body potentials used in our calculations. The core-*n* potentials used are:

(i) Woods–Saxon potential:

$$U(r_i) = \frac{V_0^n}{1+\exp\{(r_i-b)/a\}}, \quad i = 1,2,\ldots, \tag{3.1}$$

where $b = r_0 A^{1/3}$, $r_0 = 1.27$ fm, $a = 0.67$ fm and V_0^n is the depth of the potential.

(ii) One-term Gaussian [8–10]:

$$U(r_i) = V_0 \exp\{-(r_i/b)^2\}, \quad i = 1,\ 2, \tag{3.2}$$

where $b = 2.4$ fm.

(iii) Two-term Gaussian consisting of both attractive and repulsive terms: The repulsive part is used with a view to include the effect of the Pauli principle between the core- neutron and the extra-core neutron [11]

$$U(r_i) = V_r \exp\{-(r_i/a)^2\} - V_a \exp\{-(r_i/b)^2\}, \quad a = 1.2 \text{ fm},\ b = 1.9 \text{ fm}. \tag{3.3}$$

We have neglected the spin–orbit force because the recent study by Von Eiff [7] suggested a weaker spin–orbit interaction and less pronounced shell effects in neutron rich nuclei.

In the present calculation V_0^n in (3.1), V_0 in (3.2) and V_r, V_a in (3.3) are fixed such that the core $+n$ system (^{10}Li) is just unbound or nearly so and the corresponding scattering length is large and negative.

Scattering length is calculated from the asymptotic solution of two-body (core and neutron) Schrödinger equation with the core-n potential. Runge-Kutta method is used for numerical solution. We get for the Woods–Saxon potential $V_0^n = -7.0$ MeV and scattering length $= -110.82$ fm. For the one-term Gaussian potential $V_0 = -10$ MeV and scattering length $= -37.07$ fm and for the two-term Gaussian $V_r = 25.0$ MeV, $V_a = 24.7$ MeV and scattering length $= -37.08$ fm. Two versions of n-n potentials have been used:

(i) A square well radial form with depth

$$V_0 = -13.4 \text{ MeV for } r_3 < r_0, \quad \text{where } r_0 = 2.65 \text{ fm}. \tag{3.4}$$

(ii) One term Gaussian potential [13]

$$V_{nn}(r_3) = V_0 \exp\{-(r_3/a_0)^2\}, \quad V_0 = -31 \text{ MeV and } a_0 = 1.8 \text{ fm}.$$

The differential equation (2.7) is solved by the modified Numerov method and the parameter η is varied to get best fit for two neutron separation energy B_{2n} ($-E$ theory). In addition to B_{2n} we calculate the r.m.s. value of n-n separation,

$$\langle r_{nn}\rangle = (\bar{r}_3^2)^{1/2} = \left[\frac{8(\eta^2+7\eta+16)}{7(1+\eta)^2(\eta^2+5\eta+8)}\bar{R}^2\right]^{1/2}, \tag{3.5}$$

where

$$\bar{R}^2 = \int_0^\infty R^2 F^2(R)\mathrm{d}R \Big/ \int_0^\infty \mathrm{d}R F^2(R) \tag{3.6}$$

and the r.m.s. value of neutron core separation

$$\langle r_{\rm nc}\rangle = (\bar{r}_1^2)^{1/2} = \left[\frac{2\eta^4+14\eta^3+42\eta^2+70\eta+64}{7(1+\eta)^2(\eta^2+5\eta+8)}\bar{R}\right]^{1/2}. \tag{3.7}$$

A quantity of interest is the matter radius $r_{\rm m}$ of a halo nucleus of mass number A. $r_{\rm m}$ is calculated using the formula [12]

$$r_{\rm m}^2 = \frac{A-2}{A}\left\{r_{\rm c}^2 + \frac{2}{A}r_{\rm v}^2\right\}, \tag{3.8}$$

where $r_{\rm c}$ is the core radius of the mass number $A-2$ nucleus and $r_{\rm v} \equiv \langle r_{\rm nc}\rangle$ is the valence r.m.s. radius.

In the present calculation, we keep the depth of core-*n* potential at some prescribed value as discussed earlier and η is changed till the minimum of $E(\eta)$ is obtained. The minimum $E(\eta)$ occurs at $\eta = 0.7$ for the three core-*n* potentials used. Employing numerically obtained wave function, $\bar{R}^2$ is calculated according to (3.6) and finally we obtain $\langle r_{nn}\rangle$ and $\langle r_{\rm nc}\rangle$. In table 1, we compare our results with experiment. We see that eigenenergy $E_{\rm min}$ obtained in our simple model agrees reasonably well with $-B_{2n}$ where B_{2n} is the experimental two neutron separation energy. It appears that the halo phenomenon is a delicately balanced threshold effect and neglect of the details of many-body system may not be crucial.

Our neglect of the angular momentum of the extra core neutrons does not seem to affect the results in a significant way. We can infer that the choice of the depth of the core-*n* potential as put in 'by hand' in our method retains the essential physics of the problem. We have also made direct comparison of our calculations with the hyperspherical harmonics (HH) method by using exactly the same set of core-*n* (Wood–Saxon) and *n*-*n* potentials

Table 1. Halo nucleus.

(a) $^{11}_{3}$Li η for minimum energy $E = 0.7$, B_{2n} (Expt) = 0.25 MeV, $r_{\rm m}$ (Expt) = 3.1 fm, $r_{\rm c} = 2.57$ fm [12].

Core-*n* potential	Scattering length (fm)	*n*-*n* potential	Present work B_{2n} (MeV)	$\langle r_{\rm nc}\rangle$ (fm)	$\langle r_{nn}\rangle$ (fm)	$r_{\rm m}$ (fm)	Other methods B_{2n} (MeV)
Woods–Saxon	−110.82	Square-well	0.36	4.87	5.405	2.99	
		One-term Gaussian	0.305	4.87	5.408	2.99	
One-term Gaussian	−37.07	One-term Gaussian	0.26	4.797	5.33	2.971	0.25 ± 0.08 [10]
Two-term Gaussian	−37.08	One-term Gaussian	0.27	4.747	5.27	2.96	

(b) $^{19}_{5}$B η for minimum energy $E = 0.7$, B_{2n} (Expt) = 0.87 MeV, $r_{\rm c} = 3.4$ fm.

Core-*n* potential	Scattering length (fm)	*n*-*n* potential	Present work B_{2n} (MeV)	$\langle r_{\rm nc}\rangle$ (fm)	$\langle r_{nn}\rangle$ (fm)	$r_{\rm m}$ (fm)
Woods–Saxon	23.17	Square-well	0.84	4.746	5.268	3.53
		One-term Gaussian	0.82	4.75	5.27	3.53

(one-term Gaussian) in both the methods. With the HH method B_{2n} is 1.095 MeV for $K = 12$ and 1.14 MeV for $K = 16$ [14]. Compared to that our simple model gives lower binding and this trend is consistently followed when our calculations are extended to the double λ hypernucleus ${}^{\ 6}_{\lambda\lambda}$He. The large value of $(\langle r_{\rm nc}\rangle / r_{\rm c}) = 1.895$ indicates that ^{11}Li really have a halo structure. In order to test the consistency of the procedure we apply our method to study another halo nucleus $^{19}_{5}$B. For this nucleus the appropriate V_0^n is -5.5 MeV which corresponds to a moderately large positive scattering length. We get the best fit once again for the same value $(= 0.7)$ of the scaling parameter η. The predicted value of $B_{2n}(^{19}_{5}\mathrm{B}) = 0.84$ MeV which agrees with the experimental value $= 0.87$ MeV. The predicted matter radius $r_{\rm m}(^{19}_{5}\mathrm{B}) = 3.53$ fm. No experimental value for $r_{\rm m}$ is presently available. Thus with a single scaling parameter η and a proper description of binary subsystem we can explain the structural properties of a three-body system with a stable numerical procedure.

The effective potential $V_{\rm eff}(R)$, generated by the coordinate transformation is plotted as a function of R in figure 1 for ^{11}Li. The depth of $V_{\rm eff}(R)$ decreases slowly in the range (2.0 to 2.5) fm and then rises quickly up to the range (8–10) fm and once again very slowly up to 15 fm and beyond giving a long range character to the effective potential. Such a potential in our effective two-body equation gives weakly bound ground state of halo nuclei. $V_{\rm eff}(R)$ for ^{19}B has a longer range compared to ^{11}Li as expected because of its greater binding.

We also notice that the convergence of the HH expansion for the binding energy is rather slow and one needs to continue calculations up to the grand hyperangular momentum value, $K = 16$ to attain convergence of two neutron separation energy [11,14]. This involves solution of 25 coupled differential equations. In the present work we solve only a single second order differential equation and with a single value of a scaling parameter η get well-converged binding energy of both ^{11}Li and ^{19}B.

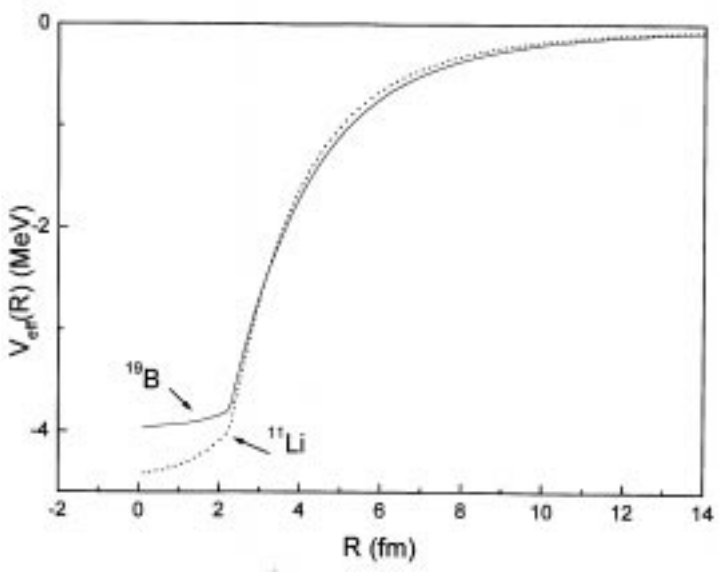

Figure 1. Effective potential $V_{\rm eff}(R)$ as a function of $R(= (r_1 + r_2 + \eta r_3)/2)$ for $^{11}_{3}$Li and $^{19}_{5}$B.

B. *Double λ hypernuclei*

We now report our results regarding the structure of the ${}^{6}_{\lambda\lambda}$He hypernucleus using the above procedure. ${}^{6}_{\lambda\lambda}$He is again treated as a three-body system consisting of the α-core and two λ hyperons. The $\alpha-\lambda$ potential to be used in the present calculations has to satisfy the experimental binding energy 3.04 MeV of ${}^{5}_{\lambda}$He hypernucleus. For the different $\alpha-\lambda$ potentials employed by us we have checked this consistency condition numerically by solving the Schrödinger equation in the centre of mass of α and λ particles. The $\alpha-\lambda$ and $\lambda-\lambda$ potentials used are:

(i) The Gaussian potential B of Tang and Herndon [15] which is $V_{\alpha\lambda}(r) = -60.17\exp(-r^2/1.273)$ MeV. This gives the binding energy of ${}^{5}_{\lambda}$He as 2.97 MeV. The corresponding $\lambda-\lambda$ singlet potential employed is the Volkov potential with a Gaussian shape having an attactive part V_{A} and a repulsive part V_{R} [16].

$$V_{\lambda\lambda}(r) = V_{\mathrm{A}}\exp(-r^2/\alpha^2) + V_{\mathrm{R}}\exp(-r^2/\beta^2),$$

$\alpha = 1.2$ fm, $\beta = 0.82$ fm, $V_{\mathrm{A}} = -93.75$ MeV and $V_{\mathrm{R}} = 148.0$ MeV. The $V_{\lambda\lambda}$ potential in the ${}^{1}S_0$ state corresponds to a scattering length $= -2.234$ fm and an effective range $= 3.302$ fm. The scaling parameter η is varied till minimum separation energy is obtained. This η is 0.3 and the $\lambda\lambda$ separation energy corresponds to 9.93 MeV which is somewhat lower than 10.8 MeV reported in [16] using the HH method.

(ii) Another one term $\alpha-\lambda$ Gaussian potential used by us is [17]

$$V_{\alpha\lambda}(r) = -43.976\exp\{-(r/1.566)^2\}\ \mathrm{MeV},$$

$\alpha-\lambda$ binding energy is 3.05 MeV in this case and the $\lambda-\lambda$ potential employed is

$$V_{\lambda\lambda}(r) = -52.25\exp\{-(r/1.034)^2\}\ \mathrm{MeV}.$$

This corresponds to a scattering length -1.75 fm and an effective range 2.082 fm. With this set of potentials we obtained as the $\lambda\lambda$ separation energy for the ${}^{6}_{\lambda\lambda}$He hypernucleus a value 10.653 MeV compared to 10.8 MeV of [17] and 11.2 MeV by the HH method [14].

(iii) Lastly we have also used $\alpha-\lambda$ potential obtained by the single-folding method [18]. The method is outlined briefly. The $\alpha-\lambda$ potential is defined by the formula:

$$V_{\alpha\lambda}(r) = \int \rho(\vec{r}\,')V_{\lambda N}(|\vec{r}-\vec{r}\,'|)\mathrm{d}\tau'. \tag{3.9}$$

Here $\rho(\vec{r}\,')$ is the nucleon density in the α-particle at a distance $\vec{r}\,'$ from its centre-of-mass. $V_{\lambda N}(|\vec{r}-\vec{r}\,'|)$ is the interaction potential between the λ particle at $\vec{r}$ and the nucleon at $\vec{r}\,'\cdot\rho(\vec{r}\,')$ is taken as $4(4\bar{\beta}/3\pi)^{3/2}\exp(-(4/3)\bar{\beta}r'^2)$ [19] after removing the centre of mass motion. $V_{\lambda N}(|\vec{r}-\vec{r}\,'|)$ is a three-term Gaussian potential

$$A\exp\{-(|\vec{r}-\vec{r}\,'|/\alpha)^2\} - B\exp\{-(|\vec{r}-\vec{r}\,'|/\beta)^2\} - C\exp\{-(|\vec{r}-\vec{r}\,'|/\gamma)^2\}.$$

The first term represents a repulsive potential and the second and third terms represent attractive potentials. The integration in (3.9) is over all space as permitted by $\rho(\vec{r}\,')$. The required normalization condition is satisfied by $\rho(\vec{r}\,')$:

$$\int \rho(\vec{r}\,')\mathrm{d}\tau' = \frac{4\left(\frac{4}{3}\bar{\beta}\right)^{3/2}}{\pi^{3/2}} \int_0^\infty \exp\left(-\frac{4}{3}\bar{\beta}r'^2\right) 4\pi r'^2 \mathrm{d}r' = 4.$$

The constant $\bar{\beta}$ is determined from the r.m.s. radius of ^{4}He. The relation is $\left(3/\sqrt{8\bar{\beta}}\right) =$ 1.47 fm (r.m.s. radius of ^{4}He). With the three-term $\lambda - N$ Gaussian potential, $V_{\alpha\lambda}(\vec{r})$ is given by

$$V_{\alpha N}(\vec{r}) = 4\left(\frac{4\bar{\beta}}{3\pi}\right)^{3/2} \int \exp\left(-\frac{4\bar{\beta}r'^2}{3}\right) (A\exp\{-(|\vec{r}-\vec{r}\,'|/\alpha)^2\} - B\exp\{-(|\vec{r}-\vec{r}\,'|/\beta)^2\} - C\exp\{-(|\vec{r}-\vec{r}\,'|/\gamma)^2\})\mathrm{d}\tau'. \tag{3.10}$$

Carrying out the integrations we arrive at the final result,

$$V_{\alpha\lambda}(r) = 4\mu^3[A_1\exp(-a_1r^2) - B_1\exp(-b_1r^2) - C_1\exp(-c_1r^2)]. \tag{3.11}$$

Here,

$$\mu^2 = \frac{4}{3}\bar{\beta}, \quad A_1 = \frac{A}{\left(\frac{4}{3}\bar{\beta}+\frac{1}{\alpha^2}\right)^{3/2}}, \quad B_1 = \frac{B}{\left(\frac{4}{3}\bar{\beta}+\frac{1}{\beta^2}\right)^{3/2}},$$
$$C_1 = \frac{C}{\left(\frac{4}{3}\bar{\beta}+\frac{1}{\gamma^2}\right)^{3/2}},$$
$$a_1 = \frac{1}{\alpha^2} - \frac{1}{\alpha^4\left(\frac{4}{3}\bar{\beta}+\frac{1}{\alpha^2}\right)}, \quad b_1 = \frac{1}{\beta^2} - \frac{1}{\beta^4\left(\frac{4}{3}\bar{\beta}+\frac{1}{\beta^2}\right)},$$
$$c_1 = \frac{1}{\gamma^2} - \frac{1}{\gamma^4\left(\frac{4}{3}\bar{\beta}+\frac{1}{\gamma^2}\right)}. \tag{3.12}$$

The three term Gaussian λN potential used here is

$$V_{\lambda N}(|\vec{r}-\vec{r}\,'|) = 919.0\exp\{-(|\vec{r}-\vec{r}\,'|/0.5)^2\} - 206.54\exp\{-(|\vec{r}-\vec{r}\,'|/0.9)^2\} - 9.62\exp\{-(|\vec{r}-\vec{r}\,'|/1.5)^2\}. \tag{3.13}$$

The different constants $A_1, B_1, \ldots, a_1, b_1, \ldots$, occurring in the folding potential expression can be calculated in a straightforward manner from (3.12) and then

$$V_{\alpha\lambda}(r) = 209.037\exp(-a_1r^2) - 178.379\exp(-b_1r^2) - 18.317\exp(-c_1r^2)\ \text{MeV}, \tag{3.14}$$

where $a_1 = 0.5915059\ \text{fm}^{-2}$, $b_1 = 0.4443259\ \text{fm}^{-2}$ and $c_1 = 0.2709581\ \text{fm}^{-2}$.

The $\alpha - \lambda$ folding potential gives a binding energy 2.94 MeV for $^5_\lambda$He which is in good agreement with the experimental value.

The $\lambda - \lambda$ potential used is a three-term Gaussian potential which is

$$V_{\lambda\lambda}(r) = 480.0\exp(-(r/0.5)^2) - 191.0\exp(-(r/0.9)^2) - 7.5\exp(-(r/1.5)^2)\ \text{MeV}.$$

This potential is characterized by a scattering length −16.346 fm with an effective range 1.902 fm. With the $\alpha - \lambda$ folding potential and the three-term $\lambda - \lambda$ potential our calculations give convergence of numerical results at the value 9.17 MeV for the $\lambda\lambda$ separation energy of ${}^{6}_{\lambda\lambda}$He. The corresponding best fit value of η is 0.75. Our calculated value of the separation energy is somewhat lower than the experimental value but such a trend is consistent with our other calculations. Table 2 summarizes the results obtained. The $\alpha - \lambda$ folding potential is shown in figure 2.

Table 2. Double-λ hypernucleus ${}^{6}_{\lambda\lambda}$He.

			$\lambda - \lambda$ potential parameters			Present work			
$\alpha - \lambda$ potential	$\alpha - \lambda$ binding energy (MeV)	Type	Scattering length (fm)	Effective range (fm)	η for E_{min}	$\langle r_{\alpha\lambda}\rangle$ (fm)	$\langle r_{\lambda\lambda}\rangle$ (fm)	$B_{2\lambda}$ (MeV)	$B_{2\lambda}^{**}$ (MeV)
Gaussian potential *B* [15]	2.97	Volkov	−2.234	3.302	0.3	2.062	2.634	9.93	10.8[a]
Gaussian potential [17]	3.051	One-term Gaussian	−1.751	2.082	0.5	2.093	2.493	10.65	11.2[b] 10.8[c]
Folding potential	2.94	Three-term Gaussian	−16.35	1.902	0.75	2.694	2.938	9.17	

**Other methods for the same potentials; [a]HH method [16]; [b]HH method [14]; [c]Method of [17].

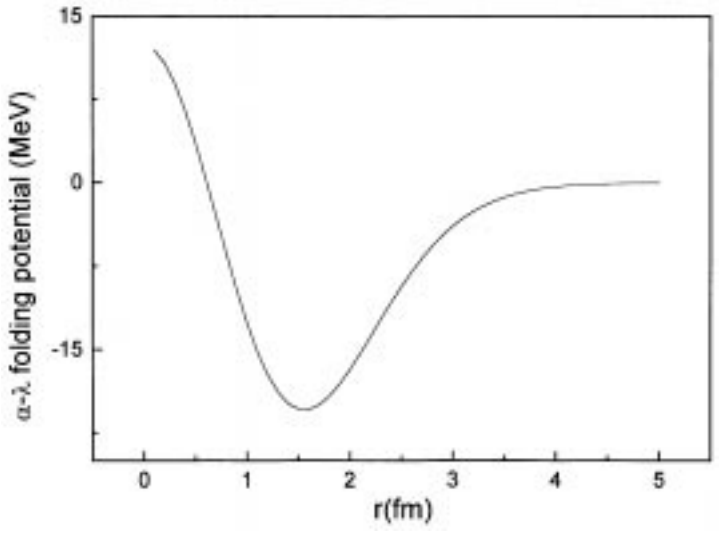

Figure 2. $\alpha - \lambda$ folding potential by the single-folding method using a three-term Gaussian $\lambda - N$ potential.

4. Conclusion

The mathematically simple variational ansatz presented here to study halo nuclei and double lambda hypernuclei proved the usefulness of the special set of coordinates, which may be used to study other asymmetric three-particle problems. Our model systems are at best only approximations of real system in nature but they are useful in extracting gross properties of physical relevance. In this context the present method may be extended to study the structural properties of some known double lambda hypernuclei beyond ${}^{\ 6}_{\lambda\lambda}$He employing the concept of folding potential.

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