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Investigation of halo structure of ^{6}He by hyperspherical three-body method

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Abstract. Hyperspherical harmonics expansion method is applied to a three-body model of two neutron halo nuclei. Convergence of the expansion has been ensured. A repulsive part is introduced in the interaction between the core and the extra-core neutron, to simulate Pauli principle. Two neutron separation energy, r.m.s. radii, correlation factor and probability density distributions have been calculated for ^{6}He. It is found that the convergence of the two neutron separation energy is relatively slow, while other quantities reach convergence quickly.

Keywords. Raynal Revai coefficient (RRC); hyperspherical harmonics expansion (HHE); hyperspherical harmonics (HH).

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1. Introduction

The study of the structure of exotic nuclei has become an area of intense activity due to the recent development in the radioactive nuclear beams (RNB) facilities [1]. This has increased the interest about the nuclei away from the stability line since the RNB facilities can reach such nuclei. Near the neutron dripline there exists nuclei having outer nuclear matter density much smaller than the magic nucleon density value of 0.17 nucleon/fm^3 [2]. The study of these nuclei yields new insight into the properties of tenuous nuclear matter and provides information about the structure and interactions of the loosely bound systems. It was observed that exotic ^{6}He shows a large electromagnetic dissociation cross-sections at high energy on heavy targets [3]. Some other exotic nuclei viz. ^{11}Li, ^{8}He, ^{14}Be also show similar results [4–6]. The elastic scattering of exotic ^{6}He on light and heavy targets results in large cross-section and fragmentation experiments show a strikingly large radius compared to neighbouring stable nuclei. Furthermore the two neutron separation energies for ^{6}He and ^{11}Li are significantly small compared to those for stable nuclei in the same mass region. These facts indicate that the exotic nuclei ^{6}He and ^{11}Li can be considered as consisting of a 'core' and two weakly bound outer neutrons, which extend to large spatial distances with a matter density much smaller than the typical nuclear density in stable nuclei. The outer neutrons thus form a 'halo' around a stable core. This leads to a simple three-body model for two neutron 'halo' nuclei with two loosely bound neutrons outside

a fairly stable core. It is interesting to note that no two-body subsystem of the three-body system is bound, which gives rise to the mythical name of 'Borromean rings' to these exotic nuclei [7]. The typical structure of Borromean three-body system resembles the heraldic symbol of the Italian Princess of Borromeo. Its crest has three rings interlocked in such a way that if one ring is removed the other two separate. The most challenging job for the study of the structure of such loosely bound three-body system is to bring out the true nature of the interactions among the core and the outer core valence nucleons. The available experimental data of ^{6}He includes a small two neutron separation energy (0.973 ± 0.04 MeV) [7] and a large r.m.s. matter radius (2.57 ± 0.1 fm) obtained in a model independent way [8] from the measeurement of total internal cross-section [9]. These data suggest that this nucleus has exotic structure of neutron halo. Beta-delayed deuteron emission from ^{6}He [10] also provided important information about the halo structure of this nucleus. Some of the gross features of 'halo' nuclei have been investigated by several authors in refs [11–22]. In ref. [17], Mazumdar *et al* have discussed some of the important features of exotic halo nuclei like ^{19}B, ^{22}C, ^{20}C etc. in the coordinate space Faddeev equation approach assuming separable *n*-*n* and *n*-core two-body potentials. They attempted to show that for such exotic halo nuclei the three-body equation with the realistic two-body potentials provide a natural framework to explain their binding energies, momentum distribution etc. They also attempted to undertake a detailed numerical analysis by computing integral equations for three-body systems to search for the occurrence of Efimov states with realistic two-body potentials as input. Thirdly they attempted to show that for the Borromean halo nuclei there is a remote possibility for the occurrence of Efimov states except when the scattering length for the virtual *n*-core system is sufficiently large (typically $\sim$ few hundered Fermi) and the three-body energy approaches zero. And on the other hand for those nuclei in which the binary subsystems are bound, the probability for the occurrence of Efimov states is quite high. Since ^{6}He nucleus is the simplest neutron rich nucleus of the so-called halo family and have a typical three-body Borromean structure having no bound binary sub-system, investigations of this nucleus may give important information about the weak binding mechanism of such three-body systems. The nucleus $^5\text{He} \Longrightarrow (n+{}^4\text{He})$ is unbound. If one neutron is taken away from ^{6}He, second neutron will also fly off immediately leaving behind ^{4}He. Thus, $^6\text{He} \Longrightarrow ({}^4\text{He} + n + n)$ is bound as a three-body system. Hyperspherical three-body model calculation for the ground state of ^{6}He nucleus has been reported by Zhukov *et al* [7]. However they did not achieve full convergence and obtained a binding energy of ~ 0.4 MeV which is less than the experimental value. Since the calculated binding energy is less than the experimental value, it seems that the interaction potentials chosen for the interacting pairs of the three-body (core $+n+n$) system are not fully correct one and the problem requires some reinvestigation. In the present work we first calculate the binding energy and some related geometrical quantities to compare our calculation with the corresponding results of Zhukov *et al* assuming the same neutron–neutron and core–neutron potential, for the ground state of ^{6}He. After that we search for a reasonable core–nucleon potential by adjusting the parameters of the potential having the same form as in the former calculation to get the converged value of the binding energy and the geometrical quantitites which are useful to study the structure of such three-body system. Finally, we compare the prediction of a three-body model of ^{6}He with experimental results. We employ hyperspherical harmonics expansion (HHE) method to solve such a three-body system. This method is a powerful tool for the *ab initio* solution of the few-body Schrödinger equation for a given set of interaction potentials among the constituent

particles. This method has been used for bound states in atomic [23–35], nuclear [36–53] and particle physics [54–56]. Attempts have been made to use it in scattering problems as well [57]. In this method, the wave function is expanded in a complete set of hyperspherical harmonics (HH), which are the six-dimensional analogue of the angular part of eigenfunctions of the 3-dimensional Laplacian operator. The resulting Schrödinger equation is a set of coupled differential equations which are solved numerically by the renormalized Numerov method (RNM) [58,59]. The HHE method is essentially an exact one and more reliable than others. It involves no approximation except an eventual truncation of the expansion basis. By gradually expanding the expansion basis and checking the rate of convergence, any desired precision in the binding energy can in principle be achieved. However the number of coupled differential equations and therefore the complexity in the numerical solution increases rapidly as the expansion basis is increased by including larger hyper-angular momentum quantum number. Computer limitations set an ultimate limit to the precision attainable. Thus in this approach the attainment of desired convergence in physical observables are of great importance. In the present work we achieved a convergence in the binding energy to within 4% by solving the three-body equations. Thereafter we utilize hyperspherical convergence formula to extrapolate the binding energy to three significant figures. In addition to the two neutron separation energy we have also studied the size, density and correlation among the ^{4}He (core) and the two valence neutrons.

This paper is organized as follows: In §2, we review the HHE method for a three-body system consisting of non-identical particles. Results of calculation and discussions are presented in §3. Finally in §4, we draw our conclusions.

2. HHE method

We label the ^{4}He-core as particle number '1' and the two valence neutrons as particles '2' and '3' respectively (see figure 1). For pairwise interactions, we can treat any of the three particles as spectator, remaining two being the interacting pair. Thus there are three possible partitions labelled i ($i = 1,2,3$). In the partition i, particle numbered i is the spectator and particles numbered j and k form the interacting pair ($i,j,k = 1,2,3$, cyclic). Now for a given partition i, the Jacobi co-ordinates (one of which is proportional to the relative separation between the interacting pair and the other is proportional to relative separation between the spectator and the centre of mass of the interacting pair respectively) are defined as

$$\left.\begin{aligned} \vec{x}_i &= \left[\frac{m_j m_k M}{m_i(m_j+m_k)^2}\right]^{\frac{1}{4}} (\vec{r}_j - \vec{r}_k) \\ \vec{y}_i &= \left[\frac{m_i(m_j+m_k)^2}{m_j m_k M}\right]^{\frac{1}{4}} \left(\vec{r}_i - \frac{m_j\vec{r}_j + m_k\vec{r}_k}{m_j+m_k}\right) \\ \vec{R} &= \frac{1}{M}\left(m_i\vec{r}_i + m_j\vec{r}_j + m_k\vec{r}_k\right) \end{aligned}\right\}, \tag{1}$$

(i,j,k=1, 2, 3 cyclic) where m_i, $\vec{r}_i$ are the mass and position of the ith particle, $M = m_i + m_j + m_k$ is the total mass and $\vec{R}$ is the centre of mass of the system. The sign of $\vec{x}_i$ is fixed by the condition that i,j,k form a cyclic permutation of (1, 2, 3). In the transformation (1) the nine-dimensional volume element is conserved (i.e. the Jacobian is unity) and the

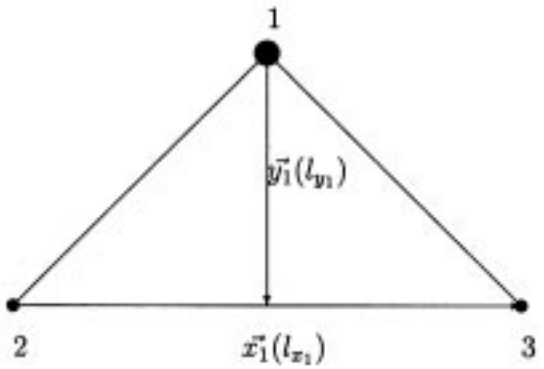

Figure 1. Choice of Jacobi coordinates for the partition '1'.

centre of mass motion is automatically separated. The relative motion of the three-body system is described by the Schrödinger equation

$$\left[-\frac{\hbar^2}{2\mu}(\nabla^2_{x_i}+\nabla^2_{y_i})+V_{jk}(\vec{x}_i)+V_{ki}(\vec{x}_i,\vec{y}_i)+V_{ij}(\vec{x}_i,\vec{y}_i)-E\right]\Psi(\vec{x}_i,\vec{y}_i)\;=\;0, \qquad (2)$$

where $\mu = \left[\frac{m_i m_j m_k}{M}\right]^{\frac{1}{2}}$ is an effective mass parameter and V_{ij} is the interaction potential between ith and jth particles. We next introduce the hyperspherical variables defined by [53]

$$\left.\begin{array}{rcl} x_i &=& \rho\cos\Phi_i \\ y_i &=& \rho\sin\Phi_i \end{array}\right\}, \qquad (3)$$

where $\rho = \sqrt{x_i^2+y_i^2}$ is the global radius (also called the hyper-radius) which is invariant under three-dimensional rotations and permutations of the particle indices. Thus ρ is the same for all three partitions. The five other hyperspherical variables include the hyperspherical angle $\Phi_i = \tan^{-1}(y_i/x_i)$ and the polar angles $(\theta_{x_i},\zeta_{x_i})$ and $(\theta_{y_i},\zeta_{y_i})$ giving orientations of $\vec{x}_i$ and $\vec{y}_i$ respectively. These are collectively denoted by

$$\Omega_i \equiv \{\Phi_i,\theta_{x_i},\zeta_{x_i},\theta_{y_i},\zeta_{y_i}\}. \qquad (4)$$

The six dimensional volume element is given by

$$\mathrm{d}V_6 = \rho^5\,\mathrm{d}\rho\;\cos^2\Phi_i\;\sin^2\Phi_i\;\mathrm{d}\Phi_i\mathrm{d}\Omega_{x_i}\mathrm{d}\Omega_{y_i}, \qquad (5)$$

where

$$\left.\begin{array}{rcl} \mathrm{d}\Omega_{x_i} &=& \sin\theta_{x_i}\,\mathrm{d}\theta_{x_i}\,\mathrm{d}\zeta_{x_i} \\ \mathrm{d}\Omega_{y_i} &=& \sin\theta_{y_i}\,\mathrm{d}\theta_{y_i}\,\mathrm{d}\zeta_{y_i} \end{array}\right\}. \qquad (6)$$

In terms of the hyperspherical variables the Schrödinger equation becomes

$$\left[-\frac{\hbar^2}{2\mu}\left\{\frac{1}{\rho^5}\frac{\partial}{\partial\rho}\left(\rho^5\frac{\partial}{\partial\rho}\right)-\frac{\hat{\mathscr{K}}^2(\Omega_i)}{\rho^2}\right\}+V(\rho,\Omega_i)-E\right]\Psi(\rho,\Omega_i)\;=\;0, \quad (7)$$

where $V(\rho,\Omega_i)=V_{jk}(\vec{x}_i)+V_{ki}(\vec{x}_i,\vec{y}_i)+V_{ij}(\vec{x}_i,\vec{y}_i)$ is the total interaction potential expressed in terms of the hyperspherical variables and $\hat{\mathscr{K}}^2(\Omega_i)$ is the square of hyper-angular momentum operator given by [53]

$$\hat{\mathscr{K}}^2(\Omega_i)\;=\;-\frac{\partial^2}{\partial\Phi_i^2}-4\cot 2\Phi_i\,\frac{\partial}{\partial\Phi_i}+\frac{1}{\cos^2\Phi_i}\hat{l}^2(\hat{x}_i)+\frac{1}{\sin^2\Phi_i}\hat{l}^2(\hat{y}_i), \quad (8)$$

where $\hat{l}^2(\hat{x}_i)$ and $\hat{l}^2(\hat{y}_i)$ are the squares of ordinary orbital angular momentum operators associated with $\vec{x}_i$ and $\vec{y}_i$ motions respectively. The operator $\hat{\mathscr{K}}^2$ satisfies an eigenvalue equation [53]

$$\hat{\mathscr{K}}^2(\Omega_i)\mathscr{Y}_{K\alpha_i}(\Omega_i)\;=\;K(K+4)\mathscr{Y}_{K\alpha_i}(\Omega_i), \quad (9)$$

where α_i is an abbreviation for the set of four quantum numbers $\{l_{x_i},l_{y_i},L,M\}$ and K, the hyper-angular momentum quantum number (which is not a conserved quantity for 3-body system) is given by $K=2n_i+l_{x_i}+l_{y_i}$ (n_i being a non-negative integer). The quantity K is the degree of the homogeneous harmonic polynomials $\rho^K\mathscr{Y}_{K\alpha_i}(\Omega_i)$ in the cartesian components of $\vec{x}_i$ and $\vec{y}_i$. Note that the quantum number K is invariant under the change of partition and hence does not involve the partition label. The eigenfunction of $\hat{\mathscr{K}}^2$ are called hyperspherical harmonics (HH) and are given by

$$\mathscr{Y}_{K\alpha_i}(\Omega_i)=^{(2)}P_K^{l_{y_i}\;l_{x_i}}(\Phi_i)\left[Y_{l_{x_i}}(\hat{x}_i)\,Y_{l_{y_i}}(\hat{y}_i)\right]_{LM}, \quad (10)$$

where

$$^{(2)}P_K^{l_{y_i}\;l_{x_i}}(\Phi_i)\;=\;N_K^{l_{x_i},\,l_{y_i}}\,(\cos\,\Phi_i)^{l_{x_i}}\,(\sin\,\Phi_i)^{l_{y_i}}\,P_{n_i}^{l_{y_i}+1/2,\,l_{x_i}+1/2}\,(\cos\,2\Phi_i). \quad (11)$$

The normalization constant $N_K^{l_{x_i},l_{y_i}}$ is given by

$$N_K^{l_{x_i},L_{y_i}}\;=\;\left[\frac{2\;n_i!(K+2)(n_i\;+\;l_{x_i}\;+\;l_{y_i}\;+\;1)!}{\Gamma(n_i\;+\;l_{x_i}\;+\;3/2)\,\Gamma(n_i\;+\;l_{y_i}+3/2)}\right]^{\frac{1}{2}} \quad (12)$$

and $P_n^{\alpha,\beta}(x)$ is the Jacobi polynomial [60].

In the present method $\Psi(\rho,\Omega_i)$ is expanded in the complete set of HH corresponding to a given partition (say partition *i*):

$$\Psi(\rho,\Omega_i)=\sum_{K\alpha_i}\frac{U_{K\alpha_i}(\rho)}{\rho^{5/2}}\mathscr{Y}_{K\alpha_i}(\Omega_i). \quad (13)$$

The factor $\rho^{-5/2}$ is included to remove the first-order derivative with respect to ρ in eq. (7). The three-body relative wave fuction Ψ can be expanded in the complete set of HH,

corresponding to any chosen partition i (i=1, 2, 3). Substitution of eq. (13) in (7) and the use of orthonormality of HH leads to a set of coupled differential equations (CDE) in ρ,

$$\left[-\frac{\hbar^2}{2\mu}\left(\frac{\mathrm{d}^2}{\mathrm{d}\rho^2}-\frac{\mathscr{L}_K(\mathscr{L}_K+1)}{\rho^2}\right)-E\right]U_{K\alpha_i}(\rho) + \sum_{K'\alpha_i'}\langle K\alpha_i \mid V(\rho,\Omega_i) \mid K'\alpha_i'\rangle U_{K'\alpha_i'}(\rho) = 0, \tag{14}$$

where $\mathscr{L}_K = K+3/2$ and

$$\langle K\alpha_i|V(\rho,\Omega_i)|K'\alpha_i'\rangle = \int_{\Omega_i}\mathscr{Y}^*_{K\alpha_i}(\Omega_i)V(\rho,\Omega_i)\mathscr{Y}_{K'\alpha_i'}(\Omega_i)\mathrm{d}\Omega_i. \tag{15}$$

Since the expansion (13) is in principle an infinite one, the CDE, eq. (14) is also an infinite set. For practical purposes, the expansion (13) has to be truncated to a finite set, leading to a finite set of CDE. Restrictions arising out of symmetry requirement and imposition of conserved quantum numbers (e.g., total angular momentum, parity etc.) can reduce the expansion basis further and consequently a smaller set of CDE is to be solved.

Evaluation of the matrix elements of the type $\langle\mathscr{Y}_{K\alpha_i}(\Omega_i) \mid V_{jk}(x_i) \mid \mathscr{Y}_{K'\alpha_i'}(\Omega_i)\rangle$ (for central interactions) are straight forward, while those for the matrix elements of the type $\langle\mathscr{Y}_{K\alpha_i}(\Omega_i) \mid V_{ki}(x_j)|\mathscr{Y}_{K'\alpha_i'}(\Omega_i)\rangle$ and $\langle\mathscr{Y}_{K\alpha_i}(\Omega_i) \mid V_{ij}(x_k) \mid \mathscr{Y}_{K'\alpha_i'}(\Omega_i)\rangle$ become very complicated even for central interactions, since x_j or x_k are expressed as linear combinations of $\vec{x}_i$ and $\vec{y}_i$, and consequently the integrand depends on the polar angles of $\vec{x}_i$ and $\vec{y}_i$ (i.e. $\hat{x}_i$, $\hat{y}_i$) (see eq. (1)). But the calculation of these matrix elements will be quite simple in the partitions j or k respectively, since in these partitions $\vec{x}_j$ or $\vec{x}_k$ are independent of $\vec{y}_j$ and $\vec{y}_k$ respectively. Since the choice of a particular partition is arbitrary, the HH basis corresponding to any chosen partition i forms a complete set spanning the same hyper-angular space. One can then relate the HH basis for two different partitions i and j through a unitary transformation. Then a particular element, $\mathscr{Y}_{K\alpha_i}(\Omega_i)$ in the partition i can be expanded in the HH basis corresponding to partition j as

$$\mathscr{Y}_{K\alpha_i}(\Omega_i) = \sum_{l_{x_j}l_{y_j}}\langle l_{x_j}l_{y_j} \mid l_{x_i}l_{y_i}\rangle_{KL}\mathscr{Y}_{K\alpha_j}(\Omega_j), \tag{16}$$

where the transformation coefficients $\langle l_{x_j}l_{y_j} \mid l_{x_i}l_{y_i}\rangle_{KL}$ are called the Raynal Revai coefficients (RRC) [61]. Since K, L and M are independent of the partition, the sum is over l_{x_j} and l_{y_j} only, subject to the restrictions $\vec{l}_{x_i}+\vec{l}_{y_i}=\vec{L}=\vec{l}_{x_j}+\vec{l}_{y_j}$. These coefficients can be computed easily [35]. Since the RRC's do not involve ρ, these are calculated once only and stored and it reduces the CPU time significantly.

In terms of RRC's the matrix elements of V_{ki} in the partition i can be written as

$$\begin{aligned}\langle\mathscr{Y}_{K\alpha_i}(\Omega_i) \mid V_{ki}(x_j) \mid \mathscr{Y}_{K'\alpha_i'}(\Omega_i)\rangle = &\sum_{l_{x_j}'l_{y_j}'l_{x_j}l_{y_j}}\langle l_{x_j}l_{y_j} \mid l_{x_i}l_{y_i}\rangle^*_{KL} \\ &\times\langle l_{x_j}'l_{y_j}' \mid l_{x_i}'l_{y_i}'\rangle_{K'L} \\ &\times\langle\mathscr{Y}_{K\alpha_j}(\Omega_j) \mid V_{ki}(x_j) \mid \mathscr{Y}_{K'\alpha_j'}(\Omega_j)\rangle.\end{aligned} \tag{17}$$

The matrix element on the right side of eq. (17) has the same form as the matrix element of V_{jk} in the partition i (preferred partition) and can be evaluated in a simple way. Thus computing the RRC's involved in eq. (17), the matrix element of V_{ki} in the partition i can be evaluated easily. Similar technique can be employed for the calculation of the matrix element of V_{ij}.

Calculation of potential matrix elements in the preferred partition can be further simplified by introducing a multipolar expansion [36] of the potential. For a matrix element in the preferred partition say partition i, the potential $V_{jk}(x_i)$, is expanded in an appropriate subset of corresponding HH,

$$V_{jk}(x_i) = \sum_{K''\alpha_i''} v^{(jk)}_{K''\alpha_i''}(\rho)\mathscr{Y}_{K''\alpha_i''}(\Omega_i), \tag{18}$$

where $v^{(jk)}_{K''\alpha_i''}(\rho)$ is called the potential multipole and can be evaluated by the use of orthogonality of HH:

$$v^{(jk)}_{K''\alpha_i''}(\rho) = \int V_{jk}(x_i)\mathscr{Y}^*_{K''\alpha_i''}(\Omega_i)\mathrm{d}\Omega_i. \tag{19}$$

The matrix element then becomes

$$\langle \mathscr{Y}_{K\alpha_i}(\Omega_i) \mid V_{jk}(x_i) \mid \mathscr{Y}_{K'\alpha_i'}(\Omega_i)\rangle = \sum_{K''\alpha_i''} v^{(jk)}_{K''\alpha_i''}(\rho)\langle K\alpha_i \mid K''\alpha_i'' \mid K'\alpha_i'\rangle, \tag{20}$$

where

$$\langle K\alpha_i \mid K''\alpha_i'' \mid K'\alpha_i'\rangle = \int \mathscr{Y}^*_{K\alpha_i}(\Omega_i)\mathscr{Y}^*_{K''\alpha_i''}(\Omega_i)\mathscr{Y}_{K'\alpha_i'}(\Omega_i)\mathrm{d}\Omega_i \tag{21}$$

is called the geometrical structure coefficients (GSC). These are independent of ρ and the interaction. Hence these coefficients need to be calculated once only and stored resulting in a fast and efficient algorithm. The GSC's involved in eq. (20) can be calculated by standard numerical integration. However they can be calculated in a very elegant manner [41] by using the completeness property of the HH. Finally the set of CDE's eq. (14) is to be solved numerically subject to appropriate boundary conditions to get the energy E and the partial waves $U_{K\alpha_i}(\rho)$.

3. Results and discussions

In the present calculation we have taken the core to be structureless and approximately incorporated the Pauli principle between the core nucleons and valence neutrons by introducing a strongly repulsive part in s-component of the core-n potential. However three-body wave function is properly antisymmetrized under the exchange of two valence neutrons.

The ground state of ^{6}He has a total angular momentum $J = 0$ and positive parity. The possible total spin of the three-body system $(\alpha + n + n)$ can take two values '0' or '1'. Thus the total orbital angular momentum L can be either '0' or '1' corresponding to $S = 0$ or 1 respectively. Hence the ground state of ^{6}He is an admixture of $^{31}S_0$ and $^{33}P_0$ states (in $^{T,S}L_J$ notation). Since the α-particle is spinless, the spin singlet state $(S = 0)$ corresponds to zero

total spin of the valence neutrons (i.e. $S_{23}=0$). Hence the spin part of the wave function is antisymmetric under the exchange of the spins of the two valence neutrons. Thus the spatial part must be symmetric under the exchange of the valence neutrons. The symmetry of the spatial part is determined by the hyper-spherical harmonics, since the hyper-radius ρ and hence the hyper-radial partial waves ($U_{K\alpha}(\rho)$) are invariant under permutation of the particles. Under the pair exchange operator P_{23} which interchanges particles 2 and 3, $\vec{x}_1 \to -\vec{x}_1$ and $\vec{y}_1$ remains unchanged (see eq. (1)). Consequently P_{23} acts like the parity operator for (23) pair only. Choosing the two valence neutrons to be in spin singlet state (spin antisymmetric), the space wave function must be symmetric under P_{23}. This then requires l_{x_1} to be even. For the spin singlet state total orbital angular momentum, $L=0$, hence we must have $l_{x_1}=l_{y_1}$ = even integer. Since $K=2n_1+l_{x_1}+l_{y_1}$, where n_1 is a non negative integer, K must be even and

$$\left.\begin{array}{rcl} l_{x_1} = l_{y_1} = & 0,2,4,\ldots,K/2 & \text{if } K/2 \text{ is even} \\ & 0,2,4,\ldots,(K/2-1) & \text{if } K/2 \text{ is odd} \end{array}\right\}. \tag{22}$$

Again for the triplet state $(S=1)$, the two valence neutrons will be in spin triplet state ($S_{23}=1$, spin symmetric). Hence the space wave function must be antisymmetric under P_{23}. This then requires l_{x_1} to be odd. For the spin triplet state the total orbital angular momentum, $L=1$, hence l_{y_1} may take values $l_{x_1}-1$, l_{x_1}, and $l_{x_1}+1$ but the parity conservation allows $l_{y_1}=l_{x_1}$ only. Again since $K=2n+l_{x_1}+l_{y_1}$, where n is a non negative integer, K must be even and

$$\left.\begin{array}{rcl} l_{x_1} = l_{y_1} = & 1,3,5,\ldots,K/2 & \text{if } K/2 \text{ is odd} \\ & 1,3,5,\ldots,(K/2-1) & \text{if } K/2 \text{ is even} \end{array}\right\}. \tag{23}$$

For practical purposes, the HH expansion basis (eq. (13)) is truncated to a maximum value ($K_{\max}$) of K. For each allowed K, all allowed values of l_{x_1} according to eqs (22) and (23) are included. This truncates eq. (14) to a set of N coupled differential equations, where

$$N = \left.\begin{array}{ll} \left(\frac{K_{\max}}{2}+1\right)\left(\frac{K_{\max}}{4}+1\right) & \text{if } K_{\max}/2 \text{ is even} \\ \left(\frac{K_{\max}+2}{4}\right)\left(\frac{K_{\max}}{2}+2\right) & \text{if } K_{\max}/2 \text{ is odd} \end{array}\right\}. \tag{24}$$

The truncated set of CDE is solved by hyperspherical adiabatic approximation (HAA) [62].

Two-body potentials

For the neutron–neutron pair we choose the GPT potential [63] and the $\alpha-n$ potential is chosen to be central but with an l-dependent strength parameter (i.e., the strength of the $\alpha-n$ potential is different for different partial wave (l)) plus a spin-orbit term [64],

$$V_{\alpha n}(\vec{r}) = \sum_l V_{\alpha n}^{(l)} \exp\left(-r^2/b_{\alpha n}^2\right) + V^{(ls)}(\vec{l}.\vec{s}) \exp\left(-r^2/b_{\alpha n}^2\right), \tag{25}$$

where $V_{\alpha n}^{(l)}$ is $+50$ MeV, -47.32 MeV, -23.0 MeV for $l=0,1$ and 2 respectively and $V^{(ls)}=-5.855$ MeV. It is to be noted that there is no angular dependence like $P_l(\cos\theta)$ in the $\alpha-n$ potential. However such angular dependence would be necessary for *non-central* potentials. The chosen range parameter for all the components of the $\alpha-n$ potential is

$b_{\alpha n} = 2.3$ fm [64]. Both the $n-n$ and $\alpha-n$ potentials fit the scattering and phase shift data well [65]. To simulate antisymmetrization under exchange of the valence nucleon and the core nucleon a repulsive component in the $l=0$ channel has been included in the $\alpha-n$ potential. This repulsive potential in effect will remove the spurious bound state of $\alpha-n$ two-body system. It is clear from eq. (25), that there will be no bound $l=0$ state, since in this case $\langle \vec{l}.\vec{s} \rangle = 0$ and the potential is repulsive. As evident from eqs (22)–(24) the number of basis states and hence the size of CDE increases rapidly as $K_{\max}$ increases. The truncated set of CDE

$$\left[-\frac{\hbar^2}{2\mu}\left(\frac{\mathrm{d}^2}{\mathrm{d}\rho^2} - \frac{\mathscr{L}_K(\mathscr{L}_K+1)}{\rho^2}\right) - E\right] U_{Kl_{x_1}LS}(\rho)$$
$$+ \sum_{K'=0,2,\ldots.}^{K_{\max}} \sum_{l'_{x_1}\,(\text{allowed})} \sum_{(L'S')=(0,0),(1,1)} \times \langle Kl_{x_1} \mid V(\rho,\Omega_1) \mid K'l'_{x_1}\rangle U_{K'l'_{x_1}L'S'}(\rho) = 0, \quad (26)$$

($l'_{x_1} = 0,2,\ldots$ only for $S=0, L=0$ otherwise $l'_{x_1} = 1,3,\ldots$ for $S=1, L=1$) has been solved by the hyperspherical adiabatic approximation (HAA) [62]. Note that the subscripts l_{y_1} (=l_{x_1}) or l'_{y_1} (=l'_{x_1}) have been suppressed for brevity. The calculated binding energies (BE) for various $K_{\max}$ values have been presented in table 1. We notice that the convergence of BE is rather slow. It is not surprising for the following reason. Although convergence of HHE method is fairly fast for short range potentials [62], it is quite slow for long range potentials [32] and is even slower for extended systems like P_S^- [33,34]. Since the halo wave function extends well outside typical nuclear dimensions, larger K-values are needed to reproduce the tail part of the wave function, making the convergence slow. Since both $\alpha-n$ and $n-n$ interactions are of Gaussian type, we expect that the convergence of BE in HH expansion will follow an exponential pattern [66]. Hence we plot $\log_e(\Delta B_{K_{\max}})$ (where $\Delta B_{K_{\max}} \equiv B_{K_{\max}+2} - B_{K_{\max}}$ and $B_{K_{\max}} = -E$ is the BE for a particular value of $K_{\max}$) against $K_{\max}$ (figure 2). As expected, we get a straight line. From this we calculated the extrapolated BE for higher values of $K_{\max}$. The results are presented in table 2. We stop at a large enough value of $K_{\max}$ such that beyond this value of $K_{\max}$, the increment in BE is smaller than the estimated error in the calculation. This criterion is satisfied for $K_{\max} \sim 80$ and we get the final extrapolated BE as 0.5854 MeV. In this way our computed extrapolated BE is correct to three significant digits. Having obtained the wave function by the HH approach and assuming the same set of two-body interactions some of the observables of the three-body system have been calculated. These include the r.m.s. radius of ^{6}He

$$R_A = \left[\frac{A_c R_c^2 + 2 * R_{\text{halo}}^2}{A_c + 2}\right]^{1/2}, \quad (27)$$

Table 1. Calculated BE and some geometrical observables for different $K_{\max}$ values.

$K_{\max}$	N	BE (MeV)	R_A (fm)	R_{halo} (fm)	$R_{(nn)c}$ (fm)	R_{nn} (fm)	η
12	28	0.4003	3.8290	6.2977	5.4972	6.1452	0.1742
14	36	0.4359	3.8039	6.2520	5.4679	6.0616	0.1752
16	45	0.4644	3.7799	6.2081	5.4392	5.9852	0.1760
18	55	0.4875	3.7587	6.1694	5.4119	5.9236	0.1767
20	66	0.5063	3.7402	6.1356	5.3871	5.8735	0.1771

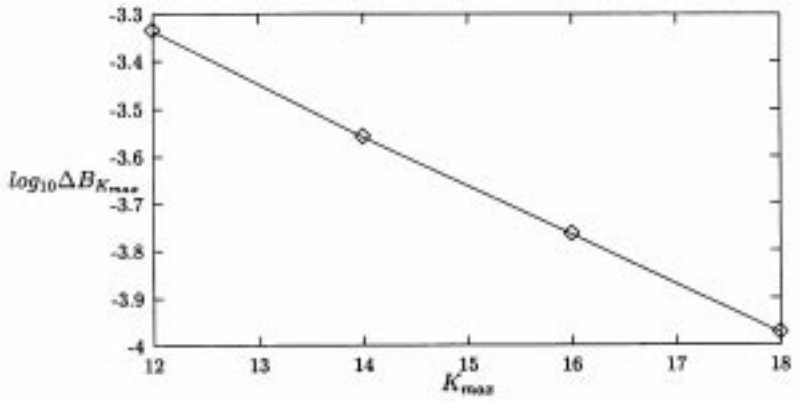

Figure 2. $\mathrm{Log}_e\,(\Delta B_{K_{\max}})$ vs $K_{\max}$ graph.

Table 2. Extrapolated BE for different $K_{\max}$ values.

$K_{\max}$	BE (MeV)	$K_{\max}$	BE (MeV)	$K_{\max}$	BE (MeV)	$K_{\max}$	BE (MeV)
20	0.5062	36	0.5710	52	0.5829	68	0.5850
22	0.5214	38	0.5738	54	0.5834	70	0.5851
24	0.5336	40	0.5760	56	0.5838	72	0.5852
26	0.5436	42	0.5788	58	0.5841	74	0.5853
28	0.5516	44	0.5793	60	0.5844	76	0.5853
30	0.5581	46	0.5805	62	0.5846	78	0.5854
32	0.5633	48	0.5815	64	0.5847	80	0.5854
34	0.5676	50	0.5822	66	0.5849	82	0.5854

where R_{halo}, the r.m.s. halo radius is defined as

$$R_{\mathrm{halo}} = \left[\frac{\langle r_{13}^2 + r_{12}^2\rangle}{2}\right]^{1/2}. \tag{28}$$

(A_c is the mass of the ^{4}He core in the nucleon mass unit and R_c is the charge radius of the core ($= 1.47$ fm).) The expectation value of the observable $\langle r_{13}^2 + r_{12}^2\rangle$ is obtained by the expression

$$\begin{aligned}\langle r_{13}^2 + r_{12}^2\rangle = & \sum_{KK' l_{x_1} LS} \int_0^\infty \rho^2 \mathrm{d}\rho U_{K l_{x_1} LS}(\rho) U_{K' l_{x_1} LS}(\rho) \\ & \times \int_0^{\pi/2} {}^{(2)}P_K^{l_{x_1}, l_{x_1}}(\Phi)\, {}^{(2)}P_{K'}^{l_{x_1}, l_{x_1}}(\Phi) \\ & \times \left[\sqrt{2/3}\,\cos^2\Phi + \sqrt{3/2}\,\sin^2\Phi\right] \cos^2\Phi \sin^2\Phi \mathrm{d}\Phi. \end{aligned} \tag{29}$$

The r.m.s. separation between the valence neutrons (R_{nn}) is given by the expression

$$R_{nn} = \left[\langle r_{23}^2\rangle\right]^{1/2}, \tag{30}$$

where

$$\begin{aligned}\langle r_{23}^2\rangle &= \sqrt{8/3}\sum_{KK'l_{x_1}LS}\int_0^\infty \rho^2 \mathrm{d}\rho U_{Kl_{x_1}LS}(\rho)U_{K'l_{x_1}LS}(\rho) \\ &\times \int_0^{\pi/2} {}^{(2)}P_K^{l_{x_1},l_{x_1}}(\Phi)\ {}^{(2)}P_{K'}^{l_{x_1},l_{x_1}}(\Phi)\ \cos^4\Phi\ \sin^2\Phi \mathrm{d}\Phi.\end{aligned} \tag{31}$$

The r.m.s. separation between the centre of mass of the valence neutrons and the core $(R_{(nn)c})$ is given by the expression

$$R_{(nn)c} = \left[\langle r_{(23)1}^2\rangle\right]^{1/2}, \tag{32}$$

where

$$\begin{aligned}\langle r_{(23)1}^2\rangle &= \sqrt{3/8}\sum_{KK'l_{x_1}LS}\int_0^\infty \rho^2 \mathrm{d}\rho U_{Kl_{x_1}LS}(\rho)U_{K'l_{x_1}LS}(\rho) \\ &\times \int_0^{\pi/2} {}^{(2)}P_K^{l_{x_1},l_{x_1}}(\Phi)\ {}^{(2)}P_{K'}^{l_{x_1},l_{x_1}}(\Phi)\ \cos^2\Phi\ \sin^4\Phi \mathrm{d}\Phi.\end{aligned} \tag{33}$$

The computed values of these observables for various $K_{\max}$ are also shown in table 1. We also calculated the contribution of various orbital angular momenta l_{x_1} to the probability distribution of ^{6}He in its ground state. The larger contribution comes from the lower orbital angular momenta. The $l = 0$ partial waves contribute more than 90% to the probability distribution. The results are presented in table 3. Finally in order to study the correlation among the constituent particles (i.e. the α-core and the valence neutrons) we computed probability density $P(r_{nn}, r_{(nn)c})$, where $r_{nn} = \left(\frac{8}{3}\right)^{1/4} x_1$ and $r_{(nn)c} = \left(\frac{3}{8}\right)^{1/4} y_1$ (see eq. (1)) are respectively the separation between the valence neutrons and the separation of the core from the centre of mass of the valence neutrons. The probability density is defined as the probability of finding the three-body system having definite separations between the constituent particles. This probability density is given by the expression

$$\begin{aligned}P(r_{nn}, r_{(nn)c}) = &\sum_{KK'l_{x_1}LS} U_{Kl_{x_1}LS}(\rho)U_{K'l_{x_1}LS}(\rho)\,{}^{(2)}P_K^{l_{x_1},l_{x_1}}(\Phi) \\ &\times {}^{(2)}P_{K'}^{l_{x_1},l_{x_1}}(\Phi)\cos^2\Phi\sin^2\Phi,\end{aligned} \tag{34}$$

Table 3. Calculated contribution of various orbital angular momenta to the probability distribution of ^{6}He ($l_x = l_{x1} = l_{y1}$).

	Partial probabilities (P_{l_x})				
$K_{\max}$	$l_x = 0$	$l_x = 1$	$l_x = 2$	$l_x = 3$	$l_x = 4$
12	0.9096	0.0157	0.0697	0.0002	0.0044
14	0.9116	0.0158	0.0677	0.0002	0.0043
16	0.9133	0.0158	0.0661	0.0002	0.0041
18	0.9146	0.0158	0.0649	0.0002	0.0040
20	0.9157	0.0158	0.0640	0.0002	0.0039

with

$$\rho = \left[\sqrt{3/8}\, r_{nn}^2 + \sqrt{8/3}\, r_{(nn)c}^2\right]^{1/2} \tag{35}$$

and

$$\Phi = \tan^{-1}\left(\sqrt{\frac{8}{3}}\,\frac{r_{(nn)c}}{r_{nn}}\right). \tag{36}$$

A plot of $P(r_{nn}, r_{(nn)c})$ as a function of r_{nn} and $r_{(nn)c}$ is shown in figure 3. The density plot exhibits two peaks, a di-neutron-like peak where the valence neutrons are located together outside the α-core ($r_{nn} < r_{(nn)c}$) and a cigar-like peak with two valence neutrons located on opposite sides of the α-core ($r_{nn} > r_{(nn)c}$). The latter peak is larger in extension than the former which indicates the possibility of existing anti-correlation among the valence neutrons. This can further be confirmed by computing a correlation coefficient defined as

$$\eta = \left[\left\langle \frac{r_{(nn)c}^2}{\rho^2} \right\rangle\right], \tag{37}$$

where

$$\begin{aligned}\left\langle \tfrac{r_{(nn)c}^2}{\rho^2} \right\rangle &= \sqrt{3/8}\sum_{KK' l_{x_1} LS}\int_0^\infty \mathrm{d}\rho\, U_{K l_{x_1} LS}(\rho) U_{K' l_{x_1} LS}(\rho) \\ &\times \int_0^{\pi/2} {}^{(2)}P_K^{l_{x_1}, l_{x_1}}(\Phi)\, {}^{(2)}P_{K'}^{l_{x_1}, l_{x_1}}(\Phi) \cos^2\Phi \sin^4\Phi \mathrm{d}\Phi.\end{aligned} \tag{38}$$

A small value of this coefficient will indicate that the two valence neutrons are situated on two opposite sides of the α-core (i.e. a cigar shape where the neutrons are anti-correlated), while a large value (≤ 1) will indicate the possibility of *nn* correlation. The computed values of this coefficient for various $K_{\max}$ is shown in the last column of table 1. As the value of η is small ($\sim$0.17), a cigar-like shape is indicated on the average. However figure 3 shows that both the cigar shape and the di-neutron-like structure are probable although the former has a larger possibility. The extrapolated value of the binding energy

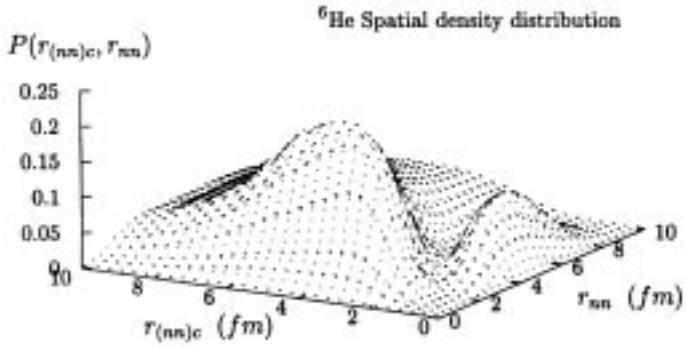

Figure 3. Correlation density plot for the ground state of ^{6}He in the *nn* and $(nn)c$ variables.

is 0.5854 MeV, which is smaller than the experimental value of 0.973±0.04 MeV. However, a similar calculation by Zhukov *et al* [7], using the same set of interaction produced only 0.4 MeV. Since the hyperspherical harmonics expansion method is an essentially exact method for the solution of the three-body problem and the convergence of binding energy with respect to the increase of $K_{\max}$ is governed by the Ritz principle, our calculated value is more reliable. Since the calculated BE is still less than the experimental value and the calculated r.m.s. matter radius (R_A) is larger (see table 1) than the experimental value ($R_{\exp} = 2.57 \pm 0.1$ fm), it appears that the actual $\alpha - n$ interaction in ^{6}He should be stronger than that used here (which was obtained from $\alpha - n$ scattering data [65]). To justify this remark we have slightly adjusted the p-component of the $\alpha - n$ potential to reproduce the experimental binding energy and a reasonable value of the various matter radii. The modified value of the strength of the $l = 1$ component of the $\alpha - n$ potential ($V_0^{l=1}$) is -49.22 MeV. However the modified $\alpha - n$ potential has not been tested for reproduction of $\alpha - n$ scattering data. The modification is purely on phenomenological basis. Since the low energy scattering data are sensitive mainly to the s-component of the potential, we do not modify this component. The small change in the strength of the p-component is likely to have minimal effect on the $\alpha - n$ scattering data. Furthermore the $\alpha - n$ potential for the ^{6}He nucleus will be an effective $\alpha - n$ potential in the presence of the other halo neutron and may be expected to be different from the free $\alpha - n$ interaction. The results are presented in tables 4 and 5. We follow the same technique as used earlier to extrapolate the actually calculated BE for $K_{\max} \leq 20$ (shown in table 4). The extrapolated BE subject to the same criterion as before is 0.9716 MeV. The calculated value of the matter radius R_A is 3.30 fm. Both these values are closer to the experimentally measured values. Thus we conclude that the effective $\alpha - n$ potential is given by eq. (25) with $V_{\alpha n}^{(l)}$ having the values $+50.0$ MeV, -49.22 MeV, -23.0 MeV for $l = 0$, 1 and 2 respectively and $V^{ls} = -5.855$ MeV.

Table 4. Calculated BE and some geometrical observables for different $K_{\max}$ values for modified $\alpha - n$ potential.

$K_{\max}$	N	BE (MeV)	R_A (fm)	R_{halo} (fm)	$R_{(nn)c}$ (fm)	R_{nn} (fm)	η
12	28	0.7491	3.3697	5.4538	4.7764	5.2647	0.1687
14	36	0.7930	3.3483	5.4140	4.7496	5.2013	0.1692
16	45	0.8284	3.3300	5.3801	4.7252	5.1453	0.1698
18	55	0.8568	3.3147	5.3516	4.7045	5.1019	0.1702
20	66	0.8795	3.3027	5.3293	4.6881	5.0685	0.1707

Table 5. Extrapolated BE for different $K_{\max}$ values for modified $\alpha - n$ potential.

$K_{\max}$	BE (MeV)	$K_{\max}$	BE (MeV)	$K_{\max}$	BE (MeV)	$K_{\max}$	BE (MeV)
20	0.8796	36	0.9559	52	0.9690	68	0.9713
22	0.8978	38	0.9591	54	0.9695	70	0.9713
24	0.9124	40	0.9616	56	0.9700	72	0.9714
26	0.9242	42	0.9636	58	0.9703	74	0.9715
28	0.9336	44	0.9652	60	0.9706	76	0.9715
30	0.9411	46	0.9665	62	0.9708	78	0.9716
32	0.9472	48	0.9675	64	0.9710	80	0.9716
34	0.9520	50	0.9683	66	0.9711	82	0.9716

4. Summary and conclusion

We have used the hyperspherical harmonics expansion (HHE) method to calculate the observables of ^{6}He considered as a bound state of $(\alpha+n+n)$. As stated earlier the convergence rate of HHE is rather slow for the binding energy (BE). On the other hand, computational requirements increase enormously as $K_{\max}$ increases. To solve this problem, we utilized the hyperspherical convergence formula, using the calculated BE for $K_{\max} \leq 20$. This then permits the extrapolation of the BE to larger $K_{\max}$ values, without actually solving the large set of coupled differential equations. By this process we achieve convergence up to the error in numerical calculations (estimated to be $\sim$0.001 MeV). With the $\alpha-n$ potential chosen as that used by Zhukov *et al*, we find that our extrapolated BE is larger (0.5854 MeV) than that calculated by Zhukov *et al* (0.40 MeV), both of which are less than the experimental value (0.973 ± 0.04 MeV). The calculated r.m.s. matter radius is about 45% more than the experimental value. We have next adjusted the $l=1$ component of the $\alpha-n$ potential slightly to get the BE within experimental error limits.

Thus we conclude that the effective $\alpha-n$ potential in ^{6}He is that given by ref. [64], (see eq. (25)) with $V_{\alpha n}^{(l=1)} = -49.22$ MeV.

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