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Neutrino anomaly and ν-nucleus interactions

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Abstract. A review of various calculations of the inclusive quasi-elastic reactions and pion production processes in neutrino reactions for various nuclei at intermediate energies relevant to solar, atmospheric and accelerator neutrinos is presented

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1. Introduction

The neutrino anomaly generally refers to the solar neutrino problem where the observed number of electron type neutrinos from the sun was found to be considerably smaller than the number predicted in standard model of particle interactions [1–2]. Similar depletion of muon type neutrinos is found in the flux of atmospheric neutrinos as observed in many experiments [3]. These experimental results on electron and muon type neutrinos are not understood theoretically but the most likely explanation seems to be the flavour oscillation of neutrinos. In this phenomenon the neutrinos of one flavour oscillate into the neutrinos of another flavour during their passage through the medium. There seems to be some evidence for this type of oscillation in an accelerator based experiment where some increase in the electron type neutrino events is observed which are supposed to come from $\nu_\mu \rightarrow \nu_e$ oscillation in ν_μ beam [4]. The medium effects also tend to increase this oscillation in some favourable conditions [5]. The physics of neutrino oscillations is very interesting and may lead us to various new processes beyond the standard model physics [6].

The interpretation of these experiments [1–4] using standard model physics involves neutrino nucleus interactions in the domain of low and intermediate energy. The nuclear physics inputs mainly enter through two types of processes. These are:

A. The nuclear processes responsible for neutrino production in the calculation of solar and atmospheric neutrino fluxes.
B. The nuclear processes in which neutrinos are detected through its neutral and charged current reaction signatures in nuclei.

In the following we discuss the neutrino nucleus interactions which are relevant in the detection processes in some detail and mention only briefly the nuclear processes involved in the production process.

2. Nuclear processes in neutrino production

Solar neutrinos are produced through a series of electroweak and strong interaction processes in the solar interior. The neutrinos which are responsible for the observation of solar neutrino fluxes in various experiments mainly come from the end reactions of pp chains which are listed below with the corresponding energies of neutrinos. The neutrinos produced in other reactions are not very important in the solar neutrino oscillation experiments either due to small neutrino flux or due to small reaction cross section [7].

		$E_{\rm max}$ (MeV)
$p+p \rightarrow d+e^{+}+\nu_e$	ppI chain	0.423
$e^{-}+ {}^{7}\mathrm{Be} \rightarrow {}^{7}\mathrm{Li}+\nu_e$	ppII chain	0.3885 (monoenergetic) 0.8631
${}^{8}\mathrm{B} \rightarrow {}^{8}\mathrm{Be}^{*}+e^{+}+\nu_e$	ppIII chain	~15

In the following, we briefly discuss the status of nuclear reactions relevant to the present experiments on solar neutrinos [1–4].

In Homestake experiment [1], where the reaction is $\nu_e+ {}^{37}\mathrm{Cl} \rightarrow e^{-}+ {}^{37}\mathrm{Ar}$ $(Q = 0.81$ MeV), ^{8}B neutrinos contribute about 77% of the events due to the high value of the reaction threshold. In KAM and SK [2] experiments, there is an experimental cut off of about 6.5 MeV on the measurement of electrons energies, in the detection reaction $\nu_e + e^{-} \rightarrow \nu_e+e^{-}$, and here again only ^{8}B neutrinos contribute. The situation is similar in the ongoing SNO experiments with D_2O target [8] where the detection reactions are $\nu+d \rightarrow e^{-}+p+p$ and $\nu + d \rightarrow \nu + n + p$. Therefore, in these reactions, the uncertainty in the determination of the flux of ^{8}B is quite important. In the production mechanism of ^{8}B neutrinos most of the electroweak and strong reactions are well understood except for the reaction $p + {}^{7}\mathrm{Be} \rightarrow {}^{8}\mathrm{B} + \gamma$ where the uncertainties in the reaction rate is as high as 8%, compared with $< 1\%$ uncertainty in the reaction $p + p \rightarrow d + e^{+} + \nu_e$. This has led to a reexamination of this reaction both theoretically and experimentally. This is one of the major activities in studying the nuclear reactions of astrophysical interest. It should be mentioned that the calculation of $e^{-}+ {}^{7}\mathrm{Be} \rightarrow {}^{7}\mathrm{Li} + \nu_e$ is also old and needs reexamination.

In SAGE and GALLEX experiments [3], the neutrino events are observed using reaction $\nu + {}^{71}\mathrm{Ga} \rightarrow e^{-}+{}^{71}\mathrm{Ge}$, where ^{8}B neutrinos contribute 10% while pp and Be neutrino contribute about 27% and 50% respectively. They are therefore relatively free from uncertainties in reaction $p + {}^{7}\mathrm{Be} \rightarrow {}^{8}\mathrm{B} + \gamma$ rate. However, there is some uncertainty related to the transition leading to higher states in the final nucleus ^{71}Ge.

In the case of atmospheric neutrinos, which are produced by the decays of pions and kaons which are produced in the proton–nucleus and nucleus–nucleus collisions, the uncertainties in these calculations correspond to [9]:

(i) initial uncertainties in the proton flux in cosmic rays,
(ii) lack of knowledge in multiplicity and energy spectrum of particle production (like pions and kaons) in the proton nucleus and nucleus–nucleus collisions.

These uncertainties affect the absolute fluxes and not the ratios of ν_e and ν_μ fluxes. Most of the calculations show that while there is an uncertainty of about 20% in the absolute fluxes, the uncertainty in the ratio of ν_e to ν_μ fluxes is less than 5% [9].

3. Neutrino processes in neutrino detection

There are mainly two types of nuclear processes which have been used in detection of ν-fluxes which are described in the following:

3.1 *Exclusive reactions*

In Homestake [1], and SAGE and GALLEX [2] experiments, the low energy solar neutrinos excite definite nuclear states in the final states. The exclusive reactions are

$$\nu + {}^{37}\mathrm{Cl} \rightarrow e^- + {}^{37}\mathrm{Ar}, \tag{1}$$

$$\nu + {}^{71}\mathrm{Ga} \rightarrow e^- + {}^{71}\mathrm{Ge}. \tag{2}$$

The ^{37}Cl$(\nu, e^-)^{37}$Ar interaction rate is well understood. In the case of, ^{71}Ga(ν, e^-) ^{71}Ge, the gs–gs transition is well understood but there are some uncertainties in the calculation of Gamow Teller strength to the excitation of $3/2^-$ and $5/2^-$ states in ^{71}Ge.

In the ongoing experiments of ICARUS [10] and SNO [8], the exclusive reactions are

	E_{th} (MeV)
$\nu_e + {}^{40}\mathrm{Ar} \rightarrow e^+ + {}^{40}\mathrm{K}^*$	5.885
$\nu_e + d \rightarrow \nu_e + n + p$	2.25
$\nu_e + d \rightarrow e^- + p + p$	1.44

All these reactions are well understood and theoretical uncertainties are less than 2.5%.

3.2 *Inclusive reactions*

When the incident neutrino energies are large as in the case of atmospheric neutrinos many nuclear states are excited and particle production processes like pion production are also possible. In this case only leptons and/or pions are measured as is done in the experiments of IMB, KAM and SK. In these experiments, the following reactions are involved:

$$\nu_e(\bar{\nu_e}) +^{16}\mathrm{O} \rightarrow e^-(e^+) + X, \tag{3}$$

$$\nu_\mu(\bar{\nu_\mu}) +^{16}\mathrm{O} \rightarrow \mu^-(\mu^+) + X, \tag{4}$$

$$\nu_e(\bar{\nu_e}) +^{16}\mathrm{O} \rightarrow e^-(e^+) + \pi + X, \tag{5}$$

$$\nu_\mu(\bar{\nu_\mu}) +^{16}\mathrm{O} \rightarrow \mu^-(\mu^+) + \pi + X. \tag{6}$$

In the case of pion production, neutral current reactions are also possible where various charged pion states can be produced in the nucleus. In the following we discuss the inclusive quasielastic and inclusive inelastic reactions.

3.2.1 *Quasi-elastic reactions*: The quasi-elastic reactions $\nu_e + n \to e^- + p$ and $\nu_\mu + n \to \mu^- + p$ taking place in nuclei dominate the fully contained events in the detectors of IMB, KAM and SK experiments. The $\nu_e + n \to e^- + p$ is the reaction which enters in the neutrino oscillation experiment of LSND. In this experiment the reaction $\nu_\mu + n \to \mu^- + p$ is used to calibrate the LSND, decay in flight (DIF) flux of muon and electron type of neutrinos. Using the flux of ν_e, the excess events in the reaction $\nu_e + n \to e^- + p$ are observed and are attributed to the $\nu_\mu \to \nu_e$ oscillation phenomenon. Therefore, in the context of fully contained events in IMB, KM, SK experiments and the excess events in LSND experiment, the inclusive cross section for intermediate energy neutrinos in ^{16}O and ^{12}C are highly desirable. Traditionally, a theoretical calculation done in a nonrelativistic Fermi gas model [11] has been used in the Monte Carlo simulations of the expected events in these experiments. While this approach is expected to give reliable results at higher energies, its predictions overstimate the cross sections at intermediate energies in the range of a few hundred MeV. For example, some recent calculation done for the ^{12}C target relevant to the LSND experiment overstimates the experimental cross section by 25%. Keeping in mind the fact that the atmospheric neutrino spectrum is peaked in the lower part of the spectrum, this method may overstimate the cross sections for the atmospheric neutrino case as well. In order to understand the uncertainty due to the nuclear structure effect, inclusive cross section calculations using other methods have been performed. In this context we would like to quote the calculations of Kolbe *et al* [12,13], Auerbach *et al* [14] but in this talk I will describe the calculations done by us [15] and compare them with the results in other calculations. Our method is an improvement over the Fermi gas model and takes into account the effect of the nuclear medium through the excitation and propagation of the particle-hole and Δ-hole correlations. In this model, the total cross section is then written as:

$$\begin{aligned}\sigma(E_\nu) = {} & -\frac{2G_F^2\cos^2\theta_c}{\pi}\int_{r_{\min}}^{r_{\max}} r^2\,\mathrm{d}r\int_{p_l^{\min}}^{p_l^{\max}} p_l^2\,\mathrm{d}p_l \\ & \times\int_{-1}^{1}\mathrm{d}(\cos\theta)\,\frac{1}{E_\nu E_l}\bar{\sum}\sum|T|^2 \\ & \times\,\mathrm{Im}U[E_\nu - E_l - Q + Q' - V_c(\vec{r}),\vec{q}\,]\,\Theta[E_l + V_c(r) - m_l],\end{aligned} \tag{7}$$

where Q is the Q value of the reaction, $Q' = E_{F_n} - E_{F_p}$ is to take into account the unequal Fermi sea in the case of $N \neq Z$ nuclei like ^{40}Ar, $V_c(\vec{r})$ is the Coulomb energy of the outgoing charged lepton and U is the Lindhard function corresponding to the particle hole excitation as shown in figure 1. $|T|^2$ is the square of the matrix elements which contains the bilinear terms in the weak coupling constants like F_A^2, $F_A F_M$ etc. which are renormalized in the medium due to the presence of strongly interacting nucleons as shown in figure 2. To take into account the effect of Δ-hole excitations, we replace the Lindhard function U by $U_N + U_\Delta$, where U_Δ is the Lindhard function corresponding to Δ-hole excitations. We present in table 1 the results for the inclusive cross sections for the reaction in the LSND experiment which is the calibration reaction for observing the excess events in the (ν_e, e^-) reaction along with other results.

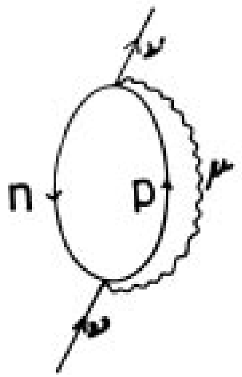

Figure 1. Feynman diagram for the neutrino induced self-energy related to $\nu n \to l^- p$ process in nuclei.

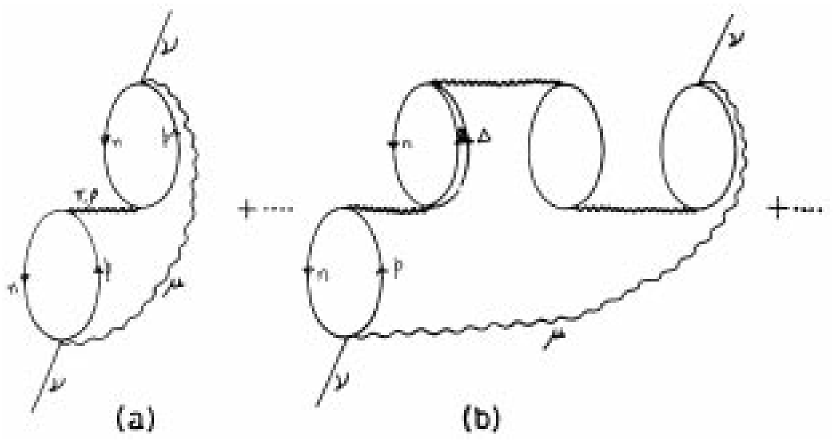

Figure 2. Many-body Feynman diagrams accounting for the medium polarization in the spin-isospin channel driven by $\nu n \to l^- p$ transition.

Table 1. Summary of the flux averaged cross sections. Experimental results are inferred by adding ground state and excited state contributions for (ν_e, e^-). Theoretical results are from Kolbe *et al*, Auerbach *et al*, Umino *et al*, and this work. The units are $10^{-42}\mathrm{cm}^2$ for (ν_e, e^-) and $10^{-40}\mathrm{cm}^2$ for (ν_μ, μ^-) cross sections.

	Kolbe *et al* [12]	Auerbach *et al* [14]	Our work [15]	Experiment
$\sigma(\nu_e)$	15.6	12.9–22.7	14.8 ± 1.13	14.8 ± 1.0 ± 1.5 15.2 ± 1.4 ± 1.8
σ_{ν_μ}	19.3–20.3	13.5–15.2	16.65 ± 1.37	11.20 ± 0.3 ± 1.8

Table 2. Summary of the ratios of flux-averaged cross sections. In columns below LDA and RPA stand for the local density approximation and random phase approximation results as compared to the Fermi gas calculations quoted by Engel *et al*. In the RPA results the ratio corresponds to the momentum spectrum over the entire range of lepton momentum.

	LDA	RPA-1	RPA-2	RPA-3
ν_e-like	0.76	0.85 – 0.89	0.84 – 1.02	0.84 – 1.07
ν_μ-like	0.80	0.87 – 0.89	0.86 – 1.02	0.84 – 0.99
Ratio	1.05	1.02	1.02	1.01

We see here a considerably large uncertainty in the inclusive (ν_μ, μ^-) reactions due to nuclear structure effects. Note that this discrepancy is over and above all the theoretical and experimental uncertainties. We present in table 2 results for the atmospheric neutrino experiment of KAMIOKANDE corresponding to the spectrum of Honda *et al* [16]. We see here also that the nuclear uncertainties are quite large in the individual reactions induced by neutrinos and amounting to 20% as reflected in the total cross sections calculated in the various approaches. However, when the ratios of these cross sections are taken this uncertainty is almost eliminated and the various approaches give ratios which agree with each other within 5%. We also present in figure 3, the effect of nuclear medium in the lepton yields for the inclusive reaction on ^{40}Ar relevant to the ICARUS experiment [17]. The results for ^{12}C and ^{16}O are quite similar.

3.3 Δ *excitation and pion production*

Assuming that the pion production is dominated by the Δ production, we estimate here the Δ production cross section. The production cross section is written as

$$\begin{aligned}\sigma(q^2, E) = \int \frac{\mathrm{d}\mathbf{r}}{4\pi^3}\frac{k'}{E_\nu}&\frac{\frac{\tilde{\Gamma}}{2} - \mathrm{Im}\Sigma_\Delta}{(W - M' - \mathrm{Re}\Sigma_\Delta)^2 + (\frac{\tilde{\Gamma}}{2} - \mathrm{Im}\Sigma_\Delta)^2}\\ &\times(\rho_n(\vec{r}) + \rho_p(\vec{r}))L_{\alpha\beta}J^{\alpha\beta},\end{aligned} \tag{8}$$

where

$$L_{\alpha\beta} = k_\alpha k'_\beta + k'_\alpha k_\beta + g_{\alpha\beta}k.k' + i\epsilon_{\alpha\beta\gamma\delta}k^\gamma k'^\delta,$$

where $\rho_n(\vec{r})$ and $\rho_p(\vec{r})$ are the neutrons and proton densities of the nucleus, $J^{\alpha\beta} = J^{\alpha\dagger}J^\beta$ and J^α describes the matrix elements for the N-Δ transition current, the detailed expression of which is given in [18]. Transition form factors for N-Δ transitions are determined from the electroproduction and neutrino production Δ resonance. For antineutrino reactions, we replace $\rho_n \to \rho_p$ and change the sign of the antisymmetric term in the leptonic tensor $L_{\alpha\beta}$.

In eq. (8), Σ_Δ describes the disappearence of Δ through ΔN-NN process while $\tilde{\Gamma}$ describes the pion produced by the remaining Δ's. These pions once produced inside the

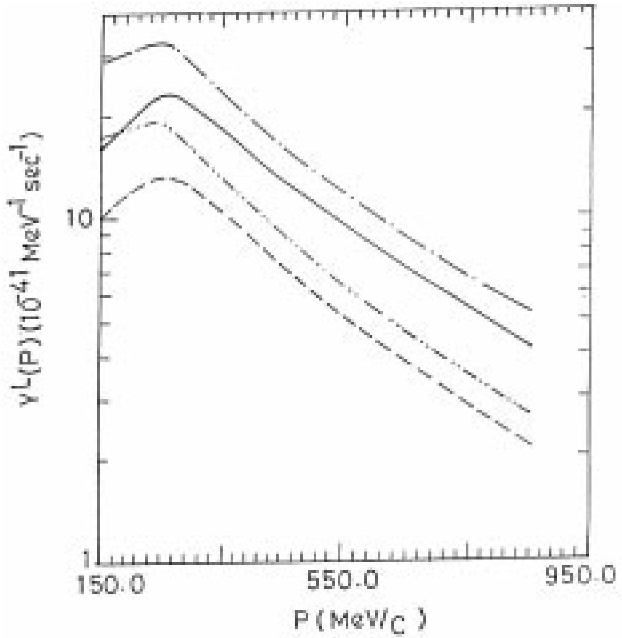

Figure 3. The charged lepton yield $Y^l(p)$ in the Fermi gas model (solid-dotted line) and in the present model (solid line) for $\nu_\mu + \bar{\nu}_\mu$ reactions. For $\nu_e + \bar{\nu}_e$ reactions $Y^l(p)$ is shown by a dashed-double dotted line in the Fermi gas model and by a dashed line in the present model.

nucleus suffer rescattering and absorption on their way out. This absorption results in the decrease of pions produced and is calculated using an eikonal approximation. Equation (8) is obtained by replacing Γ by $\tilde{\Gamma} - 2\ \text{Im}\Sigma_\Delta$ and $M' \to M + \text{Re}\ \Sigma_\Delta$ in the expression for the cross section for free Δ production. Thus the nuclear medium effects are contained in the terms $\tilde{\Gamma}$, $\text{Im}\Sigma_\Delta$ and $\text{Re}\Sigma_\Delta$.

We now summarize, the effect of nuclear structure on the pion production at 500 and 750 MeVs shown in figure 4. We find that the reduction in the Δ production cross section due to nuclear effects is about 20%. About 80% of these decay into pions out of which about 20% are absorbed by the nuclear medium. This results in the total reduction of the pion production by a factor 2 which is a large effect compared to the free production of the pions. However when the ratio of produced pions induced by ν_e and ν_μ are taken, these large effects cancel out and the ratio is about the same as it is for the free case.

In addition to the pion production mechanism discussed above, there is additional production of pions through coherent processes in nuclei, where the final nucleus stays in the ground state. These pions produced in the coherent processes are mainly produced along the forward angles. The nuclear medium effects on the coherent processes have been studied using nonrelativistic approximations by Kelkar *et al* [19] and found to be quite large in the case of ^{16}O. In figure 5 we show the numerical results where we also show that the nuclear medium effects lead to large reduction by a factor 3 to 4 in various kinematic regions of the lepton energy. These results are in qualitative agreement with the results of Kim *et al* [20] calculations using relativistic mean field theory.

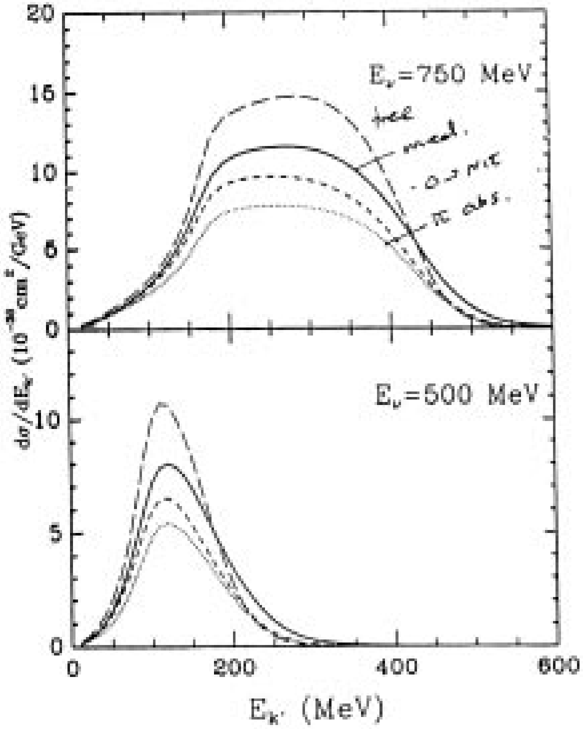

Figure 4. $(\mathrm{d}\sigma/\mathrm{d}E_{k'}) \sim E'_k$ for (ν_e, e^-) reaction on ^{16}O through Δ excitation (i) without medium effects (long-dashed); (ii) with medium effects (solid); (iii) pion production with medium effects and without absorption (short-dashed); (iv) pion production with medium effects and with absorption (dotted).

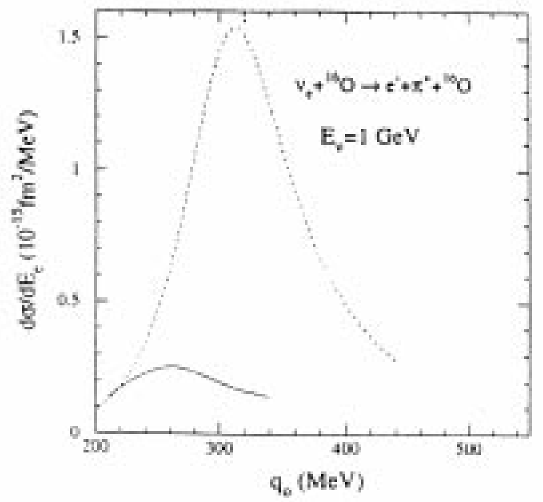

Figure 5. Energy spectrum of coherent pions produced on ^{16}O with neutrinos of beam energy 1 GeV integrated over the electron solid angle.

4. Conclusions

We conclude that

i) The nuclear effects in the processes responsible for the production of ν_e in solar neutrinos are well understood except for the reaction $p + {}^7\text{Be} \rightarrow {}^8\text{B} + \gamma$ which contribute to ^{8}B neutrinos.

ii) There are about 20% uncertainties in the absolute fluxes of ν_e and ν_μ coming due to the various nuclear processes involved in the production, but the ratios of ν_e and ν_μ fluxes are fairly constant in various approaches.

iii) The exclusive nuclear reactions relevant to the detection of solar neutrinos are well understood.

iv) The nuclear effects are quite large in the inclusive quasi-elastic neutrino reactions relevant to the detection of atmospheric neutrinos at the intermediate energies. These effects are even larger in the case of coherent as well as incoherent pion production. These nuclear effects, cancel when the ratio of ν_e and ν_μ induced reaction cross sections are taken. However if absolute cross sections are utilized as in the case of the LSND experiment to determine the excess ν_e induced events or in some of the planned long base line experiments to determine the flux through the observation of the neutral currents then the nuclear effects on these cross sections should be taken into account.

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