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Inclusion of temperature dependent shell corrections in Landau theory for hot rotating nuclei

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Abstract. Landau theory used for studying hot rotating nuclei usually uses zero temperature Strutinsky smoothed total energy for the temperature dependent shell corrections. This is replaced in this work by the temperature dependent Strutinsky smoothed free energy. Our results show that this replacement has only marginal effect for temperatures greater than 1 MeV but plays significant role at lower temperatures.

Keywords. Hot rotating nuclei; Landau theory; temperature dependent shell corrections.

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1. Introduction

Hot rotating nuclei are formed in heavy ion fusion reactions where transfer of energy and angular momentum of the relative motion excites the compound nucleus. The compound nucleus decays through particle and γ-ray emission. The properties of the hot rotating nuclei are studied usually by observing these decay patterns. One of the main expected prospect of using recently developed multi detector arrays is the more accurate study of shape transitions in hot rotating nuclei.

Theoretically the hot rotating nuclei can be studied using the Landau theory of phase transitions following Alhassid *et al* [1]. Here one expands the free energy in terms of certain temperature dependent constants which are to be extracted by fitting with the microscopic free energy calculations at fixed temperatures by the Strutinsky's method. The Strutinsky's method of shell corrections [2–4] has been successfully used in calculations of the nuclear deformation energy, with the concept of dividing the total nuclear binding energy into a smooth liquid-drop energy $E_{\rm LDM}$ and an oscillating shell correction energy δF. In the evaluation of free energy for hot nuclei, it is usual to employ the cold-nucleus approximation [1,5–7] i.e. $\delta F = (E - TS) - \widetilde{E}(T = 0)$, for obtaining the temperature dependent shell correction. This may be a good approximation which has been used for studying the shape transitions in s-d shell nuclei [6] and recently in strontium and zirconium isotopes [7]. However, it is possible to evaluate shell corrections at finite temperatures more accurately [8,9] i.e. $\delta F = (E - TS) - (\widetilde{E} - T\widetilde{S})$. In this paper we will focus on the method which we use to evaluate the shell corrections at finite temperature and the consequences of incorporating this method in our calculations.

2. Shell corrections at finite temperature

By the Strutinsky's prescription [2] the total energy is written as the sum of the liquid drop energy and the shell correction

$$F_{\rm TOT} = E_{\rm LDM} + \sum_{p,n} \delta F \ . \tag{1}$$

The shell correction is given in terms of the single particle level density as [3,4]

$$\delta F = F - \widetilde{F} = \int_{-\infty}^{\lambda} e\ g(e)\ {\rm d}e - \int_{-\infty}^{\widetilde{\lambda}} e\ \widetilde{g}(e)\ {\rm d}e, \tag{2}$$

where $g(e) = \frac{{\rm d}\mathcal{N}(e)}{{\rm d}e}$ is the single particle level density and $\mathcal{N}(e)$ is the total number of particles that can be accommodated by the energy levels with their energy $\leq e$. λ and $\widetilde{\lambda}$ are the chemical potentials corresponding to the discrete and smooth single particle distributions respectively. λ and $\widetilde{\lambda}$ can be calculated using the constrains $\mathcal{N}(\lambda) = N$ and $\widetilde{\mathcal{N}}(\widetilde{\lambda}) = N$ respectively. We can also write

$$g(e) = \left.\frac{{\rm d}\mathcal{N}(\lambda)}{{\rm d}\lambda}\right|_{\lambda=e} = \sum_{i=1}^{\infty} \left.\frac{{\rm d}n_i(e_i,\lambda)}{{\rm d}\lambda}\right|_{\lambda=e} \ . \tag{3}$$

At finite temperatures the occupation probability follows Fermi-Dirac distribution given by

$$n_i(e_i,\lambda) = \frac{1}{1+\exp\left(\frac{e_i-\lambda}{T}\right)} \ . \tag{4}$$

The single particle energies e_i have been obtained by diagonalizing the triaxial Nilsson Hamiltonian in cylindrical representation up to $N = 11$ shells [10]. From eqs (3) and (4), we can write the temperature dependent single particle level density as

$$g(e) = \sum_{i=1}^{\infty} \frac{1}{4T\cosh^2[(e-e_i)/2T]} \ . \tag{5}$$

The natural way of applying Strutinsky averaging [8] to the level density is to convolute $g(e)$ with the averaging function.

$$\widetilde{g}(e) = \frac{1}{\gamma}\int_{-\infty}^{\infty} \widetilde{f}\left(\frac{e-e'}{\gamma}\right) g(e')\ {\rm d}e' \ . \tag{6}$$

We use the averaging function

$$\widetilde{f}(x) = \frac{1}{\sqrt{\pi}}\exp(-x^2)\sum_{m=0}^{p} C_m H_m(x) \ , \tag{7}$$

where

$$C_m = \begin{cases} \frac{(-1)^{m/2}}{2^m (m/2)!} & \text{if } m \text{ is even} \\ 0 & \text{if } m \text{ is odd} \end{cases} ,$$

$x = (e - e_i)/\gamma$, γ is the smearing parameter satisfying the plateau condition $(\mathrm{d}\widetilde{F}/\mathrm{d}\gamma) = 0$, $p = 6$, is the order of smearing and H_m are the Hermite polynomials.

At finite temperatures the solution for the integral F in eq. (2) is given by

$$F = \sum_{i=1}^{\infty} e_i n_i - T \sum_{i=1}^{\infty} s_i \,, \tag{8}$$

where $s_i = -\left[n_i \ln n_i - (1 - n_i) \ln(1 - n_i)\right]$. Substituting eq. (6) in the expression for $\widetilde{F}$ in eq. (2), and utilizing the plateau condition and eq. (8), we have

$$\widetilde{F} = \sum_{i=1}^{\infty} e_i \widetilde{n}_i - T \sum_{i=1}^{\infty} \widetilde{s}_i \,, \tag{9}$$

where

$$\widetilde{n}_i = \frac{1}{\gamma} \int_{-\infty}^{\infty} \widetilde{f} \left(\frac{e - e_i}{\gamma} \right) \, n_i(e, \widetilde{\lambda}) \, \mathrm{d}e \,, \tag{10}$$

$$\widetilde{s}_i = \frac{1}{\gamma} \int_{-\infty}^{\infty} \widetilde{f} \left(\frac{e - e_i}{\gamma} \right) \, s_i(e, \widetilde{\lambda}) \, \mathrm{d}e \,. \tag{11}$$

The above integrals have been evaluated numerically. Apart from this the present method gives the free energy with exact shell correction. Using this free energy and the temperature dependent moment of inertia [7], the parameters of the extended Landau theory [5,7] have been calculated. The free energy at fixed spins have been calculated within the framework of the extended Landau theory. Thermal fluctuations have been included in the usual way [7].

3. Role of exact shell corrections in Landau theory

We have studied in this work the role of exact shell corrections in the shape evolution of Zr isotopes with $80 \leq A \leq 90$ as a function of spin and temperature. The results obtained for ^{80}Zr with the shell corrections calculated using different methods namely (a) cold nucleus approximation method and (b) the numerically exact method, for temperatures 0.5 MeV and 1.0 MeV are shown in figures 1 and 2 respectively. It is seen from the figures that the results yielded by the two methods differ marginally when the temperature is 1 MeV or greater. At $T = 1$ MeV, even though the most probable shapes may differ with the method used, such differences are suppressed by thermal fluctuations. However, when the temperature is 0.5 MeV, the results differ significantly. This scenario can be understood as a consequence of the fact that at low excitations the shell effects are predominant and have to be treated correctly. It is hence to be concluded that the cold nucleus approximation for extracting shell corrections may be good for temperatures $T \gtrsim 1$ MeV but not for lower temperatures.

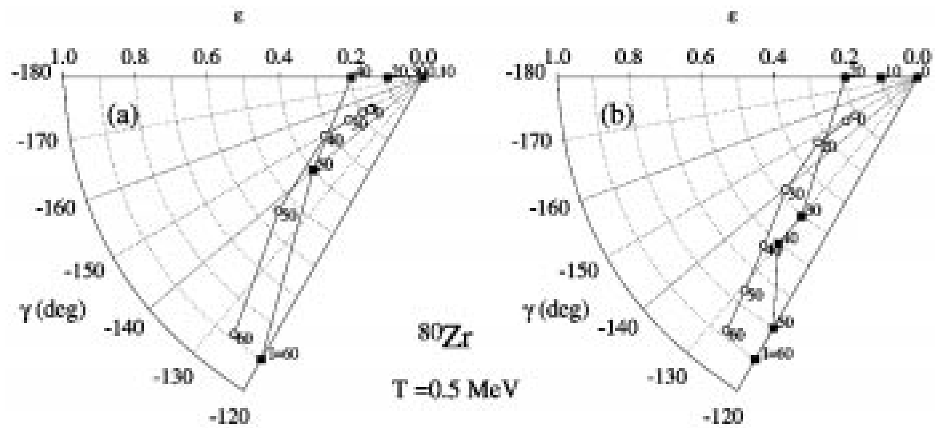

Figure 1. The shape evolution of ^{80}Zr at temperature 0.5 MeV as a function of spin by different methods (see text). The open circles represent averaged shapes and the filled squares represent most probable shapes. For shape conventions see ref. [11].

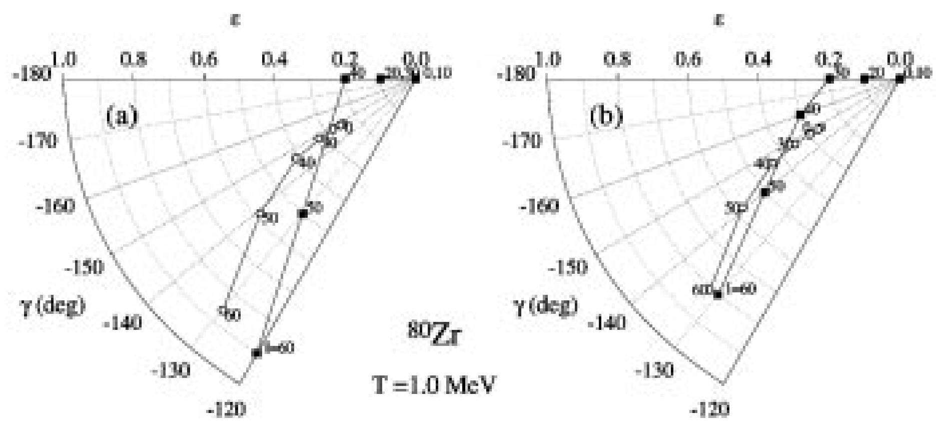

Figure 2. Same as figure 1 but at temperature 1.0 MeV.

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