Decay energy of ^{55}Fe from its inner Bremsstrahlung spectrum

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Abstract. Several measurements of decay energy using the inner Bremsstrahlung spectrum (IB) due to radiative electron capture in ^{55}Fe has been made. But the results are not uniform. Hence another attempt has been made at the same. Experimental data was obtained with a 4.445 cm. dia × 5.08 cm thick NaI (Tl) detector. It was subjected to suitable statistical treatment and various corrections using Liden and Starfelt procedure. The corrected spectrum agrees well with the Glauber and Martin theory for $1s$ electron capture beyond 100 keV. From the Jauch plot, the decay energy of 232.36±0.64 keV was obtained.

Keywords. Beta decay; radiative electron capture; inner Bremsstrahlung.

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1. Introduction

The inner Bremsstrahlung (IB) with reference to electron capture radioactive decay is a higher order process in which the orbital electron capture process will be accompanied by the emission of a continuous spectrum of electromagnetic radiation. This process is also called as 'radiative electron capture'. The energy spectrum is expected to have a characteristic shape and extend up to the energy, which is given as the difference of the decay energy and the binding energy of the orbital electron undergoing capture by the nucleus. The probability of radiative capture is expected to be 10,000 times less than the probability for the electron capture process. Theoretical calculation of the anticipated IB intensities was made by Morrison and Schiff (MS) [1]. They predicted a spectrum of the type $x(1-x)^2$ where x is the ratio of gamma ray energy to the energy released in the reaction. However the experimentally observed spectra showed much higher intensities of IB photons at lower energies than the predictions of MS theory. Glauber and Martin (GM) [2] showed through a non relativistic and approximated relativistic treatment of electrons, that the IB from the $1s$ or $2s$ electrons have the form $x(1-x)^2$ whereas the IB resulting from the $2p$ electrons

has higher intensities at low energies. GM theory also demonstrated the importance of the Coulomb effect. Martin and Glauber (MG) [3] worked out a theory taking into account the relativistic effects and all the electrostatic effects around the nucleus.

^{55}Fe decays by electron capture process with a half-life of 2.7 years. The transition takes place from $(3/2)^-$ state to $(5/2)^-$ ground state of ^{55}Mn. Bradt *et al* [4] were first to observe IB from ^{55}Fe. They used Geiger Muller counter for their experiment. Maeder and Preiswerk [5], Bell *et al* [6], Madansky and Rassetti [7], Emmerich *et al* [8], Berenyi *et al* [9] and Raj and Ramaswamy [10] measured the decay energy using NaI(Tl) crystal detector. The results were widely varying, giving the decay energy Q_{ec} from 150 keV to 248 keV. Michalowicz [11], Saraf [12] and Biavati [13] measured IB associated with $1s$ electron capture in ^{55}Fe using coincidence technique. Their measurement of the decay energies too differed from each other. Most recently Isaac, Vanin and Helene [14] measured IBEC spectrum of ^{55}Fe using a HPGe detector. The authors have adopted the response matrix method in the unfolding of the gamma spectrum. (In the present work Liden and Starfelt [15] method is used.) The HPGe detectors give high resolution but low counting efficiency as against the NaI(Tl) detectors, which give low resolution, but high counting efficiency. The decay energy obtained by them was in closer agreement with the mass difference value given by Wapstra and Audi [16]. The present work is a measurement of total IB spectrum using 4.445 cm dia × 5.08 cm thick NaI (Tl) crystal. The advantage of large crystal being its higher photo peak efficiency and hence the lesser Compton electron distribution. The ^{55}Fe does not emit any mono energetic gamma rays and the atomic x-rays are of very low energy and most of it will be absorbed by the aluminium covering of the detector.

2. Experimental details and spectral unfolding

A 10 μCi carrier free ^{55}Fe source was obtained from Bhabha Atomic Research Centre, Mumbai. The source was spread over a circular area of 1 cm diameter on a thin mylar foil and mounted on a circular plastic ring holder of outer diameter 2.5 cms. The source was calibrated at the Bhabha Atomic Research Centre, Mumbai. A NaI (Tl) crystal of size 4.445 cm dia × 5.08 cm thick was used as a detector coupled with the RCA 8053 photo multiplier tube, preamplifier, amplifier and EG&G ORTEC Model 7150 multi channel analyser. The activity at the time of conducting the experiment was 9.18 μCi.

The room temperature was maintained at $23°$C by air conditioning in order to maintain the stability of the electronic circuits. An un-interrupted stabilized power supply was used in order to keep the high tension voltage supplied to the photo multiplier tube constant. The 'source' was tested for the presence of β impurities using a beta ray spectrometer and found to be free from contamination. The measured τ spectrum in the present study did not show the presence of any gamma emitting impurity. The spectrometer was calibrated using the gamma line energy sources (^{132}Ba – 32 keV, ^{125}I – 35.5 keV, ^{170}Tm – 84 keV, ^{57}Co – 122 keV, ^{141}Ce – 145 keV, ^{203}Hg – 280 keV, ^{22}Na – 511 keV and ^{137}Cs – 662 keV). The relationship between spectrometer channel numbers and the energy peak positions was found to be linear in the region of interest. The energy calibration and the linearity being checked *before* and *after* each trial *run*. The *peak* to *total* ratios (PTR) for the various gamma lines were also determined using these sources. The values of PTR's at unknown energies were obtained from graphical interpolation and these values were used

in the spectrum unfolding. These peaks were also used to evaluate the energy resolution of the detector system by the measurement of the respective half widths. The peaks were approximated to be Gaussian. The energy resolution at energies other than known peaks was obtained by graphical interpolation. The resolving time of the detector system was measured using the pile-up counting method. i.e., if a gamma line appears at energy E (photo-peak), then a pile-up peak appears at energy $2E$. The magnitude of these pile up counts is given by

$$N(2E) = 2tN(E)/(1 - 2tN(E)),$$

where $2t$ is the resolving time of the detector (the minimum time interval required between two consecutive radiation events in order to recognize them as separate events), $N(2E)$ being the counts recorded at the energy $2E$ and $N(E)$ being the counts recorded at the energy E. For small/moderate count rates, $2tN(E) \ll 1$ and $N(2E) \approx 2tN(E)$. In the present experiment, gamma line energy sources like, ^{57}Co, ^{203}Hg and ^{137}Cs of moderate strength were used for determining the resolving time of the detector. By accumulating the counts for over 10,000 seconds, a resolving time of 9.95 micro-seconds was obtained. In the present study, the experimental source count rate was very low and the pulse pile up was negligible.

The source to detector distance was 0.33 cm and it was positioned in the center of the detector. The spectrometer calibration was 1.271 keV per channel. About 25 channels were blocked by the lower discriminator setting to eliminate instrument noise pulses. The source and the detector system including the photo multiplier tube and preamplifier assembly was shielded by lead housing which had 2 mm aluminium lining on the inner surface. The lead housing helps in reducing the background counts. Since ^{55}Fe is not emit particulate radiation and hence there is no production of unwanted external Bremsstrahlung. The backscatter from the photomultiplier tube too is negligible, as photoelectric absorption is the dominant mode of interaction. The 30 counting experiments with source, duration of each being the MCA LIVE TIME of 20,000 sec were conducted. The background measurement was done after each source counting experiment for the same duration. In any channel, the counts obtained with source (say X) include the background and the average of 30 experiments is termed as *mean source plus background*. Similarly, the average background counts in any channel is termed as *mean background*. Though the MCA had the software facility for spectrum smoothening, the spectrum obtained in individual experiments is not smoothened. Instead, the mean spectrum obtained on averaging all the 30 experiments is smoothened before subjecting it to the unfolding procedure.

The statistical strength of the data is determined by the statistical significance of the difference between the mean *source plus background* (X) and the mean *background* (B) counts. Noting that the nuclear counting follows Poisson distribution which approaches normal or Gaussian distribution when the number of measurements are large, the distribution of *sample means* too follow the normal distribution and so also the distribution of the difference between any two *sample means*. In any given channel both *sources plus background* and *background* counts are considered as the *samples* of a population for the purpose of conducting the statistical test. The standard error σ for the distribution of the *difference between any two sample means* based on Poisson distribution is given by

$$\sigma = \sqrt{[X + B]}.$$

Denoting the difference $X - B$ as I_{ib}, the statistic $t = I_{ib}/\sigma$ determines the confidence level of I_{ib}, which can be read from the standard normal curve area tables (Armitage and

Berry [17]) for which *mean* $= 0$, *standard deviation* $= 1$ and t is the abscissae extending from $-\infty$ to $+\infty$. p is the ratio of the area under the standard normal curve lying between any given value of t and $t = \infty$ to that between $t = 0$ and $t = \infty$. The p value corresponding to $t = I_{ib}/\sigma$ determines the required confidence level.

(i) If $I_{ib} \geq 3\sigma$, then the IB intensity lies in the confidence interval of 99.7–100% ($p \leq 0.003$). The data can be considered to be *excellent*.

(ii) If $2\sigma \leq I_{ib} < 3\sigma$, then the IB intensity lies in the confidence interval of 95–99.7% ($0.003 < p \leq 0.05$). The data can be considered to be *good*.

(iii) If $1\sigma \leq I_{ib} < 2\sigma$, then the IB intensity lies in the confidence interval of 68–95% ($0.05 < p \leq 0.32$). The data can be considered to be *fair*.

(iv) If $I_{ib} < 1\sigma$, then the IB intensity lies within the confidence interval of 68% ($p > 0.32$). The data is considered to be statistically *poor* and undoubtedly the *null* hypothesis to be accepted (i.e., the difference between X and B is considered insignificant and both X and B are treated as sample means coming from the same population of counts. i.e., the *background*.

If it is decided (i.e., setting a hypothesis) to accept the data up to a certain maximum value of p (for example 0.32 or 0.05 or 0.003 corresponding to the confidence intervals of 68–100% or 95–100% or 99.7–100%) which may correspond to some channel number say N, the counts in the $N + 1$th channel can be treated essentially as background only. This magnitude of counts may be subtracted from all channels. This is similar to correcting positive zero error in any measuring instrument. This procedure may also be termed as *base line shift* (BLS). The usual background subtraction is *physical background correction*, whereas the base line shift or subtraction is the *statistical background correction*. Since the counts towards the end of the experimental spectrum are orders of magnitude smaller than the counts in the major portion of the spectrum, a large part of the spectrum remains unaffected with the base line subtraction. In the present study, the magnitude of counts (15 counts) corresponding to the energy position of 222.4 keV with $p > 0.32$ (classified as *poor*) is subtracted from all channels.

In the present experimental work on ^{55}Fe, the data obtained was *excellent* up to 197 keV, *good* from 197 to 202 keV and *fair* from 202 to 212 keV and *poor* beyond 212 keV. Even though efforts are made to maintain the stability of the instruments, because of the long hours of counting and also the *background* and *source* counts being recorded at different times and the fact that the inherent fluctuations exist in both, make the difference counts I_{ib} either positive or negative towards the end of the energy spectrum, where *nil* counts are expected. This fact also justifies the BLS described above.

In our present experiment, the data up to 222.4 keV were processed with and without BLS. The unfolding of the IB intensities from the raw data was carried out adopting the step by step procedure of Liden and Starfelt [15], details of which can also be found elsewhere (Basavaraju *et al* [18] and Babu *et al* [19]). Since the count rate was low, the *dead time* correction was negligible. After initial background subtraction and pile up correction, the intensity spectrum is corrected for the finite energy resolution of the NaI (Tl) detector. The resolution function is taken as Gaussian. Next the correction due to the Compton electron distribution due to the continuous spectrum is made. Further the data is subjected to iodine K X-ray escape correction and the geometric detection efficiency of the detector-source spatial configuration including the attenuation of photons by the aluminium covering of the detector. Finally the spectrum is corrected for the photo peak efficiency of the detector.

However it is necessary to mention, the distortion caused by the detector resolution correction. The IB spectral data undergoing this correction tends to show false higher intensities in a few channels at the beginning and in a few channels towards the end. Due to the absence of counts on one side, the convolution procedure adds counts to the above said channels and hence the corrected data show false higher intensities. Except for this, most of the spectrum is well corrected within two or three iterations. The BLS method mentioned earlier overcomes this problem in the high-energy end.

3. Results and discussion

The experimental results for the ^{55}Fe isotope are represented in figures 1 to 7. The uncorrected raw IB spectrum along with the background is shown in figure 1. The background and pile up corrected data along with resolution corrected data and Compton electron distribution is given in figure 2. The data obtained after all corrections were converted to obtain the IB photon intensity/k capture/mc^2 energy range. The experimental IB intensities calculated without and with BLS of raw data are shown in figures 3 and 4, demonstrating the effect of resolution correction. The simple Jauch plot i.e., a plot of $\sqrt{(W_{ib}(E)/E)}$ versus E where $W_{ib}(E)$ is the IB intensity at the energy E, along with the least square fit line are also shown in figures 3 and 4 corresponding to the respective IB intensities computed without and with BLS. Theoretical distribution of IB photon intensity due to $1s$ electron capture as calculated by Glauber and Martin [2] taking into account the approximated relativistic Coulomb effects is also shown. Since the data points are large in number, the
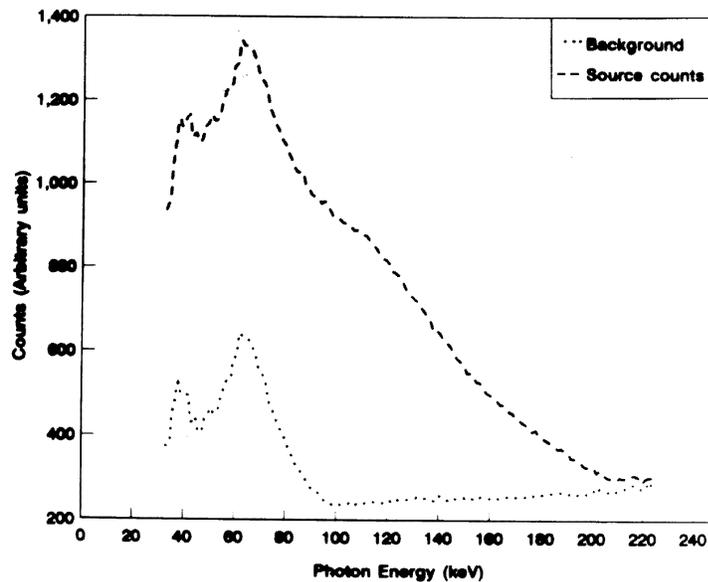

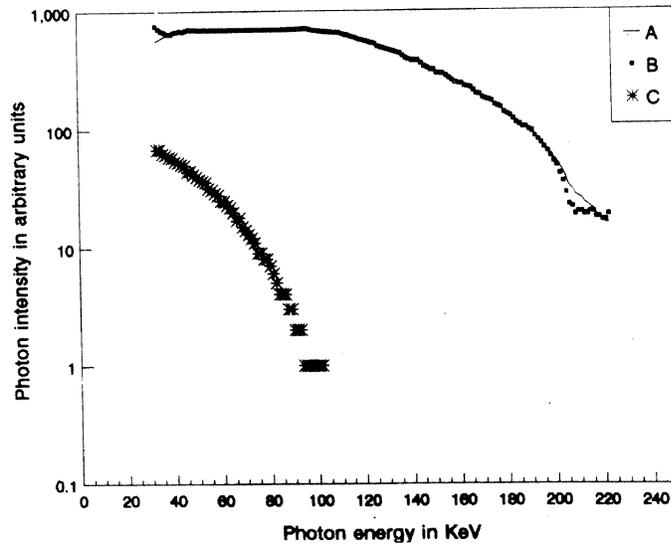**Figure 1.** Uncorrected raw IB spectrum along with the background.

Figure 2. A: Background and pile up corrected experimental spectrum of ^{55}Fe (without BLS); B: data after resolution correction of A; C: Compton electron distribution.
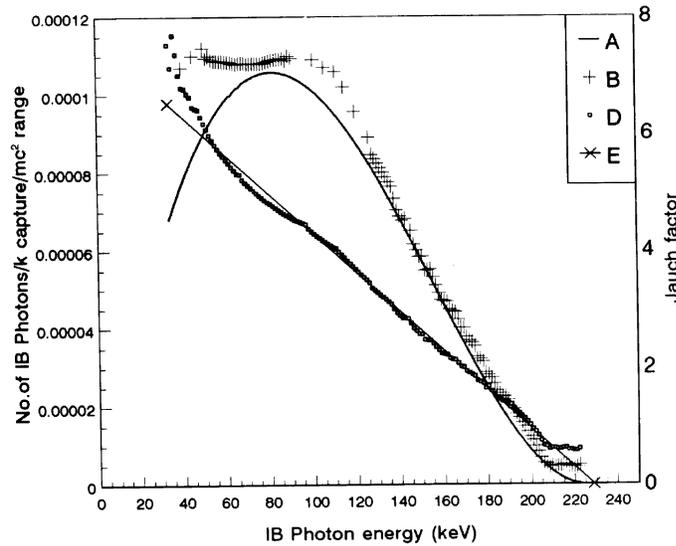

Figure 3. A: Theoretical Glauber–Martin IB spectrum following $1s$ electron capture in ^{55}Fe; B: experimental IB spectrum after all corrections without base line shift; D: Jauch plot of the data shown in B; E: Least square fit line of the Jauch plot $Y = -0.03329X + 7.61897$.

error bars are not shown, as these would affect the clarity and readability of the diagrams. Instead, the percentage errors $(\pm\sigma)$ in the data as well as in the Jauch factor are shown in figure 5. A plot of data$/x(1-x)^2$ versus IB photon energy is shown in figure 6 (for the data
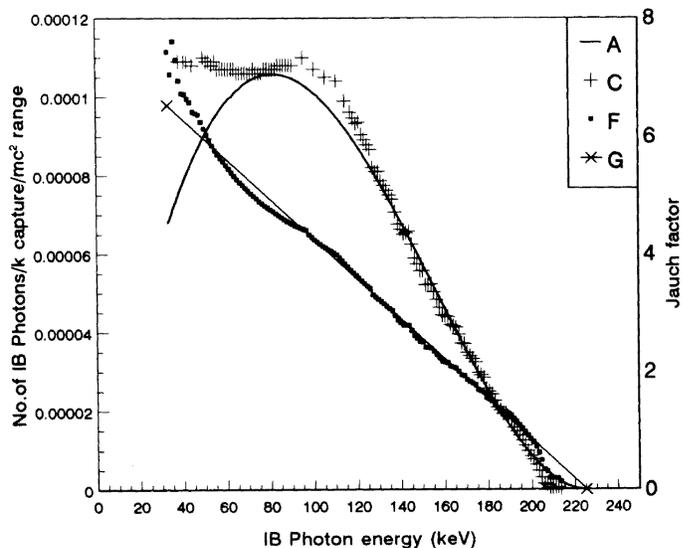
Figure 4. A: Theoretical Glauber–Martin IB spectrum following $1s$ electron capture in ^{55}Fe; C: experimental IB spectrum after all corrections with base line shift; F: Jauch plot of the data shown in B; G: least square fit line of the Jauch plot $Y = -0.03395X + 7.64788$.
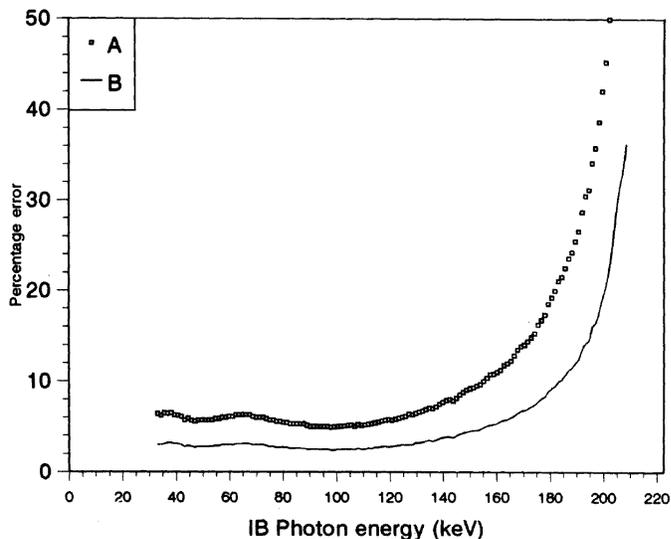

Figure 5. Percentage error $(\pm\sigma)$. A: In the IB intensity data and; B: in the Jauch factor.

computed without BLS) and in figure 7 (with BLS). The data/$x(1-x)^2$ is represented by the middle curve, whereas the top and the bottom curves represent the upper and the lower
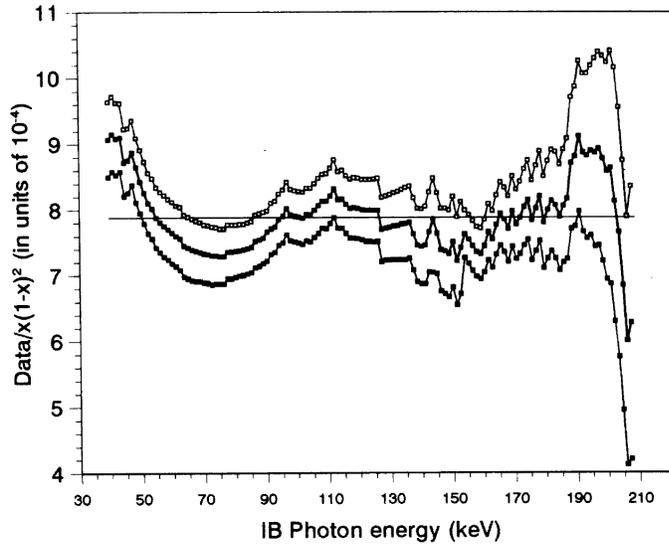
Figure 6. The middle curve represents the factor data$/x(1-x)^2$ for the case of no BLS and the upper and the lower curves give the error limits $(\pm\sigma)$. The solid straight line gives the mean factor.
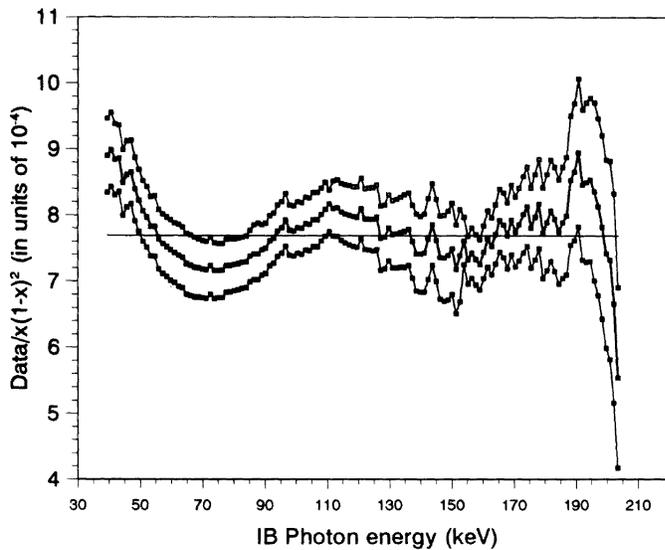

Figure 7. Same as in figure 6, for the case of data computed with BLS.

limits $(\pm\sigma)$, the solid straight line representing the mean data$/x(1-x)^2$ of all photon energies. This line lies well within the error limits indicating the dominance of $1s$ and $2s$ state electron capture.

It can be easily seen that the base line shift (BLS) of the raw data gives more realistic intensities at the high energy end of the spectrum. The IB intensities from 100 keV onwards show close agreement with the GM theory for IB resulting from the capture of $1s$ electrons, showing that the relativistic Coulomb effects are less significant in the decay of ^{55}Fe and also $1s$ capture is the dominant mode of decay. The intensities are in excess in the region 40–100 keV possibly due to the presence of $2s$ and $2p$ captures. The Jauch plot gave an end point energy of 228.84 keV (without BLS) and 225.249 keV (with BLS). The sum of the end point energy and the $1s$ electron binding energy of 7.112 keV gives the decay energy. Therefore we obtained a value of 235.95$\pm$ 0.64 keV without BLS and a value of 232.36 $\pm$ 0.64 keV with BLS. The value obtained with BLS is obviously closer to the value of 231.7 $\pm$ 0.7 keV based on mass difference given by Wapstra and Audi [16]. No attempt was made to compare the IB spectrum with other authors as the measurement of the IB spectrum in the low energy region was not possible due to the electronic noise problem where the contribution from $2s$ and $2p$ captures are likely to dominate and further $1s$, $2s$ and $2p$ capture contributions to the spectrum cannot be isolated as the present experiment is the measurement of the total spectrum only. A comparison of the decay energies (in units of keV) obtained by earlier workers is shown below.

Bradt *et al* [4]	150
Maeder and Preiswork [5]	212±10
Bell *et al* [6]	212±20
Michalowitz [11]	222±10
Madansky and Rasetti [7]	227
Emmerich *et al* [8]	232±10
Biavati [13]	227±10
Berenyi *et al* [9]	224±4
Raj and Ramaswamy [10]	248±20 (used raw IB spectrum)
Isaac, Vanin and Helene [14]	230.7±19
Present work	232.36±0.64

Acknowledgements

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