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Calculation of minor hysteresis loops under metastable to stable transformations in vortex matter

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Abstract. We present a model in which metastable supercooled phase and stable equilibrium phase of vortex matter coexist in different regions of a sample. Minor hysteresis loops are calculated with the simple assumption of the two phases of vortex matter having field-independent critical current densities. We use our earlier published ideas that the free energy barrier separating the metastable and stable phases reduces as the magnetic induction moves farther from the first order phase transition line, and that metastable to stable transformations occur in local regions of the sample when the local energy dissipation exceeds a critical value. Previously reported anomalous features in minor hysteresis loops are reproduced, and calculated field profiles are presented.

Keywords. Critical state model; metastable to stable transformations; minor hysteresis loops.

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1. Introduction

Supercooled or metastable states have been reported across first order phase transitions in vortex matter [1–12]. It has also been established that isothermal field excursions cause the metastable phase to be converted to the stable phase [6,8,9,11]. We have proposed [13] that the isothermal field variations provide a fluctuation energy that causes the metastable supercooled phase to cross the free-energy barrier and transform to the stable equilibrium phase.

The experimental techniques used to study the magnetic signatures of such transformations in vortex matter are: (i) bulk dc measurements using a SQUID or a vibrating sample magnetometer which yields the magnetization M of the entire sample [6–11]; (ii) bulk ac measurements of susceptibility which probe a region near the surface of the sample [4]; and (iii) local measurements of magnetic induction using magneto-optic or microhall probes that allow mapping the spatial profile $B(x)$ [1–3,5]. The last of these three techniques has been used recently to show that different (metastable and equilibrium) phases exist simultaneously in different regions of the sample [1–3]. Similar inference has also been drawn by studies using the resistivity [12], dc magnetization [6,7] and ac susceptibility [4] techniques. Experiments have thus shown that metastable to stable transformations occur over local regions, and prompted by these developments we have given a formalism to calculate spatially-resolved energy dissipation under an isothermal field variation [14]. In this paper we shall use this formalism to calculate how the metastable to stable transformation progresses inwards from the surface of the sample, under experimentally relevant field excursions. We shall also calculate the sample magnetization M and the spatial field profile $B(x)$ as the experimental observables.

2. Modelling the peak effect

The response of a hard superconductor to external magnetic fields is understood in terms of Bean's critical state model (CSM) [15]. Bean had assumed that the critical current density J_C is independent of field. While detailed agreement with experiments has required introduction of various functional forms of $J_C(B)$ (see e.g. ref. [16]), much of the essential physics is captured even by assuming a field-independent J_C.

Experiments have recently been addressing the region below and near the onset of a peak in $J_C(B)$ at $B = B_1$, where a first order phase transition is seen in some superconductors [2–11]. The occurrence of this 'peak-effect' has been known in various superconductors for a very long time, but attempts to have a CSM describing this $J_C(B)$ have been made only recently [17,18]. While our detailed analytical model [17] could be used for the subsequent calculations, our focus here is to understand whether qualitatively new and anomalous signatures in recent experiments [5–11] can be arising from metastable to stable transformations in vortex matter. In this paper we shall use a simple Bean-like assumption for the two phases of vortex matter viz.

$$\begin{aligned} J_C(B) &= J_1 \quad \text{for} \quad B \leq B_1 \\ &= J_2 \quad \text{for} \quad B \geq B_1. \end{aligned} \tag{1}$$

We stress that, at a fixed temperature, our model has only two constant parameters viz. (J_2/J_1) and B_1. We have assumed above that phase 1 (characterized by J_1) is the stable phase for $B(x) \leq B_1$, and phase 2 (characterized by J_2) is the stable phase for $B(x) \geq B_1$. We recognise that J_C is not a thermodynamic quantity, but is a physical property that changes discontinuously across the phase transition. Phase 2 can exist for $B(x) \leq B_1$ as a supercooled metastable phase, and we shall address this possibility in the next section. In this section we shall assume, however, that the free energy barrier surrounding the metastable phase drops very sharply as $B(x) = B_1$ is crossed, and supercooling or superheating does not occur.

To obtain magnetization-vs-field (or M-H) curves, we consider the sample to be in the form of an infinite slab in parallel field, as this geometry has the simplest algebra amongst the zero demagnetization factor cases of infinite cylinders in parallel field. We shall also continue with Bean's simplifying assumption of $H_{C1} = 0$ followed usually in the CSM [15–17].

We follow standard procedures [16,17] to solve the CSM, and show in figure 1 the envelope M-H curves obtained with eq. (1), with the parameters $B_1 = 1500$ mTesla, $J_1R = 4$ mTesla, and $J_2R = 10$ mTesla. Here the slab has surfaces at $x = \pm R$, is infinite along the y and z directions, and the magnetic field is applied along the z-axis. (We shall consider only positive values of x in this paper; there is a symmetry about $x = 0$.) Note that the first order transition shows different widths in M vs H when measured along the field-increasing and along the field-decreasing directions. This is because

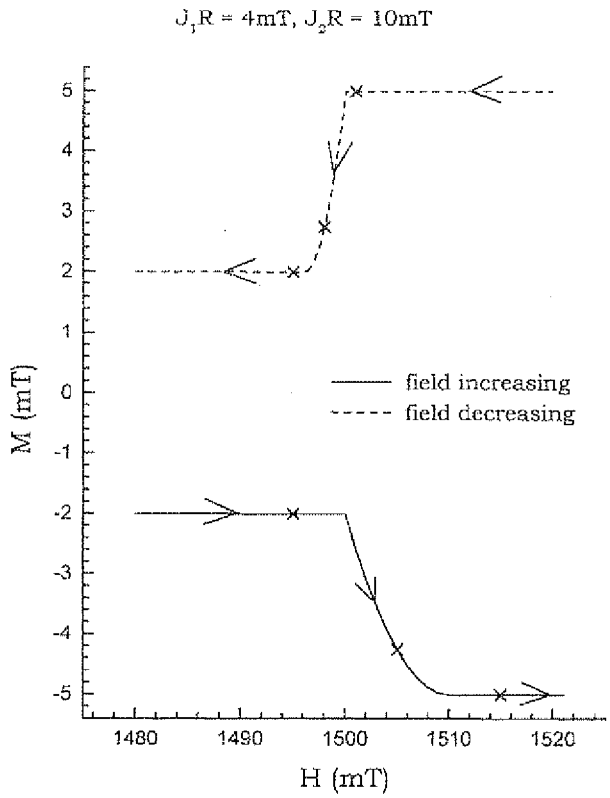

Figure 1. Field-increasing and field-decreasing envelope M-H curves are shown, following the model of eq. (1), with $B_1 = 1500$ mTesla. The crosses indicate applied field values at which $B(x)$ profiles are shown in figure 2.

the shielding current density at x is dictated [15–18] by the local magnetic induction $B(x)$ through eq. (1), and $B(x)$ is different from the applied field as well as different in the field-increasing and field-decreasing cases. In figure 2 we plot $B(x)$ for some values of applied field H corresponding to the field-increasing and field-decreasing cases. We note that phase 1 and 2 exist simultaneously in two different regions of the sample. We emphasize that there is no metastability because the stable phase 1 exists wherever $B(x) \leq B_1$ and the stable phase 2 exists wherever $B(x) \geq B_1$.

The calculation above is for some fixed temperature T_1, and we note that the phase transition field B_1 falls as the temperature T_1 rises [19].

3. Supercooling and metastable-to-stable transformations

We now consider that we have applied a field H_1 which is smaller than $B_1(T_1)$. But we apply this field at a much higher temperature T_2 such that H_1 is much larger than $B_1(T_2)$.

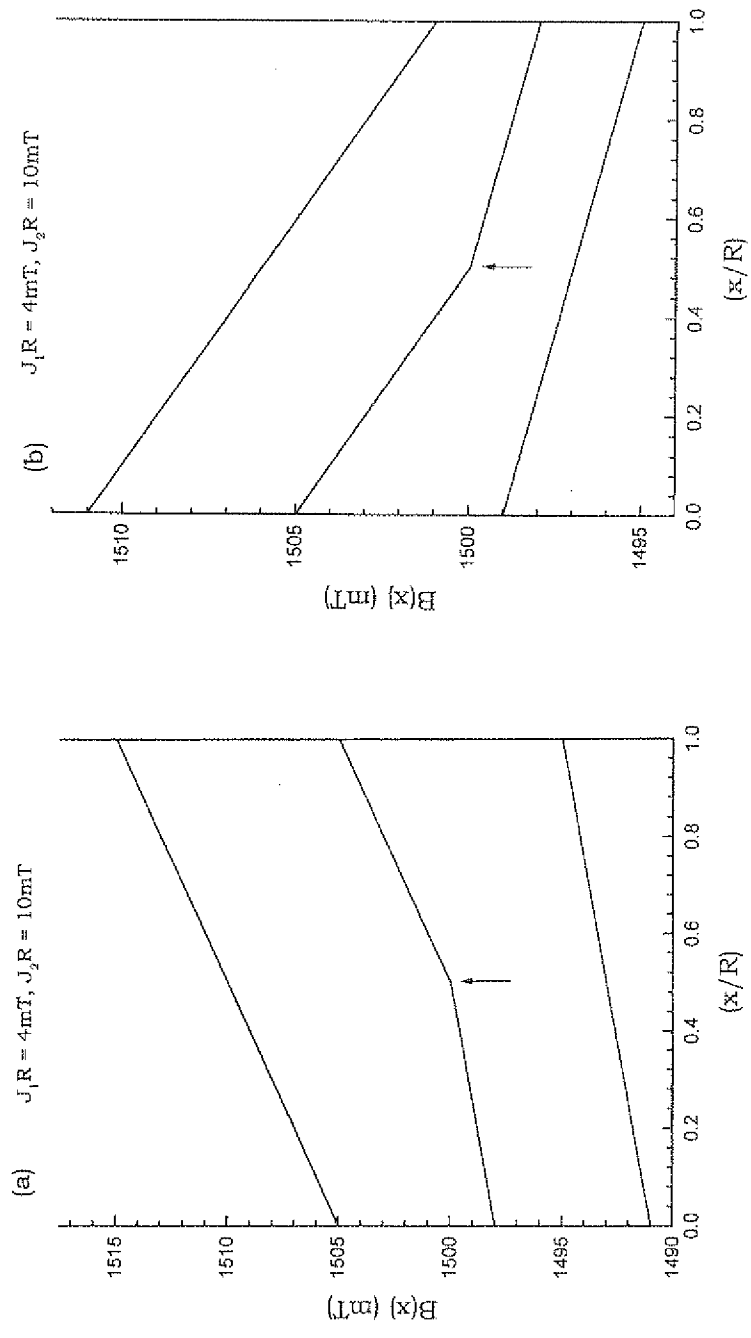

Figure 2. $B(x)$ profiles are shown for (a) field-increasing case at applied fields of 1495, 1505, and 1515 mTesla; and (b) field-decreasing case at applied fields of 1501, 1498, and 1495 mTesla. The arrows indicate the x at which $B(x) = 1500$ mTesla.

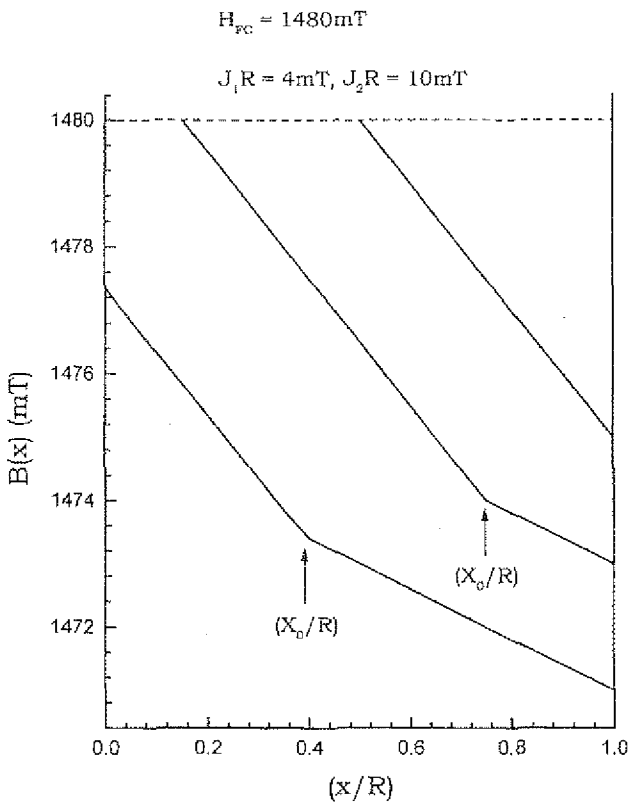

Figure 3. The sample is field-cooled in 1480 mTesla, when $B(x)$ is constant as shown by the dashed line, and vortex matter is in the metastable phase 2. $B(x)$ are shown as the applied field is lowered isothermally to 1475, 1473, and 1471 mTesla. In the last two fields the vortex matter has transformed to the stable phase 1 at $x > x_0$, where x_0 is indicated by an arrow.

So, $B(x)$ throughout the sample is larger than B_1 at that temperature, and the entire sample is in phase 2. We assume further that $B(x)$ is constant at T_2. This happens if the critical current density J_2 in phase 2 vanishes at T_2. One can, however, also achieve a constant $B(x)$ by applying an external field $H_1 + h\cos(wt)$, with $(H_1 - B_1(T_2)) > h > J_2(T_2)R$, and then slowly reducing the amplitude h to zero [16,20].

We now lower the sample temperature (i.e. field-cool) to T_1 such that the sample is supercooled and is metastable in phase 2. As discussed in references [13,14,19], there is a free energy barrier $f_{\rm B}(T)$ that keeps phase 2 metastable, where $f_{\rm B}(T)$ is determined uniquely by $B(x)$ and T. The vortex matter in the neighbourhood of x will transform to phase 1 when the fluctuation energy $P_d(x)$ created by an isothermal field variation is larger than $[f_{\rm B}(T) - kT]$.

The field profile $B(x)$ in the field-cooled sample is constant at H_1 (see figure 3), and we now start lowering the applied field H with the temperature fixed at T_1. Since the sample is in the supercooled phase 2, the shielding currents set up initially will have a magnitude

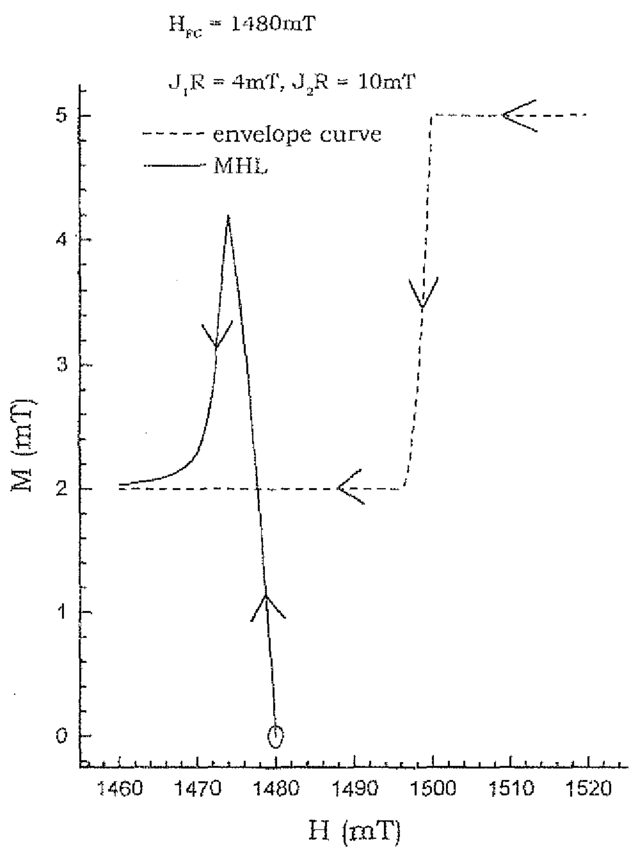

Figure 4. MHL obtained after field-cooling at 1480 mTesla is shown by the solid line. It overshoots the envelope curve, and merges with it slowly from above. The circle indicates the starting point of the MHL, with $M = 0$ corresponding to the constant $B(x)$.

J_2. The variations in field will cause a fluctuation energy $P_d(x)$ given by eqs (4) and (5) of ref. [14], and the vortex matter in the neighbourhood of $x = x_0$ will transform to the stable phase at $P_d(x_0) = f_B(T) - kT = P_0$. This transformation is triggered from the surface [14] and the shielding current magnitude will drop to J_1 for $x > x_0$. The point x_0 moves from $(x/R) = 1$ to $(x/R) = 0$ as the applied field is lowered, and $B(x)$ are shown in figure 3 for representative values of the applied field. We have used $H_1 = 1480$ mTesla, and $P_0 = 18$ (mTesla)^2. The large (small) slopes of $B(x)$ correspond to large (small) magnitudes of the shielding current density, and thus to vortex matter being in phase 2 (phase 1). From these $B(x)$ one can readily calculate [15–17] the sample magnetization as the field H is lowered. In figure 4 we show the minor hysteresis loop (MHL) obtained as the applied field is lowered after field-cooling. We show also the field-decreasing envelope curve from figure 1. Note that the MHL first shoots out above the envelope curve, and then slowly merges from above. This nature is in qualitative agreement with published data

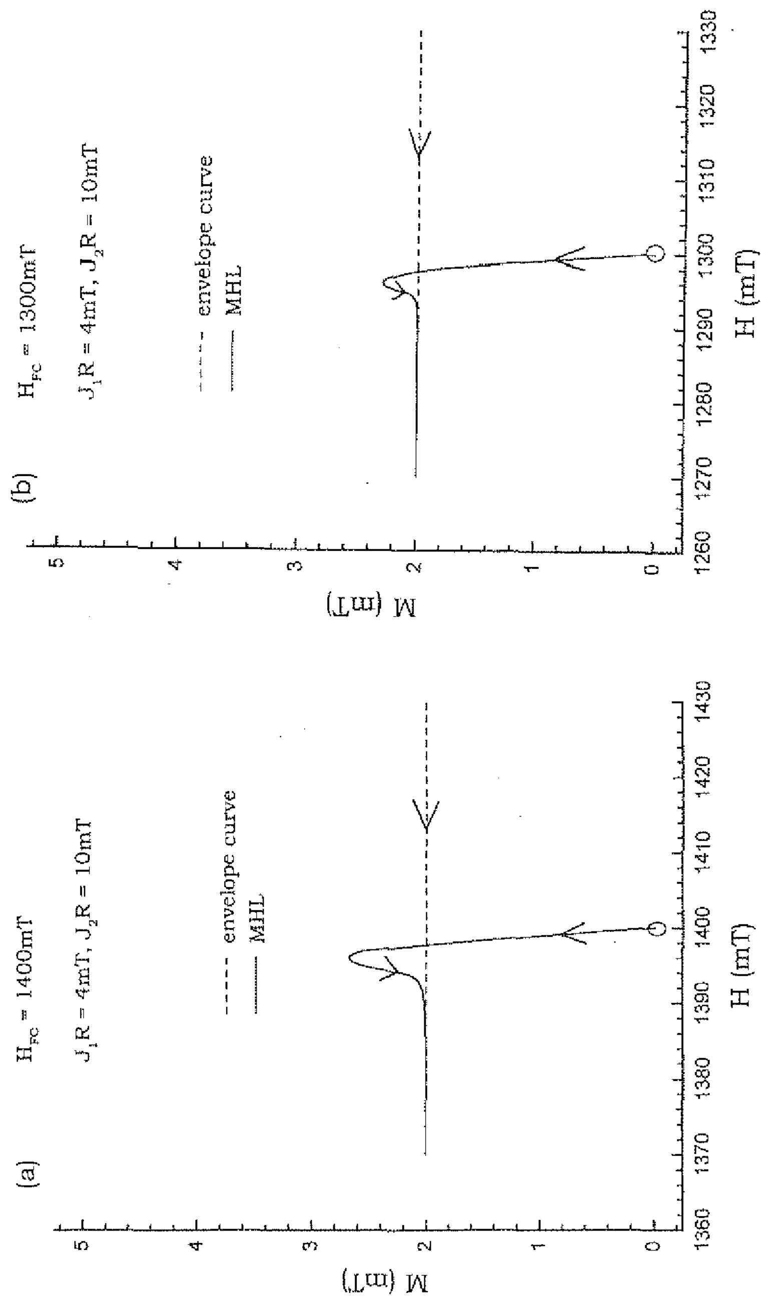

Figure 5. Same as figure 4 except that field-cooling was done at (a) 1400 mTesla; and (b) 1300 mTesla.

[8,10,11,21,22]. If we had used the detailed $J_C(B)$ of ref. [17] to model the peak-effect, instead of the simple model of eq. (1), the peak of the MHL would be less sharp and the merger with the envelope curve would be slower. The simple model used to obtain figure 4 brings out the qualitative behaviour observed and captures the essential underlying origin of anomalous MHLs as being due to phase 2 being supercooled and the transformation from the metastable phase 2 to the stable phase 1 occurring progressively deeper into the sample.

We have assumed that P_0 (and thus $f_B(T_1)$) is constant at 18 (mTesla)^2. $f_B(T_1)$ is actually dictated by $B(x_0)$, and falls monotonically as $B(x_0)$ moves farther from the phase transition line $B_1(T)$. As is seen in figure 3, $B(x_0)$ varies only by less than a few mTesla as the MHL merges with the envelope curve. The assumption of a constant P_0 over an MHL is thus justified. If, however, we field-cool to the same temperature T_1 at a lower field H_2, then $f_B(T)$ will be lower [19]. This implies that $P_0(H_{FC} = H_2)$ will be smaller than $P_0(H_{FC} = H_1)$. We show, in figure 5a, the MHL for the case when the sample was field-cooled to $H_2 = 1400$ mTesla where P_0 is taken to be 4.5 (mTesla)^2. In figure 5b we have taken $H_{FC} = H_3 = 1300$ mTesla, where f_B must be still lower and is taken as $P_0 = 2$ (mTesla)^2. The MHLs again shoot out of the envelope curve, but to peak values progressively smaller than in figure 4. The merger of the MHLs with the envelope curve also occurs over a progressively narrower range of field reduction than in figure 4. This qualitative change in the nature of the MHLs with reduction of H_{FC} is also consistent with published data [8,10,11,21,22].

4. Conclusion

We have used the ideas developed in references [13,14,19] to calculate the isothermal field-cooled MHLs, and the spatial field profiles $B(x)$. The model calculation was done, in the spirit of Bean's original work [15], with field-independent critical current densities. The only parameters were (J_2/J_1), and the onset field B_1 at which the peak effect starts in the field increasing case. We used the fact [19] that f_B becomes smaller as $B(x)$ falls below B_1, and that metastable to stable transformations occur in local regions of the sample [14].

The formalism of ref. [14] can similarly be used to calculate MHLs after different thermomagnetic histories. We assert here that the simple model of eq. (1) reproduces qualitative features of various observations [6–11,21,22] of anomalous MHLs. As was stated in the introduction, more detailed tests of the extent of phase coexistence are possible and calculated $B(x)$ can be compared with field profiles measured with local probes. Our model also predicts the spatial region over which the two phases coexist, and the evolution of these regions under isothermal field variation.

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