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Complex wave-interference phenomena: From the atomic nucleus to mesoscopic systems to microwave cavities

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Abstract. Universal statistical aspects of wave scattering by a variety of physical systems ranging from atomic nuclei to mesoscopic systems and microwave cavities are described. A statistical model for the scattering matrix is employed to address the problem of quantum chaotic scattering. The model, introduced in the past in the context of nuclear physics, discusses the problem in terms of a prompt and an equilibrated component: it incorporates the average value of the scattering matrix to account for the prompt processes and satisfies the requirements of flux conservation, causality and ergodicity. The main application of the model is the analysis of electronic transport through ballistic mesoscopic cavities: it describes well the results from the numerical solutions of the Schrödinger equation for two-dimensional cavities.

Keywords. Mesoscopic physics; information theory; chaotic scattering.

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1. Introduction

Wave scattering by complex systems has been an interesting chapter of physics for a long time. Let us mention, for instance, the problem of multiple scattering of waves, which has been of great relevance to optics [1]. Recently, a revival of interest in this problem has been seen, both for electromagnetic waves and for electrons, motivated by the phenomenon of localization, which gives rise to a great many exciting effects [2–4].

The field of nuclear physics, with typical dimensions of a few fm ($1\,\mathrm{fm} = 10^{-15}$ m), offers excellent examples – dating as far back as the 1930's, when compound-nucleus resonances were discovered – of quantum-mechanical scattering by 'complex' many-body systems and of its *statistical* aspects.

Interestingly enough, features similar to the ones observed in complex nuclear problems have also been found in certain 'simple' systems. While the geometry of these systems is apparently very simple, their classical dynamics is fully chaotic. In contrast to the complex cases, these systems are amenable to exact (numerical) calculation; however, when the results are analyzed *statistically*, they appear to have a close relation with those of the complex systems. Experimentally, two types of situations involving simple scattering systems have been studied: electron transport through microstructures called 'ballistic quantum dots', whose dimensions are of the order of $1\,\mu$m, and microwave scattering from metallic cavities, with typical dimensions of 0.1 m.

The 'universal' statistical properties of wave-interference phenomena observed in systems whose dimensions span about 14 orders of magnitude turn out to depend on very general physical principles and constitute the central topic of this talk [5]. Our aim is to provide a theoretical framework to describe this physical situation. The description of the general statistical features of chaotic scattering processes is achieved in this presentation through a *random-matrix* model that was first introduced in the past in the context of nuclear physics using an *information-theoretic* approach [6].

In order to emphasize the generality of the ideas involved we present in §2 some of the features of statistical nuclear reactions – with a brief reference to microwave cavities – and, in §3, some characteristics of the problem of electronic conduction through ballistic quantum dots. The latter, in fact, is the field of main interest in this talk.

In §4 we define the scattering problem and introduce the basic object we shall work with, the scattering or S matrix of the problem. Throughout the talk we shall treat ensembles of systems in terms of ensembles of S matrices. For that purpose, we first explain how to 'weigh' S matrices: this is done through the notion of the 'invariant measure', the mathematical concept that corresponds to the intuitive idea of 'equal-*a-priori*-probabilities'. In §5 we introduce the information-theoretic model and, in §6, its consequences for the problem of electronic transport through quantum dots are discussed.

2. The atomic nucleus and microwave cavities

Figure 1 shows the excitation function associated with the reaction $^{35}\mathrm{Cl}(p,\alpha_0)^{32}\mathrm{S}$ as a function of the incident proton energy: this is on the order of 10 MeV, so that the excitation energy in the compound system ^{36}A is about 18 MeV. At these energies, the width-to-spacing ratio of nuclear resonances is large, so that the peaks seen in the figure do not correspond to individual resonances, but rather to the coherent superposition of many resonances, giving rise to the phenomenon known as *Ericson fluctuations* [7].

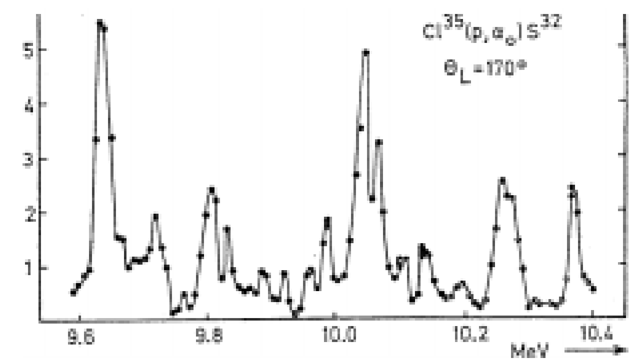

Figure 1. The excitation function for the reaction $^{35}\mathrm{Cl}(p,\alpha_0)^{32}\mathrm{S}$, in which the residual nucleus $^{32}\mathrm{S}$ is left in its ground state; the observation is made at 170° in the laboratory. The abscissa indicates the energy of the incident proton. The observed peaks are not individual resonances in the compound nucleus ^{36}A, but statistical fluctuations produced by the coherent superposition of many resonances. (From ref. [7].)

The system involved in the experiment just described is a 'complex' many-body one. There does not exist a theory of the nucleus capable of reproducing the individual peaks shown in figure 1 and, even if there should be one such theory, it would be of very little interest. A statistical analysis of the data is much more interesting and significant: for instance, it is of great importance to know the average cross section, its fluctuations and correlations in a certain energy interval.

One of the most successful models in nuclear physics, called the 'optical model' of the nucleus, was invented by Feshbach, Porter and Weisskopf [8] in the 1950's. That model describes the scattering of a nucleon by an atomic nucleus – a complicated many-body problem – in terms of *two distinct time scales*:

1. A *prompt* response related with *direct processes*, in which the incident nucleon feels a mean field produced by the other nucleons. This response is described mathematically in terms of the *average* over an energy interval around a given energy E of the actual scattering amplitudes: these averaged, or *optical*, amplitudes vary much more slowly with energy than the original ones.
2. A *delayed*, or *equilibrated*, response, related to the formation and decay of the compound nucleus. It is described by the difference between the exact and the optical scattering amplitudes: it varies appreciably with the energy and is studied with *statistical* concepts using techniques known as random matrix theory [9].

In the field of statistical mechanics time averages are very difficult to construct and hence are replaced by *ensemble averages* using the notion of *ergodicity*; similarly, in the present context one finds it advantageous to study energy averages in terms of ensemble averages through an ergodic property [10].

The relation of the above problems with the theory of waveguides and cavities was proposed very clearly by Ericson and Mayer–Kuckuk [7] more than thirty years ago: 'Nuclear-reaction theory is equivalent to the theory of waveguides . . .. We will concentrate on processes in which the incident wave goes through a highly complicated motion in the nucleus . . .. We will picture the nucleus as a closed cavity, with reflecting but highly irregular walls.'

Recent experiments with microwave cavities have indeed shown features similar to those that had been observed in the nuclear case. Now we know that the 'irregular walls' mentioned by Ericson and Mayer–Kuckuk for the nuclear case are *not* necessary in order to see these features: the analogy between nuclear reaction theory and the theory of cavities and waveguides holds for smooth cavities as long as the corresponding classical dynamics is chaotic. This has been the idea behind several experiments involving microwave scattering from metallic cavities [11,12]. Very important were also various experiments on microwave scattering by a disordered dielectric medium (see, for instance, ref. [13]).

3. Ballistic mesoscopic cavities

As we mentioned in the introduction, features similar to those described in the last section also occur in quantum-mechanical single-particle scattering problems that involve cavities whose classical dynamics is chaotic. Ballistic microstructures, or quantum dots, are an experimental realization of such systems. One finds that, for sufficiently low temperatures and for spatial dimensions of the order of 1 μm or less, the phase coherence length l_ϕ

exceeds the system dimensions, so that the phase of the single-electron wave function – in an independent-electron approximation – remains coherent across the system of interest. Under these conditions, these systems are called *mesoscopic*. The elastic mean free path $l_{\rm el}$ also exceeds the system dimensions: impurity scattering can thus be neglected, so that only scattering from the boundaries of the system is important. (For a review, see [4].) In these systems, the dot acts as a resonant cavity and the leads as electron waveguides and we are back to the description indicated in the previous section.

Theoretically, the problem of quantum transport through ballistic quantum dots has been described in the past in the semiclassical [14,15], field-theoretic [16] and random-matrix [16–20] approaches.

Experimentally, an electric current is established through the leads that connect the cavity to the outside, the potential difference across the cavity is measured and the conductance G is then extracted. In an independent-electron picture, the two-terminal conductance is given by [21]

$$G = 2\frac{e^2}{h}T, \tag{1}$$

$$T = \mathrm{tr}\left(tt^\dagger\right), \tag{2}$$

t being the matrix of transmission amplitudes t_{ab}, with a, b denoting final and initial channels. In this 'scattering approach' to electronic transport one thus aims at understanding the quantum-mechanical single-electron scattering by the cavity in question.

It is the multiple scattering of the waves reflected by the various portions of the cavity that gives rise to interference effects. When the external magnetic field B, or the Fermi energy $\epsilon_{\rm F}$, or the shape of the cavity are varied, the relative phase of the various partial waves changes and so do the interference pattern and the conductance. This sensitivity of G to small changes in parameters through quantum interference is called *conductance fluctuations*.

Representative experiments on quantum dots are given in ref. [22]. They report cavities in the shape of a stadium, for which the single-electron classical dynamics would be chaotic, as well as experimental *ensembles* of shapes. Averages of the conductance, its fluctuations and its full distribution were obtained over such ensembles.

4. The scattering problem

Very generally, a quantum-mechanical scattering problem is described by its scattering matrix, or S matrix, whose rows and columns label the so called *open channels*.

In our application to mesoscopic physics we consider a system of noninteracting 'spinless' electrons and study the scattering of an electron at the Fermi energy $\epsilon_{\rm F}$ by a $2D$ microstructure, connected to the outside by L leads of width W_l, $l = 1, \ldots, L$. Along each lead l we define an axis x_l pointing *away* from the cavity, the origin being assumed at the place where the lead is connected to the cavity. The general situation is indicated in figure 2; from now on, though, we restrict the discussion to two leads only, i.e. $L = 2$, with equal widths W.

The *transverse modes*, or channels, originate from the confinement of the electron in the transverse direction in the leads. For the incident Fermi momentum $k_{\rm F}$ there are

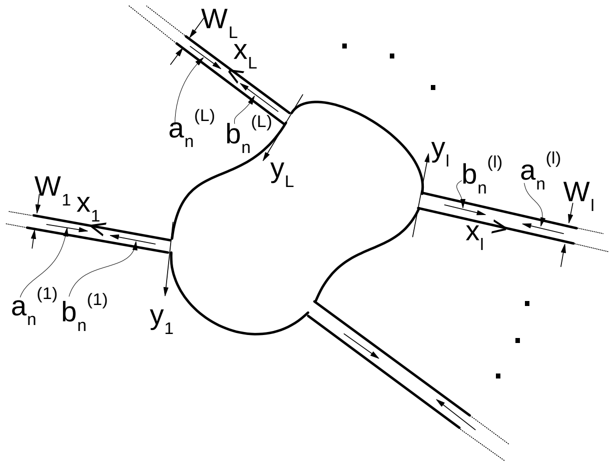

Figure 2. The 2D cavity referred to in the text. The cavity is connected to the outside via L waveguides. The arrows inside the waveguides indicate incoming or outgoing waves. In waveguide l there can be N_l such incoming or outgoing waves: this is indicated in the figure by the amplitudes $a_n^{(l)}$, $b_n^{(l)}$, respectively, where $n = 1, \ldots, N_l$.

$$N = k_{\mathrm{F}} W/\pi \tag{3}$$

transmitting or running modes (open channels) in each of the two leads. The wavefunction in lead l ($l = 1, 2$ for the left and right leads, respectively) is written as the N-dimensional vector

$$\Psi^{(l)}(x_l) = [\psi_1^{(l)}(x_l), \ldots, \psi_N^{(l)}(x_l)]^T, \tag{4}$$

the m-th component being a linear combination of unit-flux plane waves, i.e.

$$\psi_m^{(l)}(x_l) = a_m \frac{\mathrm{e}^{-ik_m x_l}}{(\hbar k_m/\mu)^{1/2}} + b_m \frac{\mathrm{e}^{ik_m x_l}}{(\hbar k_m/\mu)^{1/2}} \qquad m = 1, \ldots, N. \tag{5}$$

In (5), k_m, the 'longitudinal' momentum in channel m, is such that

$$k_m^2 + \left[\frac{m\pi}{W}\right]^2 = k_{\mathrm{F}}^2. \tag{6}$$

The $2N$-dimensional S matrix relates the incoming to the outgoing amplitudes as

$$\left[\begin{array}{c} b^{(1)} \\ b^{(2)} \end{array}\right] = S \left[\begin{array}{c} a^{(1)} \\ a^{(2)} \end{array}\right], \tag{7}$$

where $a^{(1)}$, $a^{(2)}$, $b^{(1)}$, $b^{(2)}$ are N-dimensional vectors. The matrix S has the structure

$$S = \left(\begin{array}{cc} r & t' \\ t & r' \end{array}\right), \tag{8}$$

where r, t are the $N \times N$ reflection and transmission matrices for particles from the left and r', t' for those from the right. This thus defines the transmission matrix t used earlier in eq. (2).

The requirement of *flux conservation* (*FC*) implies *unitarity* of the S matrix [1], i.e.

$$SS^{\dagger} = I. \tag{9}$$

This is the only requirement in the absence of other symmetries. For a *time-reversal invariant* problem (as is the case in the absence of a magnetic field) and no spin, the S matrix, besides being unitary, is *symmetric* [1,23]:

$$S = S^{T}. \tag{10}$$

In numerical simulations of quantum scattering by $2D$ cavities with a chaotic classical dynamics one finds that the S matrix and hence the various transmission and reflection coefficients fluctuate considerably as a function of the incident energy E (or incident momentum k), because of the resonances occurring inside the cavity [17,24]. These resonances are moderately overlapping for just one open channel and become more overlapping as more channels open up. An example is illustrated in figure 3.

One is led to a statistical analysis of the problem, just as was explained in §2 in connection with the nuclear problem. The statistical properties of the quantities of interest, sampled along the energy axis, will be represented by a statistical ensemble of systems, in a way analogous to that followed in classical statistical mechanics, where time averages are represented by ensemble averages, employing an ergodic hypothesis. In the latter field, the starting point is the notion of equal-*a-priori*-probabilities in phase space. The equivalent notion in the present context is provided by the *invariant measure* in the space of S matrices, to be described below.

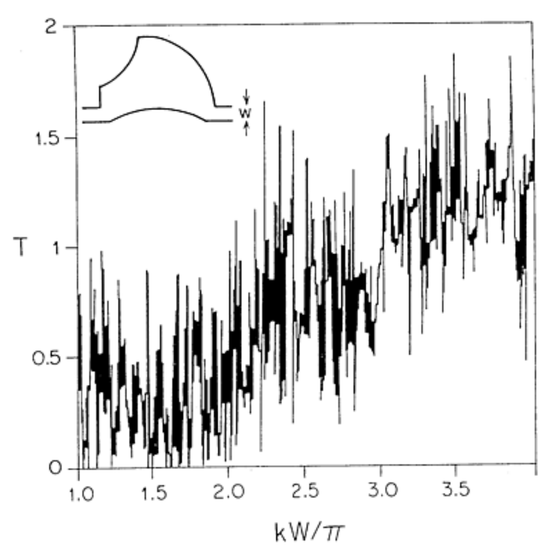

Figure 3. Total transmission coefficient of eq. (2) as a function of incident momentum, obtained by solving numerically the Schrödinger equation for the system indicated in the upper left corner. (From ref. [24].)

By definition, that measure remains invariant, for a given symmetry class, under an automorphism of that class of matrices unto itself [23,25], i.e.

$$\mathrm{d}\mu^{(\beta)}(S) = \mathrm{d}\mu^{(\beta)}(S'). \tag{11}$$

For unitary symmetric S matrices we have

$$S' = U_0 S U_0^T, \tag{12}$$

U_0 being an arbitrary, but fixed, unitary matrix: (12) represents an automorphism of the set of unitary symmetric matrices unto itself. This is the *orthogonal* case, also known as the $\beta = 1$ case. For unitary, not necessarily symmetric, S matrices

$$S' = U_0 S V_0, \tag{13}$$

U_0 and V_0 being now arbitrary fixed unitary matrices. For this *unitary*, or $\beta = 2$ class, the resulting measure is the well known Haar's measure of the unitary group and its uniqueness is well known [26,27]. Uniqueness for the $\beta = 1$ class (and $\beta = 4$) was established in ref. [23]. The invariant measure, eq. (11), defines the *circular* (*orthogonal*, *unitary*) *ensemble* (*COE, CUE*), for $\beta = 1, 2$ respectively.

The S-matrix of eq. (8) can be expressed in the so-called 'polar representation' [28–30] as

$$S = \begin{bmatrix} v^{(1)} & 0 \\ 0 & v^{(2)} \end{bmatrix} \begin{bmatrix} -\sqrt{1-\tau} & \sqrt{\tau} \\ \sqrt{\tau} & \sqrt{1-\tau} \end{bmatrix} \begin{bmatrix} v^{(3)} & 0 \\ 0 & v^{(4)} \end{bmatrix}, \tag{14}$$

where τ stands for the $N \times N$ diagonal matrix of the eigenvalues τ_a of the Hermitean matrix $tt^\dagger$. The $v^{(i)}$ are arbitrary unitary matrices for $\beta = 2$, with $v^{(3)} = (v^{(1)})^T$ and $v^{(4)} = (v^{(2)})^T$ for $\beta = 1$. The trace $\sum_a \tau_a$ is the total transmission T, eq. (2).

In the polar representation the invariant measure can be expressed as [17,20]

$$\mathrm{d}\mu^{(\beta)}(S) = p^{(\beta)}(\{\tau\}) \prod_a \mathrm{d}\tau_a \prod_i \mathrm{d}\mu(v^{(i)}), \tag{15}$$

where the joint probability density of the $\{\tau\}$ is

$$p^{(\beta)}(\{\tau\}) = C_\beta \prod_{a<b} \mid \tau_a - \tau_b \mid^\beta \prod_c \tau_c^{(\beta-2)/2}. \tag{16}$$

In eq. (15), $\mathrm{d}\mu(v^{(i)})$ denotes the invariant, or Haar's, measure [26,27] on the unitary group $U(N)$ and C_β is a normalization constant.

The invariant measure introduced above is the starting point for the development to be carried out in the next section.

5. The information-theoretic model

As was mentioned in the §2, the prompt response associated with direct processes is described by the energy-averaged, or optical, S matrix [8], which, through the ergodic property of the ensemble, is represented as the ensemble average $\langle S \rangle$. For the invariant measure of the previous section we clearly have $\langle S\rangle = 0$. Ensembles in which $\langle S\rangle$ is nonzero contain more *information* than the circular ensembles and will be constructed as

$$\mathrm{d}P^{(\beta)}_{\langle S\rangle} = p^{(\beta)}_{\langle S\rangle}(S)\ \mathrm{d}\mu^{(\beta)}(S)\ . \tag{17}$$

The Shannon information $\mathcal{I}$ associated with the above probability distribution is defined as [6,9,31,32]

$$\mathcal{I}[p_{\langle S\rangle}] \equiv \int p_{\langle S\rangle}(S)\ \ln[p_{\langle S\rangle}(S)]\ \mathrm{d}\mu(S)\ . \tag{18}$$

Far from channel thresholds the S-matrix is *analytic* in the upper half of the complex-energy plane (causality). This requirement, plus that of ergodicity (so that energy averages can be calculated as ensemble averages), we call the *analyticity-ergodicity requirement* (*AE*). AE implies the *reproducing property* [6,25]

$$f(\langle S\rangle) = \int f(S)\mathrm{d}P_{\langle S\rangle}(S)\ \ , \tag{19}$$

for a function $f(S)$ analytic in its argument. The probability density

$$p^{(\beta)}_{\langle S\rangle}(S) = V_\beta^{-1}\frac{[\det(I-\langle S\rangle\langle S^\dagger\rangle)]^{(\beta n+2-\beta)/2}}{\mid \det(I-S\langle S^\dagger\rangle)\mid^{\beta n+2-\beta}}, \tag{20}$$

known as *Poisson's kernel (*V_β is a normalization constant and $n = 2N$), satisfies the reproducing property, eq. (19) [25]. It can also be shown [6] that the information associated with Poisson's kernel is less than or equal to that of any other probability density satisfying the AE requirements for the same $\langle S\rangle$. Thus, Poisson's kernel describes those physical situations in which, having fulfilled the requirements of flux conservation, time-reversal invariance (when applicable), and AE, *the details are irrelevant except for the average S matrix.*

6. The information-theoretic model and the problem of transport through ballistic chaotic cavities

We first consider the particular case in which *direct processes are absent*. In this case the optical matrix $\langle S\rangle$ vanishes and eqs (17) and (20) show that the distribution for the S matrix reduces to the invariant measure, i.e.

$$\mathrm{d}P^{(\beta)}_{\langle S\rangle=0} = \ \mathrm{d}\mu^{(\beta)}(S)\ . \tag{21}$$

Averages of products of S and S^* matrix elements for the CE's can be calculated without using an explicit form for $\mathrm{d}\mu_\beta(S)$, but solely its invariant properties [32,33]. Using the notation

$$\langle f\rangle_0^{(\beta)} = \int f\mathrm{d}\mu^{(\beta)}(S), \tag{22}$$

with

$$\int \mathrm{d}\mu^{(\beta)}(S) = 1, \tag{23}$$

one finds, for example (for $\beta = 1, 2$) [17]

$$\left\langle | \ t_{ab} \ |^2 \right\rangle_0^{(\beta)} = \frac{1}{2N + 2 - \beta} \ , \tag{24}$$

$$\left\langle | \ t_{ab} \ |^2 | \ t_{cd} \ |^2 \right\rangle_0^{(\beta)} = \frac{2(N + 2 - \beta)(1 + \delta_{ac}\delta_{bd}) - \delta_{ac} - \delta_{bd}}{2N(2N + 1)(2N + 7 - 4\beta)}. \tag{25}$$

We can obtain the first and second moments of the transmission coefficient T of eq. (2) by performing, in the above equations, the sum over channels, with the results [17]

$$\langle T \rangle_0^{(\beta)} - \frac{N}{2} = -\delta_{1\beta} \frac{N}{4N + 2} \to -\frac{1}{4}\delta_{1\beta} \tag{26}$$

$$\mathrm{var}(T) = \frac{N(N + 1)^2}{(2N + 1)^2(2N + 3)} \to \frac{1}{8}, \qquad COE \tag{27}$$

$$= \frac{N^2}{4(4N^2 - 1)} \to \frac{1}{16}, \qquad CUE \tag{28}$$

where the limit is as $N \to \infty$. In this limit, the *weak-localization correction* (26) and the magnitude of the *conductance fluctuations*, $\mathrm{var}(T)$, become *universal*; the latter is twice as large in the presence of time-reversal symmetry ($\beta = 1$) as in the absence of such symmetry. Comparison with numerical simulations of the type already discussed in §4 is presented in figure 4. We see that the agreement with the theoretical results of eqs (26)–(28) is very good.

The distribution of T can be obtained by direct integration of eq. (15). For instance, for $N = 1$ and $\beta = 1$ one obtains

$$w(T) = \frac{1}{2\sqrt{T}}, \tag{29}$$

and, for $\beta = 2$

$$w(T) = 1. \tag{30}$$

These results are compared in figure 5 with the numerical simulations discussed above. Again we conclude that the CE gives a very good description of the data.

When *direct processes are present*, we have to use eq. (20) for the distribution of the S matrix. A number of cases have been worked out analytically for the distribution of the transmission coefficient T. For instance, for $\beta = 2$ and in the presence of direct reflection only (i.e., no direct transmission), one finds

$$\begin{aligned} w(T) = {} & (1 - X^2)^2(1 - Y^2)^2\{(1 - X^4Y^4)(1 - X^2Y^2) \\ & -(1 - T)[(X^2 + Y^2)(1 - 6X^2Y^2 + X^4Y^4) + 4X^2Y^2(1 + X^2Y^2)] \\ & +(1 - T)^2[(1 + X^2Y^2)(6X^2Y^2 - X^4 - Y^4) - 4X^2Y^2(X^2 + Y^2)] \\ & +(1 - T)^3(X^2 + Y^2)(X^2 - Y^2)^2\} \times \{(1 - X^2Y^2)^2 - 2(1 - T) \\ & \times[(1 + X^2Y^2)(X^2 + Y^2)4X^2Y^2] + (1 - T)^2(X^2 - Y^2)^2\}^{-5/2}. \end{aligned} \tag{31}$$

Here, $X = | \langle S_{11} \rangle |$, $Y = | \langle S_{22} \rangle |$.

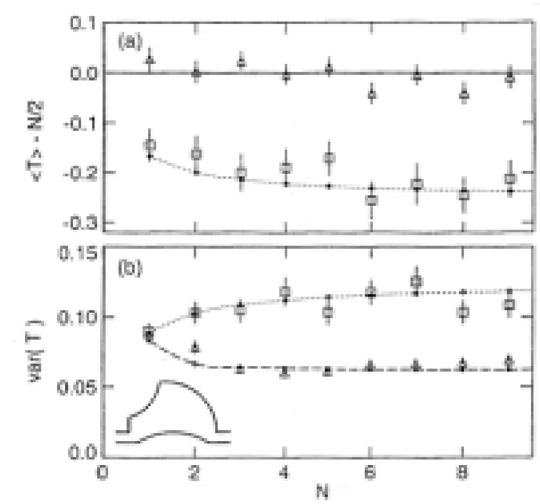

Figure 4. (**a**) The weak-localization correction and (**b**) the variance of the conductance, obtained solving numerically the Schrödinger equation for the system indicated in the lower left corner. The numerical results (with the statistical error bars) are indicated with squares (for $B = 0$) and triangles (for $B \neq 0$). The dotted line is the theoretical result obtained from the COE, and the dashed line the one obtained from the CUE. The abscissa is the number of channels. The agreement is excellent. (From ref. [17].)

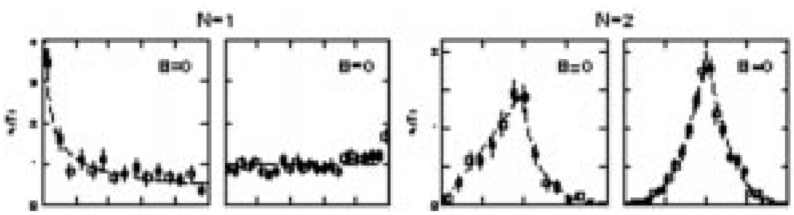

Figure 5. The probability density of the total transmission coefficient T, obtained numerically for the same system as in figure 4. In each panel the number of open channels N is fixed. The dotted lines represent the theoretical results from COE (left column) and CUE (right column). (From ref. [17].)

A number of computer simulations have been performed, solving numerically the Schrödinger equation for 2D structures, in order to compare with our theoretical results. The optical S was extracted from the numerical data and used as an input to eq. (20), in order to find the distribution of T from our model, which thus becomes a *parameter-free prediction*. The results are presented in figure 6. The agreement between the numerical solutions of the Schrödinger equation and our information-theoretic model is, generally speaking, found to be very good [19]. The magnetic field was increased as much as to pro-

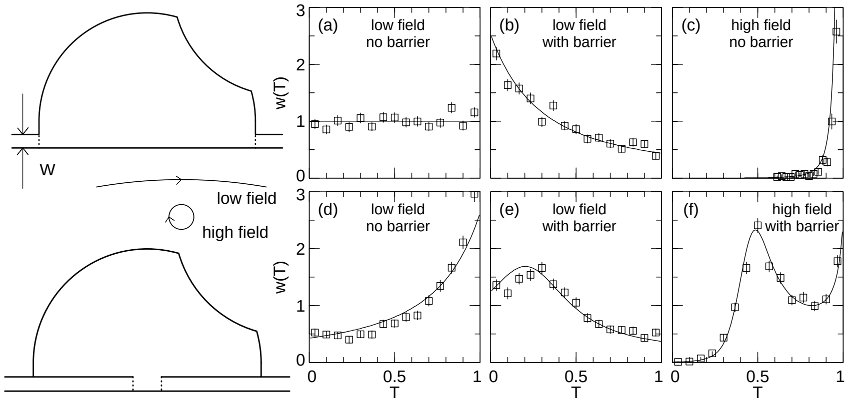

Figure 6. The probability density of the total transmission coefficient T obtained integrating numerically the Schrödinger equation for the billiards shown on the left side of the figure for $N = 1$. The results are indicated with squares, that include the statistical error bars. The curves are obtained from Poisson's kernel, eq. (20), with $\langle S \rangle$ extracted from the numerical data. The agreement is very good. (From ref. [19].)

duce a cyclotron radius smaller than a typical size of the cavity, and about twice the width of the leads, and the agreement was still found to be good. The above predictions for $w(T)$ in the presence of direct processes should be experimentally observable in microstructures where phase breaking is small enough, the sampling of the conductance distribution being performed by varying the energy or shape of the structure with an external gate voltage.

Before closing, we remark that other definitions of information have been proposed [34], in addition to Shannon's definition (18) that was employed in the above analysis. However, the consequences of these other definitions in the present context have not been investigated, and they thus constitute, at present, an open question.

Finally, we wish to point out that the philosophy behind the use of the information-theory criterion is best illustrated by William's of Occam's (1300–1349) famous dictum: 'Essentia non sunt multiplicanda praeter necessitatem', known as the 'Occam razor'. Occam's statement, which, literally, means: 'Entities do not have to be multiplied beyond necessity', was rephrased by Bertrand Russel [35] as: ' If in a certain science everything can be interpreted without a certain hypothesis, there is no reason to use it'.

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