PRAMANA — journal of physics | © Indian Academy of Sciences | Vol. 56, Nos 2 & 3 Feb. & Mar. 2001 pp. 393–401

Interference due to coherence swapping

ARUN KUMAR PATI[1,2] and MAREK ZUKOWSKI[3]
[1]Quantum Optics and Information Groups, School of Informatics, Dean Street, University of Wales, Bangor LL 57 1UT, UK
[2]Theoretical Physics Division, 5th Floor, Central Complex, Bhabha Atomic Research Centre, Mumbai 400 085, India
[3]Instytut Fizyki Teoretycznej i Astrofizyki, Uniwersytet Gdanski, PL-80-952 Gdansk, Poland
[1]Email: akpati@sees.bangor.ac.uk; apati@apsara.barc.ernet.in

Abstract. We propose a method called 'coherence swapping' which enables us to create superposition of a particle in two distinct paths, which is fed with initially incoherent, independent radiation. This phenomenon is also present for the charged particles, and can be used to swap the effect of flux line due to the Aharonov–Bohm effect. We propose an optical version of experimental set-up to test the coherence swapping. The phenomenon, which is simpler than entanglement swapping or teleportation, raises some fundamental questions about the true nature of wave-particle duality, and also opens up the possibility of studying the quantum erasure from a new angle.

Keywords. Quantum interference; qubit; measurement; Aharonov–Bohm effect.

PACS Nos 03.65.Bz; 03.67.Hk

1. Introduction

One of the basic mysteries in quantum theory is the phenomenon of interference of a quantum particle if it has more than one alternatives to choose from [1]. The famous Young's double slit experiment can be demonstrated with single photon or electron, which vividly shows this interference effect in nature. In fact, most of the physical phenomena in micro world can be attributed to the quantum interference. In recent years the quantum interference effects have been exploited to achieve parallelism in quantum computers, which cannot be performed with a classical computer [2,3]. The basic unit in quantum computer 'qubit' is nothing but a linear superposition of two distinct bits which is capable of showing interference effect. In quantum interference (first order) the important requirement is the coherence of a quantum state, which usually we tend to associate with a particle if it has come from a single source and made to pass through a double slit or through a suitable device such as a beam splitter (as in a Mach–Zehnder interferometer). But can we imagine a situation where we can observe interference between two particles coming from independent sources and passing through two independent double slits?

In this paper we propose a scheme called 'coherence swapping' (CS) which enables us to create superposition of a particle in two distinct paths, which is fed with initially incoherent, independent radiation. This allows us to observe interference between two independent sources, which can be quite far apart. In case of charged particles, the coherence swapping method can be used to swap a flux line in Aharonov–Bohm setting. For example, the effect of flux line can be transferred from one set of interfering paths to another set of paths which may be far apart and even may not see the presence of vector potential. The coherence swapping will have application in quantum computer [2,3] where one can create a qubit from independent outputs after some information processing inside two independent quantum computers. For example, one can take one branch of the computational state from one computer and other branch from another computer and then by performing CS operation one can create a new computational state.

The process of 'coherence swapping' is analogous in spirit to the recently proposed scheme 'entanglement swapping' [4,5], but they are different. The difference is that entanglement swapping can be used to create an 'ebit', whereas coherence swapping can be used to create a 'qubit'. The entanglement swapping has also been verified experimentally [6,7] using parametric down conversion sources. The coherence swapping provides us a means to have *interference 'out of nothing'* (or more precisely, *selected out of a complete noise*). The idea of creating correlation for independent emissions goes back to the fundamental paper by Yurke and Stoler [8]. However, the scheme we propose has not been realized before. This raises some question as to what is the true nature of wave-particle duality of a quantum particle. We hope coherence swapping will open up possibility of studying this duality and in particular the quantum erasure problem [10,11] from a new perspective.

2. Coherence swapping

Let us consider the interference phenomenon in a typical Mach–Zehnder interferometer. A particle comes from a source S_1 and falls on a 50-50 beam splitter (BS). After the action of BS the particle is in a superposition of being in the outputs a and b:

$$|\Psi\rangle = \frac{1}{\sqrt{2}}(a^* + b^*)|0\rangle, \tag{1}$$

where we have used the convention that the particle creation operators in the state of being in the given beam, a^* and b^*, bear the same name as the beam, and $|0\rangle$ is the vacuum state. As the later effect will be due to the particle indistinguishability we assume that we deal with bosons (but similar effects are expected for fermions). The interference arises due to the fact that without some additional measurements towards establishing in which path the particle is, its interaction with the beam splitter does not reveal this information. This can be seen by allowing these two paths to recombine and superpose on the exit beam splitter (after passing through a phase shifter PS_1 in say the arm a) and then putting two detectors the exit ports. After passing through phase shifters the state becomes

$$|\Psi\rangle = \frac{1}{\sqrt{2}}(e^{i\phi}a^* + b^*)|0\rangle, \tag{2}$$

after they are recombined at BS_2 the state becomes (we drop the over all phase)

$$\left(\cos\frac{\phi}{2}a^* + i\sin\frac{\phi}{2}b^*\right)|0\rangle, \tag{3}$$

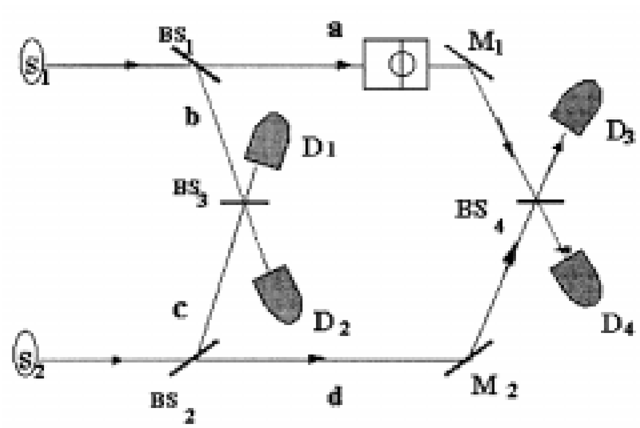

Figure 1. Basic interferometer set-up to observe coherence swapping.

where we used the convention that the transmitted beams are denoted by the same letter as the input ones. This shows that the detector 0 clicks with probability $\sin^2(\frac{\phi}{2})$ and the detector 1 clicks with probability $\cos^2(\frac{\phi}{2})$. This is the interference phenomenon.

Let us consider two independent sources S_1 and S_2 each emitting one particle. Let particle 1 pass through a beam splitter BS_1 as in a Mach–Zehnder interferometer. It splits coherently to two paths a and b. We can write the state of the particle after passing through beam splitter as given in (1). Now suppose the second particle coming from the source 2 passes through another beam splitter BS_2. After passing through BS_2 the state of particle 2 can be similarly written as

$$|\Psi'\rangle = \frac{1}{\sqrt{2}}(c^* + d^*)|0\rangle, \tag{4}$$

where c, d have the same meaning for the 2nd particle. If we allow the paths to recombine then particle 2 will show the interference as before. Let us modify the interferometer set-up in a slightly different way. Instead of recombining path a, b and c, d let us recombine the particles in the path b, c and a, d by introducing suitable beam splitters BS_3, BS_4 and mirrors M_1, M_2 as shown in the figure 1. Now the combined state of the system in the interferometer setup can be written as

$$|\Phi, \Phi'\rangle = \frac{1}{2}(a^* + b^*)(c^* + d^*)|0\rangle. \tag{5}$$

The important observation is that if paths b and c lead to a 50-50 beam splitter BS_3, the following unitary transformation links the inputs and the outputs (denoted by the subscript out)

$$b_{\rm out} = \frac{1}{\sqrt{2}}(b + c)$$

and

$$c_{\text{out}} = \frac{1}{\sqrt{2}}(b - c).$$

Therefore behind the beam splitter BS_3 the state reads as

$$\frac{1}{2}\left(a^* + \frac{1}{\sqrt{2}}(b^*_{\text{out}} + c^*_{\text{out}})\right)\left(\frac{1}{\sqrt{2}}(b^*_{\text{out}} - c^*_{\text{out}}) + d^*\right)|0\rangle. \quad (6)$$

This can be rewritten as

$$\frac{1}{2}\left(a^*d^* + b^*_{\text{out}}\frac{1}{\sqrt{2}}(d^* + a^*) + c^*_{\text{out}}\frac{1}{\sqrt{2}}(d^* - a^*) + \frac{1}{2}(b^{*2}_{\text{out}} + c^{*2}_{\text{out}})\right)|0\rangle. \quad (7)$$

Therefore we see that if one registers a *single* photon in the b output of the mixing beam splitter, the other photon continues its propagation in the state

$$\frac{1}{\sqrt{2}}(d^* - a^*)|0\rangle,$$

i.e. it is in a *coherent* superposition of being in the beam d and a, whereas if a photon is registered in the c output, the other one continues in a superposition, which is orthogonal to the previous one namely

$$\frac{1}{\sqrt{2}}(d^* + a^*)|0\rangle.$$

This shows that the particles passing through the path a and d are now in a pure state conditioned on the detection result behind the mixing beamsplitter. If say only events of single particle counts at b are selected the other particle can reveal interference phenomena (if say we put a phase shifter into the beam d, and then superpose the beams a and d on another beam splitter). This is a method to swap coherence from the primary pairs of possible paths to another pair paths. Such a process of *creating first order single particle interference involving two initially incoherent independent paths we call coherence swapping.* The coherence swapping creates a 'qubit' out of complete random noise, which is the key result of the paper.

3. Coherence swapping and Aharonov–Bohm effect

It is known that if a charged particle encircles a flux line in a Young's double slit experimental set-up then we observe a shift in the interference pattern even though there is no magnetic field present along the path [12]. The field is present *only* inside the solenoid. But since the vector potential is present along the path, the electron wavefunction is affected and that gives rise to a phase shift in the interference pattern. In the standard description of Aharonov–Bohm effect the electron wavefunction is split coherently into $\Psi(r) = \Psi_1(r) + \Psi_2(r)$. In the presence of vector potential the wavefunction is modified to

$$\Psi(r) = \mathrm{e}^{\frac{ie}{\hbar c}\int_{\mathrm{path1}} A.\mathrm{d}x}\Psi_1(r) + \mathrm{e}^{\frac{ie}{\hbar c}\int_{\mathrm{path2}} A.\mathrm{d}x}\Psi_2(r). \tag{8}$$

When they are recombined the interference pattern depends on $\Phi = e/\hbar c \oint \vec{A}\mathrm{d}\vec{x}$, which is the dimensionless flux enclosed by the solenoid, which the electron has never seen. This is a non-local magnetostatic effect.

If in the interferometer of figure 1 one has a magnetic flux in the internal area (bounded by the internal paths of the device), the interference due to the above described coherence swapping depends on the total flux passing thorough this internal area. Therefore one can concentrate all flux in, say just a corner of the device (e.g. close to the beamsplitter BS_1) and still due to the coherence swapping observe interference at the other end of the device, behind the beam splitter BS_4 (of course conditioned on the events behind BS_3). We skip the calculational steps, but one can easily check that the final interference after BS_4 will depend on the flux enclosed by the solenoid. This can be called flux swapping. Thus, flux swapping allows us to transfer the effect of flux line from one set of interfering paths to another set of paths which have no common origin.

4. Optical experiment to test coherence swapping

In this section we propose an optical experimental set-up to test the coherence swapping (which can be done with current technology).

4.1 *Summary of physics of parametric down conversion*

If one shines a strong linearly polarised monochromatic laser beam, or a quasi-monochromatic laser light pulse, on a suitably cut and oriented birefringent crystal endowed with a high quadratic nonlinearity some pump photons spontaneously fission into pairs of photons of lower frequency (for historical reasons called signal and idler). The process is quasi elastic. Thus the frequencies of pump photon, ω, signal, ω_s, and idler, ω_i, satisfy $\omega \approx \omega_s + \omega_i$ (for the pulsed pump this relation still holds, however in this case the pulse frequency is not precisely defined). Photons can be observed only at so-called phase matched directions at which all emission processes within the full illuminated zone of the crystal interfere constructively: $\mathbf{k}_p \approx \mathbf{k}_s + \mathbf{k}_i$, i.e., the emissions are strongly correlated directionally (again, for the pulsed case $\mathbf{k}_p$ is not precisely defined). Due to the dependence of the speed of light on frequency, phase matching within a crystal cannot occur for all frequencies, and all emission directions, and thus into a given direction only specific frequencies are emitted.

Consider a pulsed pump. We assume that the probability of a multiple emission from a single PDC is low, the laser pulse is not too short, i.e., the nonmonochromaticity of the pulse will not blur too much the strong angular correlation of the emissions (due to the effective energy and momentum conservation within the crystal). Thus, the photons can be still described as emitted in specified, very well defined directions.

Under the approximations that: (i) one has perfect phase matching and only two phase matched directions are singled out; (ii) the idler and signal frequencies satisfy perfect energy conservation conditions with the pump photons, which is described by a sharply peaked at the origin function $\Delta_L(\omega - \omega_i - \omega_s)$ which approaches Dirac's delta for $L \to \infty$

(L symbolically represents the crystal's size); (iii) the pump pulse is described as a classical wave packet (no-depletion) with one single direction for all wave vectors (the frequency profile of the pulse will be denoted by $V(\omega)$), the state of the photon *pair* emerging from the PDC source (plus the filtering system) via the beams a and d can be written as

$$|\psi_{ad}\rangle = \int \mathrm{d}\omega_1 \mathrm{d}\omega_2 \mathrm{d}\omega_0 \Delta_L(\omega_0 - \omega_1 - \omega_2) g(\omega_0) \times f_a(\omega_1) f_d(\omega_2) |\omega_1; a\rangle |\omega_2; d\rangle, \quad (9)$$

where, e.g., the ket $|\omega; e\rangle$ describes a single photon of frequency ω in the beam e, the function g represents the spectral content of the pulse, f_e is the transmission function of the filtering system in the beam e (a pinhole and/or a filter).

The amplitude to detect a photon at time $t_{x'}$ by a detector monitoring the beam x' and another one at time $t_{y'}$ by a counter in the beam y', provided the initial photon state was, say, $|\psi_{xy}\rangle$, can be written as $A_{xy}(t_{x'}, t_{y'}) = (\langle t_{x'}; x'|\langle t_{y'}; y'|)|\psi_{xy}\rangle$, where $|t; b\rangle = \frac{1}{\sqrt{2\pi}} \int \mathrm{d}\omega \mathrm{e}^{i\omega t} |\omega; b\rangle$ [4]. The elementary amplitudes of the detection process have a simple, intuitively appealing, form [5]

$$A_{xy}(t_x, t_y) = \frac{1}{\sqrt{2\pi}} \int \mathrm{d}t G(t) F_x(t_x - t) F_y(t_y - t), \quad (10)$$

where the functions denoted by capitals are the Fourier transforms: $H(t) = \frac{1}{\sqrt{2\pi}} \int \mathrm{d}\omega \mathrm{e}^{i\omega t} h(\omega)$, for $h = f$ or g. The convolution of the filter functions in (10) reveals one of the basic properties of the PDC radiation: the time correlation between the detection of the idler and the corresponding signal photon is entirely determined by the bandwidth of the detection system. For example, this implies that in the limit of no filtering, when the functions $F(t)$ are approaching $\delta(t)$, the time correlation is extremely sharp (of the order of femtoseconds), what can be illustrated by somewhat mathematically incorrect limiting case of (10), namely $G(t_x)\delta(t_x - t_y)$, (in reality, one also has to take into account the phase matching function Δ, and this imposes a sharp but finite time correlation for the PDC process (Ou *et al* 1999)). However, just a single filter will blur this correlation to around the inverse of the filter's bandwidth, $\Delta T \approx 1/\Delta\nu$ (the coherence time of the filtered radiation). The function $G(t)$ represents the temporal shape of the laser pulse and its presence in the formula simply indicates that (barring retardation) the signal and idler can be produced only when the pulse is present in the crystal.

If the birefringent crystal is cut in such a way that the so called type I phase matching condition is satisfied, both PDC photons are of the same polarisation (if the pump beam is an ordinary wave the down converted photons are extraordinary). Due to the phase matching condition (4) (single) photons of the same frequency are emitted into cones centred at the pump beam. By picking photons from a specially chosen cone one can have PDC radiation with both photons of equal frequency $\frac{1}{2}\omega_p$. The selection can be done by a suitable pinhole arrangement in a diaphragm behind the crystal and/or with the use of filters.

4.2 *Proposed observation of coherence swapping*

We take two separate down conversion crystals, A, B, however pumped by the same pulsed laser (see figure 2). The pump beam is beam-split in such a way that the pulses enter both

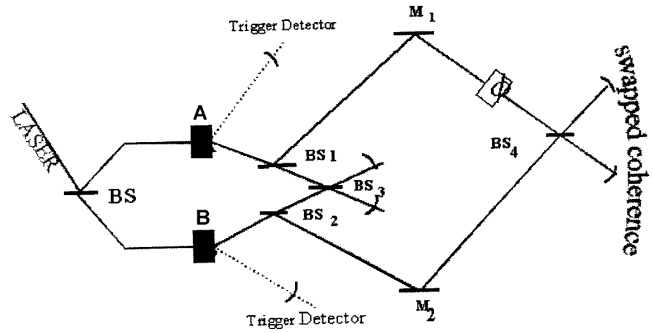

Figure 2. Experimental set up to observe coherence swapping.

crystals at exactly the same moment of time. We choose emission directions from the two crystals in such a way that the emitted photons are degenerate in frequency.

From time to time both sources emit spontaneously a pair of photons each. We direct the idlers from each source to two trigger detectors. The role of these trigger firings is the following one: if both trigger detectors fire we know that the sources emitted additionally one signal photon each. In front of both trigger detectors one finds two identical filters. Their role is to define the coherence time of the detected radiation.

We place two 50-50 beam splitters, BS_1, BS_2, and direct to the first one the signal of the source A, and to the second one the signal of the source B. Each photon enters its beam splitter via one input port. Of course, upon leaving the beam splitters, both photons are in a coherent superposition of being in one or the other exit beam of the beam splitter.

We direct one output of each beam splitter into one of the input ports of yet another beam splitter, BS_3, behind which we place two detectors. If only one of the detector fires, and this firing is due to one photon only, then the other photon is in a coherent superposition of being in one or the other exit beam (not fed into BS_4) of the primary beam splitters BS_1, BS_2. If one wants to have just one superposition state in beams a and d, one should introduce a phase shift of 0 or π in one of the beams, depending on which of the detectors behind the beam splitter BS_3 fired (this can be thought to be the 'classical communication' component here, if one seeks parallels of this experiment with the teleportation process).

The source must be pulsed in order to warrant that the parts of wave packets of the two photons that impinge on the BS overlap in the exit beams of it. Only in this way have the required total erasure of the information: which of the sources contribute to the click at behind BS_3. This warrants the required superposition (coherence) to form in the other two beams. One can test all this in an interference experiment on the beams.

The visibility of the obtained interference behind a device which is composed of a phase shifter and a beam splitter, BS_4, can be obtained in the following simple way. One assumes that the filters in beams leading to the trigger detectors (behind BS_3) are identical, and that the functions have the following structure: $F_f(t) = \mathrm{e}^{-\frac{i}{2}\omega_p^o t}|F_f(t)|$, $G(t) = \mathrm{e}^{-i\omega_p^o t}|G(t)|$, where ω_p^o is the central frequency of the pulse, then interference fringes in the joint probability to have joint counts in four detectors, that is the two trigger detectors, one of the detectors behind BS_3, and one of the detectors behind BS_4, behaves as $1 - V\cos(\phi)$ with the visibility V given by

$$V = \frac{\int \mathrm{d}^4 t |A_{ad}(t_a, t_d) A_{bc}(t_b, t_c) A_{bd}(t_b, t_d) A_{ac}(t_a, t_c)|}{\int \mathrm{d}^4 \mathrm{d}t |A_{ad}(t_a, t_d) A_{bc}(t_b, t_c)|^2}, \tag{11}$$

where $\mathrm{d}^4 t = \mathrm{d}t_a \mathrm{d}t_b \mathrm{d}t_c \mathrm{d}t_d$.

If one specifies, for simplicity, all the functions as gaussians, $\exp\left[-\frac{1}{2}(\omega - \Omega)^2/\sigma^2\right]$, where Ω is the mid frequency and σ the width, the formula for the visibility reads:

$$V = \left(\frac{\sigma_p^2}{\sigma_p^2 + \sigma_f^2}\right)^{\frac{1}{2}}, \tag{12}$$

where σ_p^2 is the pulse frequency spread, σ_f^2 is the width of the filters.

The feasibility of this scheme has already been tested in the teleportation and entanglement swapping experiments [13,6,7].

5. Conclusions

To conclude, we have proposed a new way of generating coherence between independent paths of two different quantum systems. The process of measurement, which was thought to be a hindrance to coherence, can be suitably designed to create coherence. Using coherence swapping method, one can swap the effect of flux line to a distant site. Finally, we have proposed an optical realisation of coherence swapping method. The phenomenon seems to be the simplest possible process involving interference of independent sources of quantum particles. Much simpler than entanglement swapping or teleportation, however it shares with them the basic property: possibility of a state preparation at a remote place with the use of wave vector collapse and classical transfer of information (this latter feature is inherently associated with the conditional nature of the interference behind BS_4). One can generalise the concept of coherence swapping for multi path and muliti particle interference set ups similar to the idea of entanglement swapping for multiparticles [14]. We hope that this will open up the possibility of studying the wave particle duality and quantum erasure for independent particles.

Acknowledgement

AKP wishes to thank Sam Braunstein and P van Loock for discussions. He thanks EPSRC for the financial support to carry out the research. MZ was supported by the University of Gdansk, Grant No. BW/5400-5-0032-0.

References

[1] R Feynman, R Lighton and M Sands, *The Feynman lectures on physics* (Addision Wesley, Reading, 1965) vol. III
[2] D Deutsch, *Proc. R. Soc. London* **400**, 97 (1985)
[3] S L Braunstein, *Encylopedia of applied physics* (Wiley Publication, 1999) pp. 239–255
[4] M Zukowski, A Zeilinger, M A Horne and E Ekert, *Phys. Rev. Lett.* **A71**, 4278 (1993)

[5] M Zukowski, A Zeilinger and H Weinfurter, *Fundamental problems in quantum theory* (Annals of New York Academy of Sciences, 1995)
[6] J-W Pan, D Bouwmeester, H Weinfurter, A Zeilnger, *Phys. Rev. Lett.* **80**, 3891 (1998)
[7] D Bouwmeester, K Mattle, J-W Pan, H Weinfurter, A Zeilnger, M Zukowski, *Appl. Phys.* **B67**, 749 (1998)
[8] B Yurke and D Stoler, *Phys. Rev.* **A46**, 2229 (1992)
[9] B Yurke and D Stoler, *Phys. Rev. Lett.* **68**, 1251 (1992)
[10] M O Scully, B-G Englert and H Walther, *Nature* **351**, 111 (1991)
[11] F Herbut and M Vujicic, *Phys. Rev.* **A56**, 931 (1997)
[12] Y Aharonov and D Bohm, *Phys. Rev.* **115**, 485 (1959)
[13] D Bouwmeester, J-W Pan, K Mattle, M Eible, H Weinfurter and A Zeilinger, *Nature* **390**, 575 (1997)
[14] S Bose, V Vedral and P Knight, *Phys. Rev.* **A57**, 822 (1998)