Teleportation via decay

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Abstract. We present a rare example of a decay mechanism playing a constructive role in quantum information processing. We show how the state of an atom trapped in a cavity can be teleported to a second atom trapped in a distant cavity by the joint detection of photon leakage from the cavities. The scheme, which is probabilistic, requires only a single three level atom in a cavity. We also show how this scheme can be modified to a teleportation with insurance.

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1. Introduction

An undetected decay is a mechanism for coherence loss in a quantum system. A decay therefore normally plays a negative role in quantum information processing [1]. However, a detected decay is a measurement on the state of the system from which the decay ensues. Could the detection of a decay be used in a fruitful way for quantum information processing? Two recent papers [2,3] illustrate that this is possible by showing how the detection (or the non detection) of decays can be used to entangle the states of distinct atoms. In this paper we will show that this approach is not limited to the establishment of entanglement. It can, in fact, be used for *complete* quantum communication protocols such as *teleportation* [4].

In our teleportation proposal, the states to be teleported (the 'stationary qubits') are internal states of an atom, ideal for the storage of quantum information. Quantum information is physically transferred from place to place via photonic states (the 'flying qubits' [5]), which are the best long distance carriers of quantum information. In all experimental implementations of teleportation to date [6–8], and in some related proposals [9], the stationary qubits have been of optical origin. Optical qubits are very difficult to hold in a place and light trapped in a cavity eventually leaks out. These are thus not ideal for the long term storage of quantum information. There have, of course, been earlier proposals for atomic state teleportation [10], which use the ideal stationary qubit. But in these proposals, the flying qubits, used to establish entanglement prior to teleportation, have been atomic states as well. However, atoms move slowly and also interact strongly with their environment and are not ideal for long distance transfer of quantum information. Our scheme differs from

these earlier experiments and proposals in using both the ideal stationary qubit (atomic state) and the ideal flying qubit (optical state).

Our scheme also differs crucially from the much studied quantum communication setup in which a photon leaks out of one cavity with quantum information and transfers this quantum information to another atom in a distant cavity by getting into this distant cavity from outside [5,11–14]. However, though light quickly leaks out of a cavity, it is very difficult to get it into a good cavity from outside. Feeding a photon into a cavity from outside is thus an extremely sophisticated task [11,12]. Our scheme dispenses with this requirement. It does not require a direct carrier of quantum information between distant atoms. The joint detection of photons leaking from distinct cavities constitutes a measurement that enables a *disembodied* transfer of quantum information from an atom in one of the cavities to an atom in the other.

2. Outline of the scheme

The setup consists of two optical cavities A and B (supporting cavity modes A and B) respectively. Each containing a single trapped Λ three level atom, as shown in figure 1. Atoms in cavities A and B are designated 1 and 2 respectively. The photons leaking out from both the cavities impinge on the 50–50 beam splitter S and are detected at the detectors D_+ and D_-. For simplicity, in this article, we will assume unit efficiency of photon detection (a slightly more detailed analysis including finite efficiency detectors can be found towards the end of ref. [15]). The cavity A, atom 1, beam splitter S and the detectors D_+ and D_- belong to Alice. The cavity B with atom 2 belongs to Bob. We require both the cavities to be *one sided* so that the only leakage of photons occur through the sides of the cavities facing S. By following our teleportation protocol, Alice can teleport an unknown state of her atom 1 to the atom 2 held by Bob in three stages.

The first stage is the preparation stage in which Alice maps her atomic state to her cavity state [16]. While Alice is doing this, Bob creates a maximally entangled state of his atom and his cavity mode. The next stage is the detection stage which lasts for a predetermined *finite* period of time. During this stage Alice simply waits for either or both of her detectors to click. If any one of the detectors register a single click during this time period, then the protocol is successful. Otherwise Alice informs Bob that the protocol has failed. This protocol can be related to the standard teleportation protocol [4] by noting that the beam splitter and the detectors constitute a device for measurement of the joint state of the two cavities in the basis $\{|0\rangle_A |0\rangle_B, |1\rangle_A |1\rangle_B, \frac{1}{\sqrt{2}}(|0\rangle_A |1\rangle_B + |1\rangle_A |0\rangle_B), \frac{1}{\sqrt{2}}(|0\rangle_A |1\rangle_B - |1\rangle_A |0\rangle_B)\}$. Here $\{|0\rangle_A, |1\rangle_A\}$ and $\{|0\rangle_B, |1\rangle_B\}$ are photon number states in cavities A and B respectively. The teleportation is probabilistic, because it is successful only for the pair of Bell state outcomes of the above measurement. We will describe later how to convert this to a *reliable* state transfer protocol. At the end of the detection period, if the protocol has been successful, Alice lets Bob know whether D_+ or D_- had clicked. This corresponds to the classical communication part of the standard teleportation protocol [4]. Dependent on this information Bob applies a local unitary operation to his atom to obtain the teleported state. We call this the post-detection stage.
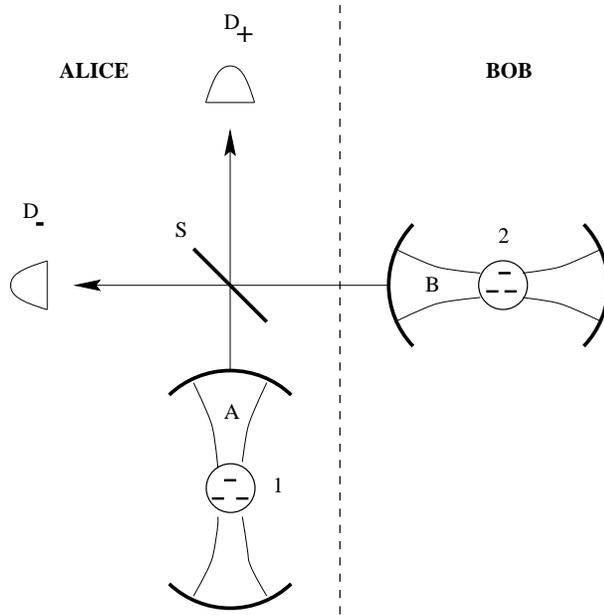
Figure 1. Our setup for atomic state teleportation. Cavity A, atom 1, the beam splitter S and the detectors D_+ and D_- belong to Alice. Cavity B and atom 2 belong to Bob.

3. Detailed analysis

We now proceed to present a detailed analysis of the scheme. We wish to look at single realizations conditioned on detection (or not) of cavity decays. For such an analysis, the ideal unraveling of the system's evolution is through the quantum jump approach [17]. Let the photon decay rate from both the cavities be κ. While Alice/Bob is applying a Hamiltonian H to her/his atom-cavity system, its evolution subject to no detector click, is governed by the effective Hamiltonian (with $\hbar = 1$)

$$H_{\text{eff}} = H - i\kappa c^\dagger c, \tag{1}$$

where $c^\dagger$ and c are the creation and the destruction operators for the cavity mode under consideration. The coherent evolution due to H_{eff} is interrupted by quantum jumps when there is a click in either the detector D_+ (corresponds to an action of the operator $(c_A + c_B)/\sqrt{2}$ on the joint state vector of the pair of atom-cavity systems, c_A and c_B being the lowering operators for modes A and B respectively) or the detector D_- (corresponds to an action of the operator $(c_A - c_B)/\sqrt{2}$ in the same way).

The three level atoms have two ground states $|g\rangle$ and $|e\rangle$ (e.g. hyperfine ground states) and an excited state $|r\rangle$ (with a spontaneous decay rate γ) as shown in figure 2. Alice and Bob use two types of time evolutions of the atom-cavity system as their basic local operations. The first type an adiabatic evolution (shown in figure 2) which is initiated by switching on a classical laser field which drives the $|e\rangle \rightarrow |r\rangle$ transition with a coupling constant Ω. The $|r\rangle \rightarrow |g\rangle$ transition is driven by the quantized cavity mode of

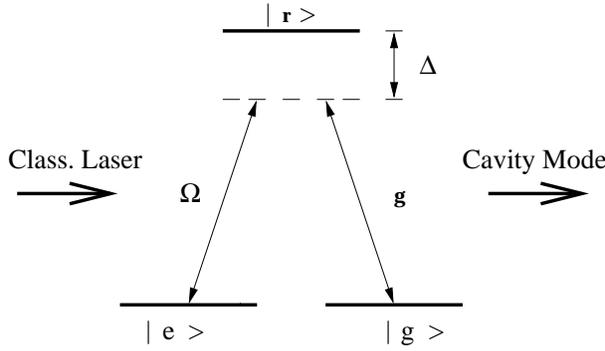**Figure 2.** The level configuration of the trapped atom showing the fields responsible for the adiabatic evolution. The $|e\rangle \to |r\rangle$ transition being driven by a classical laser field of coupling Ω and the $|r\rangle \to |g\rangle$ transition being driven by the quantized cavity mode of coupling g. Δ is the detuning of both the classical laser field and the quantized field mode from their respective transitions.

coupling g. Both the classical laser field and the cavity modes are assumed to be detuned from their respective transitions by the same amount Δ. As the atom is trapped in a specific position in the cavity, we can assume that the couplings Ω and g remain constant during the interaction. We choose parameters such that $g\Omega/\Delta^2 \ll 1$ (the upper level $|r\rangle$ can then be decoupled from the evolution) and $\Delta \gg \gamma$ (the spontaneous decay rate from $|r\rangle$ can be neglected). The Hamiltonian for the evolution of the system under such conditions (and assuming $g = \Omega$ for simplicity), is given by

$$H^{(1)} = E|e\rangle\langle e| + E|g\rangle\langle g| + E(c|e\rangle\langle g| + c^\dagger|g\rangle\langle e|), \tag{2}$$

where $E = g\Omega/\Delta$ [12]. The other local operation accessible to Alice and Bob is the Zeeman evolution used to give an arbitrary phase shift of the level $|e\rangle$ relative to the level $|g\rangle$. The Hamiltonian for this evolution is

$$H^{(2)} = \delta E|e\rangle\langle e|, \tag{3}$$

where δE is an energy difference.

Let the unknown state of the atom 1 which Alice wants to teleport be

$$|\Psi\rangle_1^I = a|e\rangle_1 + b|g\rangle_1, \tag{4}$$

where the superscript I in $|\Psi\rangle_1^I$ stands for input and a and b are complex amplitudes. We will assume that the initial state of Alice's cavity is $|0\rangle_A$ and the initial state of Bob's atom-cavity system is $|e\rangle_2|0\rangle_B$. At first, Alice maps the state of atom 1 onto the cavity mode A by switching the Hamiltonian $H^{(1)}$ on for a period of time t_I given by $\tan\frac{\Omega_\kappa t_I}{2} = -\frac{\Omega_\kappa}{\kappa}$ where $\Omega_\kappa = \sqrt{4E^2 - \kappa^2}$. Subject to no decay being recorded in the detectors, the cavity state is given by

$$|\Psi\rangle_A^I = \frac{1}{\sqrt{|a|^2\alpha^2 + |b|^2}}(a\alpha|1\rangle_A + b|0\rangle_A), \tag{5}$$

where $\alpha = (\frac{e^{-\frac{\kappa}{2}t_I}}{\Omega_\kappa} 2E \sin \frac{\Omega_\kappa t_I}{2})$. The probability that no photon decay takes place during this evolution is given by $P_{ND}(\mathrm{A}) = (|a|^2\alpha + |b|^2)$. Meanwhile, Bob also switches on the Hamiltonian $H^{(1)}$ in his cavity for a shorter length of time t_E given by $\tan \frac{\Omega_\kappa t_E}{2} = -\frac{\Omega_\kappa}{2E-\kappa}$. His atom-cavity system thus evolves to the entangled state

$$|\Psi\rangle_{2,\mathrm{B}}^E = \frac{1}{\sqrt{2}}(|e\rangle_2|0\rangle_\mathrm{B} + i|g\rangle_2|1\rangle_\mathrm{B}). \tag{6}$$

The probability that no photon decay takes place during this evolution is given by $P_{ND}(\mathrm{B}) = |\beta|^2$ where $\beta = \frac{e^{-\frac{\kappa}{2}t_E}}{\Omega_\kappa} 2\sqrt{2}E \sin \frac{\Omega_\kappa t_E}{2}$. For simplicity, we assume that Alice and Bob synchronize their actions such that the preparation of the states $|\Psi\rangle_\mathrm{A}^I$ and $|\Psi\rangle_{2,\mathrm{B}}^E$ terminate at the same instant of time. This concludes the preparation stage of the protocol. The probability that this stage is a success is the probability that no photon decays from either cavity during the preparation. This is given by $P_{\mathrm{suc}}(\mathrm{prep}) = P_{ND}(\mathrm{A})P_{ND}(\mathrm{B})$. We will choose $\Omega_\kappa >> \kappa$ which makes $P_{\mathrm{suc}}(\mathrm{prep}) \sim 1$.

Now comes the detection stage, in which Alice simply waits for any one of the detectors D_+ or D_- to click. She waits for a finite detection time denoted by t_D. Alice and Bob reject the cases in which Alice does not register any click or registers two clicks. The joint state of Alice's and Bob's system at the beginning of the detection stage is

$$|\Phi(0)\rangle = |\Psi\rangle_\mathrm{A}^I \otimes |\Psi\rangle_{2,\mathrm{B}}^E. \tag{7}$$

Assume Alice registers a single click at a time $t_j \leq t_D$. The joint state of Alice's and Bob's system evolves as $|\Phi(t)\rangle_{\mathrm{A},2,\mathrm{B}} = |\Psi(t)\rangle_\mathrm{A}^I \otimes |\Psi(t)\rangle_{2,\mathrm{B}}^E$ [17], where $|\Psi(t)\rangle_A^I = (a\alpha e^{-\kappa t}|1\rangle_\mathrm{A} + b|0\rangle_\mathrm{A})/\sqrt{|a\alpha|^2 e^{-2\kappa t} + |b|^2}$ and $|\Psi(t)\rangle_{2,\mathrm{B}}^E = (|e\rangle_2|0\rangle_\mathrm{B} + i e^{-\kappa t}|g\rangle_2|1\rangle_\mathrm{B})/\sqrt{1 + e^{-2\kappa t}}$. The registering of a click at one of the detectors corresponds to the action of the jump operators $(c_\mathrm{A} \pm c_\mathrm{B})/\sqrt{2}$ on the state $|\Phi(t_j)\rangle_{\mathrm{A},2,\mathrm{B}}$. Then the resultant joint state of Alice's and Bob's system becomes

$$\begin{aligned}|\Phi(t_j)\rangle_{\mathrm{A},2,\mathrm{B}}^{J\pm} = &\frac{1}{\sqrt{P_{ND}(\mathrm{A}) + 2|a|^2\alpha^2 e^{-2\kappa t_j}}}\{(a\alpha|e\rangle_2 \\ &\pm ib|g\rangle_2) \otimes |0\rangle_\mathrm{A}|0\rangle_\mathrm{B} \\ &+ e^{-\kappa t_j} a\alpha|g\rangle_2 \otimes (|1\rangle_\mathrm{A}|0\rangle_\mathrm{B} \pm |0\rangle_\mathrm{A}|1\rangle_\mathrm{B})\}.\end{aligned} \tag{8}$$

$|\Phi(t_j)\rangle_{\mathrm{A},2,\mathrm{B}}^{J\pm}$ corresponds to the click being registered in $D_\pm$ and the superscript J stands for jump. At the end of a successful detection stage the joint state of the cavities A, B and atom 2 will be $|\Phi(t_D)\rangle_{\mathrm{A},2,\mathrm{B}}^{J\pm}$. In the post detection stage, Bob uses $H^{(2)}$ to give $|g\rangle_2$ an extra phase shift with respect to $|e\rangle_2$. This phase shift is $-i$ if D_+ had clicked and i if D_- had clicked. This concludes the entire protocol.

4. Fidelity and success probability

We now proceed to estimate the fidelity of the teleported state generated at Bob's end with respect to Alice's input state $|\Psi\rangle_1^I$. First we must note that though the field *continues to decay* even after the protocol is over (i.e Alice has ceased to

keep track of detector clicks), the reduced density matrix of atom 2 remains unchanged, as this atom no longer interacts with the cavity field. Thus the average density matrix of Bob's atom generated due to our teleportation procedure is given by $\rho_2^{\text{Tel}} = \{P_{ND}(A)|\Psi\rangle_2\langle\Psi|_2 + 2|a|^2\alpha^2 e^{-2\kappa t_D}|g\rangle_2\langle g|_2\}/\{P_{ND}(A) + 2|a|^2\alpha^2 e^{-2\kappa t_D}\}$, where $|\Psi\rangle_2 = (a\alpha|e\rangle_2 + b|g\rangle_2)/\sqrt{|a|^2\alpha^2 + |b|^2}$. The fidelity of this state with respect to the input state is $F(t_D, a, b) = \{P_{ND}(A)(|a|^2\alpha + |b|^2) + 2|a|^2\alpha^2 e^{-2\kappa t_D}|b|^2\}/\{P_{ND}(A) + 2|a|^2\alpha^2 e^{-2\kappa t_D}\}$. We see that apart from the system parameters κ and Ω_κ, the fidelity of the generated state also depends on the detection time t_D and the modulus of the amplitudes a and b of the initial state. It is a teleportation protocol with a *state dependent fidelity*. The fidelity does not depend on $P_{ND}(B)$ because the initial state $|\Psi\rangle_{2,B}^E$ prepared by Bob is independent of the decay rate of his cavity.

We plot the variation of the average fidelity of teleportation over all possible input states as a function of the detection time t_D in figure 3. We see that the fidelity increases with increasing detection time. This happens because increasing the detection time decreases the proportion of $|g\rangle_2\langle g|_2$ in the teleported state ρ_2^{Tel} and brings it closer to the initial state $|\Psi\rangle_1^I$ of Alice's atom. The parameter regime used for figure 3 $\{(g : \Omega : \kappa : \gamma : \Delta)/2\pi = (10 : 10 : 0.01 : 1 : 100) \text{ MHz}\}$, is carefully chosen to satisfy all our constraints $(g\Omega/\Delta^2 \ll 1, \Delta \gg \gamma, \Omega_\kappa \gg \kappa)$. This regime could be approached, for example, by increasing the cavity finesse of ref. [18] by an order of magnitude and increasing the length of that cavity to about a millimeter while keeping the beam waist and other parameters constant. As evident from figure 3, the average fidelity exceeds 0.99 for a detection time of about half the cavity life time.

The total probability of success of the protocol is also state dependent and given by $P_{\text{suc}} = P_{\text{suc}}(\text{prep}) \times P_{1D}(0, t_D) = (P_{ND}(A) + 2|a|^2\alpha^2 e^{-2\kappa t_D})P_{ND}(B)(1 - e^{-2\kappa t_D})/2$,
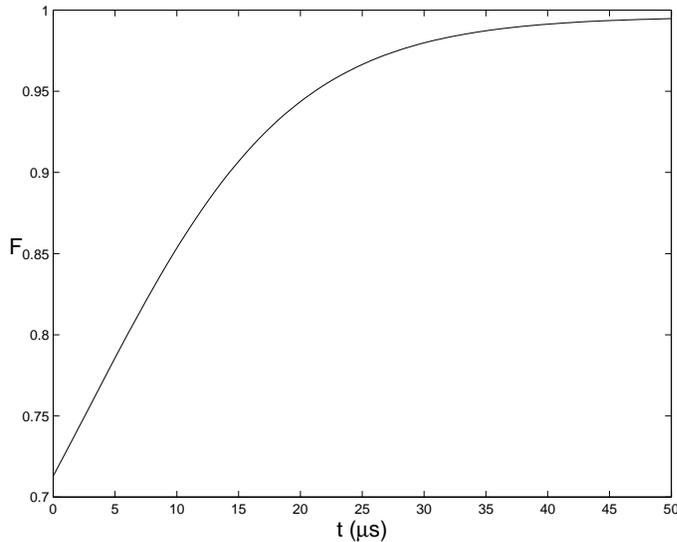

Figure 3. The improvement of average teleportation fidelity with the length of the detection stage. The parameter regime is $(g : \Omega : \kappa : \gamma : \Delta)/2\pi = (10 : 10 : 0.01 : 1 : 100)$ MHz

where $P_{1D}(0, t_D)$ is the probability of a single decay during the detection period. In the parameter domain under consideration, for $t_D = 50 \; \mu$s, we find that the average of the probability of success over all input states is about 0.49. This is a little lower than the ideal success probability of 0.5 (for Alice registering any of the pair of Bell state outcomes) because the preparation stage has an extremely small, but finite, chance of failure.

5. Teleportation with insurance

Consider the situation in which Alice has been supplied with only *one* copy of atom 1 in an unknown state and asked to transfer it to Bob. If Alice attempts our teleportation protocol and fails, the only copy of the unknown state gets destroyed. How to prevent this? The solution is to slightly modify our probabilistic protocol so that in the cases when the protocol is unsuccessful, the original state of Alice's atom 1 is not destroyed, but mapped onto another reserve atom r trapped in Alice's cavity. We will call this *teleportation with insurance*. To accomplish this, Alice has to follow the *local redundant encoding* of ref. [13] and code her initial state $|\Psi\rangle_1^I$ as $a(|e\rangle_1|g\rangle_r + |g\rangle_1|e\rangle_r) + b(|g\rangle_1|g\rangle_r + |e\rangle_1|e\rangle_r)$. After this, she just follows the same protocol as before. But in cases when the protocol is unsuccessful, she is left with either the state $a|g\rangle_r + b|e\rangle_r$ or a state that can be converted to $a|g\rangle_r + b|e\rangle_r$ by a known unitary transformation. She can now exchange the roles of atom 1 and atom r and try to teleport the state $|\Psi\rangle_1^I$ again. She can repeat this procedure until teleportation is successful.

6. Other potential schemes involving the same setup and cavity decays

The most straightforward extension of the applications of the above setup is the teleportation of entanglement (or entanglement swapping [19,20]). The states of the atom and the cavity mode are entangled in each cavity and a joint detection of the state of the two cavity modes at the beam splitter entangles the atoms due to entanglement swapping. The type of entanglement generated in this way has been discussed in ref. [15] (where its value, according to ref. [21] has also been computed). If there were a multiple number of cavities, if atom-cavity mode entangled states were generated in each of these, and if a joint detection of the state of all the modes was done in a multiport beam splitter, an entangled cat state of all the atoms would be generated. This follows from multiparticle entanglement swapping described in ref. [20]. One could also demonstrate the local concentration of shared entanglement via entanglement swapping [22] using our setup. In this case Alice and Bob have to prepare a certain type of less than maximally but equally entangled state of their respective atoms and cavity modes. Then the usual joint detection of decays will concentrate the shared entanglement for those cases when a single click is recorded in Alice's detectors. One could also do dense coding [23] with our setup. The atoms in distant cavities are first entangled by teleporting entanglement as described above. This acts as the prior entanglement shared in the dense coding protocol. Now Bob performs the standard rotations on the state of his atom to encode messages for Alice. Both he and Alice then map their atomic states onto their cavity fields. Alice can distinguish three possible joint states of the two cavity modes by detector clicks (the states

$\frac{1}{\sqrt{2}}(|0\rangle_A|0\rangle_B \pm |1\rangle_A|1\rangle_B)$, $\frac{1}{\sqrt{2}}(|0\rangle_A|1\rangle_B + |1\rangle_A|0\rangle_B)$ and $\frac{1}{\sqrt{2}}(|0\rangle_A|1\rangle_B - |1\rangle_A|0\rangle_B)$ correspond to none or double detector clicks, single click on D_+ and single click on D_- respectively). Thus the purpose of dense coding is achieved in the sense that Bob conveys three possible messages by sending a single qubit to Alice.

7. Conclusions

In this paper, we have presented a simple scheme for atomic state teleportation, which could be implemented by trapping single three level atoms in a cavity. Moreover, by adding one more atom to Alice's cavity, it can be converted to a *reliable* state transfer protocol. This state transfer protocol can be viewed as an *alternative* to designer laser pulses for transferring (refs [11,12]) quantum information into a cavity from outside. This state transfer should work for distances of the order of the absorption length scales of a fiber. The model independent portions of the analysis of communication through a noisy quantum channel [13,14,24] should carry over to this decay-induced scenario of state transfer. The scheme described here is also a rare example of a decay playing an useful role in quantum information processing. We have also mentioned some other potential applications of our setup and leave the detailed consideration of these other applications for the future.

Acknowledgements

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