PRAMANA — journal of physics
© Indian Academy of Sciences
Vol. 56, Nos 2 & 3
Feb. & Mar. 2001
pp. 239–243

A verification of quantum field theory – measurement of Casimir force

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Abstract. Here we review our work on measurement of the Casimir force between a large aluminum coated a sphere and flat plate using an atomic force microscope. The average statistical precision is 1% of the force measured at the closest separation. We have also shown nontrival boundary dependence of the Casimir force.

Keywords. Casimir force; zero point energy; atomic force microscope.

PACS Nos 12.20.Fv; 61.16.Ch

1. Introduction

Casimir force [1,2] results from the alteration by the metal boundaries of the zero point energy $E = \sum_n^\infty 1/2\hbar\omega_n$, where $\hbar\omega_n$ is the photon energy in each allowed photon mode n. The regularized zero point energy per unit area between two parallel plates of infinite conductivity separated by a distance z is given by [1,2]

$$U(z) = -\frac{\pi^2\hbar c}{720}\,\frac{1}{z^3}. \tag{1}$$

This results in a Casimir force per unit area $F/A = -\delta U/\delta z = -(\pi^2\hbar c/240)(1/z^4)$. It is a strong function of z and is measurable only for $z < 1\ \mu$m. Experimentally it is hard to configure two parallel plates uniformly separated by distances less than a micron. However, when one plate is spherical eq. (1) can be modified; and the force for this geometry can be obtained by the use of proximity force theorem [3]. The modified Casimir force in plate-sphere geometry is given by

$$F_c^0(z) = -\frac{\pi^3}{360}R\,\frac{\hbar c}{z^3}, \tag{2}$$

where R is the radius of curvature of the spherical surface. The finite conductivity of the metal leads to a correction which for a given metal plasma frequency ω_p is [4,5]

$$F_c(z) = F_c^0(z)\left[1 - 4\frac{c}{z\omega_p} + \frac{72}{5}\left(\frac{c}{z\omega_p}\right)^2\right]. \tag{3}$$

Such a correction also can be accomplished through use of Lifshitz theory [6]. Using macroscopic fluctuation theory Lifshitz generalized the Casimir force between any two infinite dielectric half spaces as the force between fluctuating dipoles induced by the zero point electromagnetic fields and obtained the same results as Casimir for two perfectly reflecting (infinite conductivity) flat plates. For a metal with a dielectric constant the force between a large sphere and flat plate is given by

$$F^0(z) = -\frac{R\hbar}{\pi c^3}\int \mathrm{d}z \int_0^\infty \int_1^\infty p^2\xi^3 \mathrm{d}p\mathrm{d}\xi \times \left\{ \left[\frac{(s+p)^2}{(s-p)^2}\mathrm{e}^{\frac{2p\xi z}{c}} - 1\right]^{-1} + \left[\frac{(s+p\epsilon)^2}{(s-p\epsilon)^2}\mathrm{e}^{\frac{2p\xi z}{c}} - 1\right]^{-1} \right\}, \tag{4}$$

where $s = \sqrt{\epsilon - 1 + p^2} \cdot \epsilon(i\xi) = 1 + \frac{2}{\pi}\int_0^\infty \frac{\omega\epsilon''(i\xi)}{\omega^2+\xi^2}\mathrm{d}\omega$ is the dielectric constant of the metal, and ϵ'' is the imaginary component of ϵ. ξ is the imaginary frequency given by $\omega = i\xi$. There are at least two other corrections to the Casimir force resulting from 1) the stochastic roughness of the surface, which changes the surface separation [7,8] and, 2) thermal correction [9].

If we replace the flat plate by a sinusoidally corrugated (period $= \lambda$) plate the averaged regularized energy per unit area from eq. (1) will be modified to

$$\left\langle U\left(z + A\sin\frac{2\pi x}{\lambda}\right)\right\rangle = -\frac{\pi^2\hbar c}{720}\frac{1}{z^3}\sum_m C_m\left(\frac{A}{z}\right)^m, \tag{5}$$

where $\langle\ \rangle$ stands for average over the size L of the plate, and A is the amplitude of the sinusoidal corrugation. Using poximity force theorem the corresponding force is

$$F = 2\pi R\langle U\rangle. \tag{6}$$

In practice one needs to replace z in above eqs (1)–(6) by $z + z_0$, where z_0 is the mean surface separation after contact due to the stochastic roughness (+ the periodic corrugation for corrugated surface) of the metal coating. For the corrugated surface the origin of the measurement of z_0 is taken such that the mean oscillation of the corrugation is zero. In the above, $\lambda \ll L$ and $z + z_0 > A$ have been used (perturbative approximation). The nonzero even power coefficients in eq. (5) are $C_0 = 1, C_2 = 3, C_4 = 45/8, C_6 = 35/4$.

The Casimir force has been measured between two flat plates [10], and a large sphere and flat plate [11–13]. Spring balances were used in [10] and torsion pendulum in [11]. We have measured the Casimir force using an atomic force microscope (AFM). Our experimental results agree with the theory with an average rms deviation 1% [12,13]. Theoretical treatments of the Casimir force have shown that it is a strong function of the boundary geometry [14,15]. Using periodic uniaxial corrugated surface (PUSC) we have demonstrated the nontrivial boundary dependence of the Casimir force [16].

2. Experiments

Schematic of the experiment is shown in figure 1. We used standard atomic force microscope (AFM) to measure the force between a metal coated flat (or corrugated) plate and a sphere at a pressure below 50 m Torr and at room temperature. The detail of the experiments are discussed in refs [12,13]. Polysterene sphere of 200 ± 4 μm diameter were mounted on the tip of 320 μm long cantilevers with Ag epoxy. An optically polished sapphire plate of diameter 10 mm was used to represent the flat plate. To get PUSC surface a diffraction grating with a uniaxial sinusoidal corrugation of period $\lambda = 1.1$ μm and amplitude of 90 nm was used. The radius of the sphere is much greater than the periodicity and the surface separation. The cantilever (with sphere), corrugated plate, and flat plate were coated with 250 nm of Al (measured with AFM) in a thermal evaporator. Al metal was chosen because of its high reflectivity at short wavelengths (corresponding to close surface separations). Alternatively, as Al is a simple metal and its ϵ can be well represented with the free electron model of metals with a plasma frequency ω_p, as it does not have inter-band resonances in the region of interest [17]. In such representation $\epsilon(i\xi) = 1 + \frac{\omega_p^2}{\xi^2+\gamma\xi}$ is the dielectric constant of Al, ξ is the imaginary frequency given by $\omega = i\xi$. The wavelength, λ_p, corresponding ω_p is 100 nm [17]. γ is the relaxation frequency corresponding to 63 meV [17]. Both representations of ϵ, lead to similar results for surface separations and experimental uncertainties being reported. To minimize any space charge effect due to patch oxidation of the Al surfaces all surfaces are then coated with 8 nm layer (measured by AFM) of 60% Au/40% Pd, to form a nonreactive and conductive top layer. For this thickness of Au–Pd coating the measured transparency is 90% for $\lambda < 300$ nm. The sphere diameter, measured by scanning electron microscope (SEM), is 194.6 ± 0.5. For the PUSC surface the amplitude of the corrugation of the metal coating is measured to be 59.4 ± 2.5 nm (using both AFM and SEM). The rms roughness for the metallized surface is 4.7 nm (measured using AFM). The roughness correction for our experiment was 1.3% of the maximum measured force. The temperature correction is less than 1% of the Casimir force for the surface separations reported here.

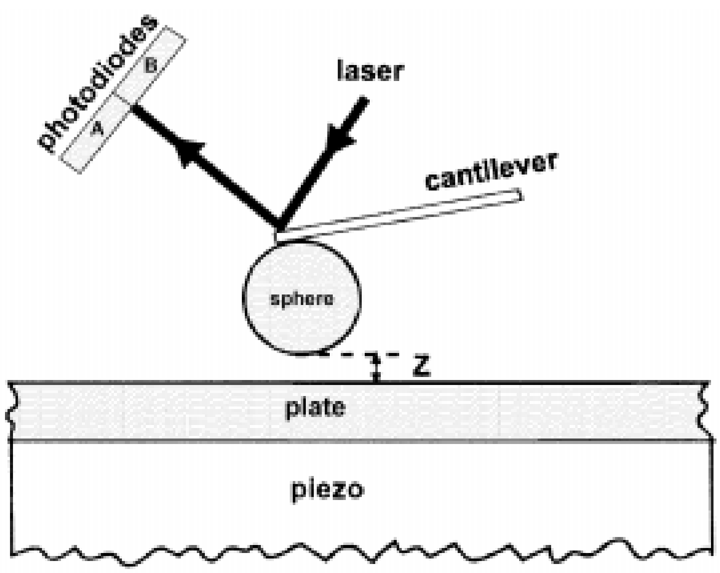

Figure 1. Schematic of the experiment.

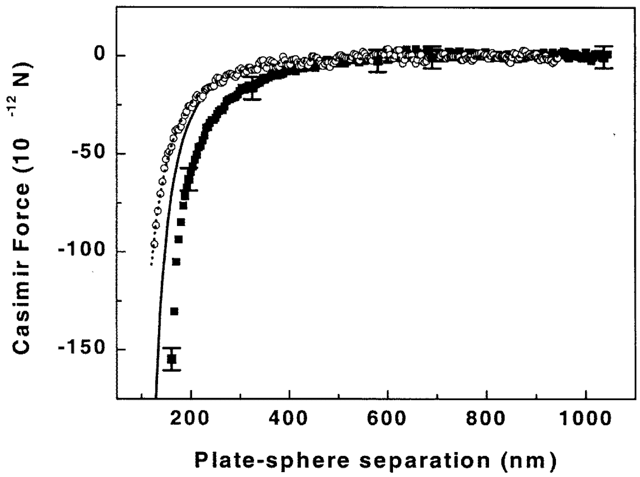

Figure 2. Open circles are the measured Casimir force between a flat plate and sphere. Solid squares are the Casimir force between corrugated surface and the same sphere. The solid line is the corresponding theoretical Casimir force using eq. (6) for latter configuration. Dashed line is the theoretical Casimir force obtained by setting $A = 0$ in eq. (6) for the former configuration.

3. Results and discussion

A force between sphere and the plate causes the cantilever to tilt. This tilt is detected by the deflection of the laser beam leading to a difference signal between photodiodes A and B. This force and the corresponding cantilever deflection are related by Hooke's law: $F = kz$ where k is the force constant and z is the cantilever deflection. The cantilever is calibrated and the residual potential difference between the grounded sphere and plate is measured using the electrostatic force between them. The detail of the calibration procedure is elaborated in refs [12,13]. This residual potential arises from the different materials used to fabricate the sphere and the flat plate. The correction due to piezo hysteresis and cantilever deflection were applied as reported in refs [12,13]. We have also measured z_0, the average surface separation on contact of the two surfaces, by electrostatic means [13]. To measure z_0 the plate is connected to a dc voltage supply (calibrated against the voltage standards) while the sphere remains grounded. The value of z_0 is found out to be 49±4 nm for the flat plate and 132±5 nm for PUSC surface. Given 8 nm Au/Pd coating on each surface this would correspond to an average surface separation 132±5+8+8=148 nm for the case of Casimir force measurement for PUSC surface and 49±4+8+8 for flat plate.

The main systematic error corrections to the total force curve are due to 1) the electrostatic force between sphere and the plate, 2) linear coupling of scattered light from the moving plate into the diodes. Each force scan was fitted with the function form $F = F_c(z + z_0) + F_e(z + z_0) + cz, F_c$ is the Casimir force contribution to the total force, F_e is the electrostatic correction (1), and the third term is correction due to scattered light (2). Only the points above 400 nm are used in this fit as Casimir force is insignificant at a large distance and electrostatic force dominates. After subtraction of the systematic error corrections from the net force curve one obtains the measured Casimir force as $F_{c-m} = F_m - F_e - cz$. The experiment is repeated for 15 scans and the average Casimir force measured is shown as open circles in figure 2. The experiment and the analysis were repeated for same sphere and an identically coated corrugated plate. The average measured Casimir force of 15 scans is shown by solid squares in figure 2.

The theoretical curve for the force between sphere and the corrugated plate given by eq. (6) with no adjustable parameters is shown as a solid line in the same figure. As seen in the figure there is a significant deviation between measured force from perturbative theory. The theoretical Casimir force due to flat plate and the sphere obtained by setting $A = 0$ in eq. (6) is shown by dashed line, which is in good agreement with experiment. We have shown elsewhere [12,13] using eqs (3) and (4) one gets the fit with experimentally measured Casimir force between Al-coated sphere and the flat plate within 1% accuracy.

4. Conclusion

The Casimir force was measured between sphere and the flat plate. The statistical measure of the experimental precision is of the order of 1% of the force at the closest separation. The measured Casimir force between large sphere and the corrugated plate is significantly different from the perturbative theory which account only for changes in separation between surfaces, neglecting the diffractive effect associated with the corrugated surfaces. More detailed understanding is in progress at UC, Riverside.

References

[1] H B G Casimir, *Proc. Kon Akad. Wet.* **51**, 793 (1948)
[2] P W Milonni, *The quantum vacuum* (Acad. Press, San Diego, CA, 1994)
V M Mostepanenko and N N Trunnov, *The Casimir effect and its applications* (Clarendon Press, 1997)
[3] J Blocki, J Randrup, W J Swaitecki and C F Tsang, *Ann. Phys. (NY)* **105**, 427 (1977)
[4] J Schwinger, L L De Radd Jr and K A Milton, *Ann. Phys. (NY)* **115**, 1 (1978)
[5] V B Bezerra, G L Klimchitskaya and C Romero *Mod. Phys. Lett.* **A12**, 2613 (1997)
[6] E M Lifshitz, *Sov. Phys. JETP* **2**, 73 (1956)
[7] M Bordag, B Geyer, G L Klimchitskaya and V M Mostepanenko, *Phys. Rev.* **D58**, 75003 (1998)
[8] G L Klimchitskaya, Anushree Roy, U Mohideen and V M Mostepanenko, *Phys. Rev.* (1999)
[9] J Mehra, *Physica (Amsterdam)* **37**, 145 (1967)
L S Brown and G J Maclay, *Phys. Rev.* **184**, 1272 (1969)
[10] M J Sparnaay, *Physica* **24**, 751 (1958)
[11] S K Lamoreaux, *Phys. Rev. Lett.* **78**, 5 (1997)
[12] U Mohideen and Anushree Roy, *Phys. Rev. Lett.* **81**, 4549 (1998)
[13] Anushree Roy, C Y Lin and U Mohideen, *Phys. Rev.* **D60**, 111101–1 (1999)
[14] R Balian and B Duplantier, *Ann. Phys. (NY)* **112**, 165 (1978)
[15] R Golestanian and M Kardar, *Phys. Rev. Lett.* **78**, 3421 (1997)
[16] Anushree Roy and U Mohideen, *Phys. Rev. Lett.* **82**, 4380 (1999)
[17] *Handbook of optical constants of solids* edited by E D Palik (Academic Press, New York, 1985)