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Unsharp spin observables, non-locality and Fry, Walther and Li experiment

SISIR ROY
Indian Statistical Institute, 203, B.T. Road, Calcutta 700 035, India

Abstract. Recently it has been demonstrated that Bell inequalities for spin 1/2 particles must be modified if unsharp spin observables are considered, and furthermore, the modified Bell inequalities may not be violated by quantum mechanics if the observables are sufficiently unsharp. In case of massive particles there may be more imperfection than seems to appear in the photon EPR experiments. So the experiment proposed by Fry, Walther and Li can place experimental limits on the unsharpness of spin variables. It sheds new light on the much debated issues like non-local correlations in quantum mechanics.

Keywords. Unsharp observables; non-locality; detection loophole.

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1. Introduction

In 1965 Bell derived mathematical inequalities [1] which must be satisfied by physical theories that assume realism and locality, and he also showed that conventional quantum mechanics can violate these inequalities. These inequalities and their subsequent generalizations have become known as Bell inequalities. Since the 1960s a number of experimental tests have been conducted, and the results [2–4] have been in excellent agreement with quantum mechanics and have shown violations of Bell inequalities. Because of the Bell inequalities and experiments by Clauser [2], Aspect, Dalibard and Roger [3], and others [4], most physicists have concluded that quantum mechanics requires nonlocal correlations between pairs of particles in entangled systems. More recently a systematic presentation of quantum theory called operational quantum physics [5] has been developed, and this newer presentation uses the concept of unsharp physical observables and may permit a different interpretation of the experimental data. It has been demonstrated [6] that the Bell inequalities for spin 1/2 particles must be modified if unsharp spin observables are considered, and furthermore, the modified Bell inequalities may not be violated by quantum mechanics if the observables are sufficiently unsharp. The main intention was to see whether the quantum mechanical predictions necessarily yield a conflict with Bell inequality. It should be mentioned that Mittlestaedt and Stachow [7] derived Bell inequality within sharp quantum logic. They used a formalization of the EPR reality crieterion as well as locality within the abstract quantum language and deduced a contradiction between these assumptions and the validity of quantum mechanics. Busch [8] tried to investigate whether this contradiction survives in case of unsharp observables. The answer is no if the unsharpness is sufficiently large; the resulting probabilities do satisfy Bell's inequality.

Fry, Walther and Li [9,10] proposed an experiment which puts experimental limits on the unsharpness of spin observables because the experiment will permit detection of pairs of particles with very high quantum efficiency detectors.

In the traditional presentation of quantum mechanics each physical observable is represented by a self-adjoint operator [11]. A mathematically equivalent statement is that the projection valued (PV) spectral measure of a self-adjoint operator is associated with each physical observable. In operational quantum physics a physical observable is represented by a positive operator valued (POV) measure. In many cases the POV measure is obtained from a convolution of the spectral measure of a self-adjoint operator with a confidence function. In operational quantum physics the traditional concept of a physical observable as a self-adjoint operator still is permitted as a special case, but the more general concept of an unsharp physical observable as a POV measure is often useful.

The spin values of particles continue to have the normal integer or half integer values in operational quantum physics. Probabilities also have the standard properties that they are nonnegative and can be integrated to yield the value of one. A very significant difference between unsharp and sharp observables is illustrated in the case of a singlet state. If there are two electrons in a singlet state, which is considered within the context of unsharp observables, and one electron is observed to have 'spin up', then there is a high probability, but not 100% certainty, that the spin measurement of the other electron will be down. In other words, there is not a strict anticorrelation of the measured spins of the two electrons in a singlet state because of the unsharpness. A closely related corollary is that there is a small but nonzero probability that the spin measurements for the two electrons in the singlet state will point in the same direction in the case of unsharp observables. These results seem to be very unusual to physicists who are accustomed only to sharp spin observables, and indeed, these ideas require a modification [5,6] of the Bell inequalities.

Unsharp observables and POV measures are useful mathematical concepts in a rigorous presentation of the foundations of quantum mechanics, but it also must be emphasized strongly that unsharp spin observables are closer to the actual measurements of a Stern-Gerlach device than are the sharp spin observables in idealized textbook treatments of the experiment. In a simplified understanding of a Stern-Gerlach experiment, if a beam of silver atoms is sent into the apparatus, then the beam is split into two well-separated parts. The atoms with spin up are detected in the upper part of a screen, and the atoms with spin down are found in the lower part of the screen. If the spin observables were perfectly sharp, then there would be a sharp demarcation on the screen between the atoms with spin up and the atoms with spin down, but in practice, the situation is somewhat more complicated. Busch, Grabowski, and Lahti [5] used real Gaussian wave functions for the incident silver atoms and showed that, because of the spreading of the wave packets, it is not possible to divide the detection screen sharply so that only spin up particles are on one side of the dividing line and spin down on the other side. In other words, the measurement is somewhat unsharp, but Busch *et al* [8] acknowledge that the degree of unsharpness can be minimized by the choice of certain initial states.

Another important physical reason for unsharp spin measurements in a Stern-Gerlach experiment is the actual form of the magnetic field used in the measurement. Let $\mathbf{e}_1$, $\mathbf{e}_2$, and $\mathbf{e}_3$ be unit vectors along the three cartesian axes. It is common [5] to represent the magnetic field $\mathbf{B}$ in a Stern-Gerlach experiment by the idealized mathematical model in eq. (1). This idealized magnetic field is a linear function of the position variable z and parallel to the vector $\mathbf{e_3}$, but the magnetic field components along the $\mathbf{e_1}$ and $\mathbf{e_2}$ directions are neglected.

$$\mathbf{B} = (B_0 - bz)\mathbf{e_3}. \tag{1}$$

The magnetic field $\mathbf{B}$ is spatially uniform if $b = 0$, and the magnetic moment of an atom in a uniform field experiences a torque that tends to align the atom's moment with the field. A nonuniform $\mathbf{B}$ field exerts on an atom's magnetic moment a force that causes translation of the atom, which is necessary for a Stern-Gerlach measurement. A serious difficulty with eq. (1) is that it is inconsistent with the Maxwell equation $\nabla \cdot \mathbf{B} = 0$ except for the uninteresting case when $b = 0$. A more plausible Stern-Gerlach magnetic field which is consistent with the Maxwell equations is given in eq. (2).

$$\mathbf{B} = bx\mathbf{e_1} + (B_0 - bz)\mathbf{e_3} = (B_1, B_2, B_3). \tag{2}$$

Busch *et al* [5] have calculated the properties of a Stern-Gerlach pattern for the magnetic field given by (1), and even if a particle initially has spin up, there is a nonzero probability of detecting the particle in the bottom part of the screen [5] because of the particle's interaction with the magnetic field. In other words, the unsharpness of spin measurements in Stern-Gerlach experiments can be understood, in part, when realistic magnetic fields are considered.

In the preceding paragraphs the spreading of wave packets and the form of the magnetic fields have been discussed as physical causes of unsharp measurements in Stern-Gerlach experiments, but Busch *et al* [5] have examined additional reasons for unsharp observables in Stern-Gerlach measurements, optical polarization experiments, measurements of the position of an atom, and many other experiments. Unsharp observables have been discussed qualitatively so far in this paper, but to determine quantitatively the importance of unsharp spin observables in Bell-type experiments, it is helpful to use detectors with very high quantum efficiencies. The suggestion by Fry, Walther and Li [9] is the most fully developed example of such a proposed experiment. In §2 of this paper the Clauser–Horne version of a Bell inequality will be reviewed because it is important in understanding the Fry–Walther–Li experiment. In §3 of this paper the Fry–Walther experiment [10] will be discussed in the context of unsharp observables, and a final discussion is given in §4.

Before proceeding to the next section it is important to emphasize that the relatively low efficiency of standard photodetectors has been recognized as a 'loophole' of Bell's theorem for many years [12–14]. This is known as 'detection loophole'. This is based on the fact that in real experiments the efficiencies of the detectors are such that the set of detected events is significantly smaller than the set of tested quantum systems. One has to assume that the sample over which the statistics is measured is a fair sample. From quantum mechanical point of view this assumption is almost trivial. However, from the local hidden variable point of view the opposite assumption is equally almost trivial [15]. In hidden variable theory, it is very plausible that the actual value of these variables affects the probability to trigger a detector. Gisin *et al* [15] studied a local-hidden variable model which is shown to be compatible with both CHSH and CH Bell inequalities. However, the detector efficiency is limited to 75%.

Garg and Mermin [14] considered Bell inequalities for conventional sharp spin observables and calculated that the quantum efficiencies of the detectors must be at least 82.84% to eliminate this 'loophole'. The net effect of unsharp observables is twofold. The use of unsharp observables introduces another parameter in the calculations, and so the quantum efficiencies of the detectors must be at least 95% to close the loophole. Second, in the past Bell-type experiments not all of the light quanta have been detected because of the photodetector inefficiencies, and the correlations have been measured using only those events in which both quanta in the pair were observed. It has been natural to assume for sharp spin observables, with the strict anticorrelation of the spins of the two particles in a singlet state, that the detected pairs of particles were representative of the entire ensemble. On the other hand, when unsharp spin observables are introduced and the strict anticorrelation is no longer valid, it may not be appropriate to assume that events for which both particles in a pair are detected are representative of the entire ensemble. It immediately raises some interesting questions regarding the existence of non-local correlation in quantum mechanics which will be discussed in the final section of this paper.

2. Clauser–Horne inequality

Clauser and Horne [12] proved Bell's theorem for general realistic theories which include also inherently stochastic theories. It should be noted that they proved in such a way so as to define an experiment which might be actually performed and does not require the auxiliary assumptions. Their proof can be visualised in the following manner.

Suppose that the source emits, say, N of the two-component systems of interest. Let $N_1(a)$ and $N_2(b)$ be the number of counts at detectors 1 and 2 respectively, and $N_{12}(a,b)$ is the number of simultaneous counts from the two detectors (coincidence counts). For sufficiently large N, the probabilities for these results for the whole ensemble (with due allowance for random errors) are given by

$$p_1(a) = \frac{N_1(a)}{N}; p_2(b) = \frac{N_2(b)}{N}; p_{12} = \frac{N_{12}(a,b)}{N}. \tag{3}$$

CH derived the inequality containing the ratios of probabilities rather than their absolute magnitudes so as to avoid the influence of N. After some algebraic calculations we get the following inequality

$$\frac{p_{12}(a,b) - p_{12}(a,b') + p_{12}(a',b) + p_{12}(a',b')}{p_1(a') + p_2(b)} \leq 1. \tag{4}$$

It is now clear that this is now independent of N. Using the above eq. (4) and defining $R(a,b)$ as the rate of coincident detections, and $r_1(a)$ and $r_2(b)$ as the rate of single-particle detections by either apparatus, the inequality (4) turns out to be

$$\frac{R(a,b) - R(a,b') + R(a',b) + R(a',b')}{r_1(a') + r_2(b)} \leq 1. \tag{5}$$

Several authors [5,6] discussed CHSH and Bell inequalities for unsharp spin observables. We have already mentioned that the concept of unsharp spin observable is more nearer to the realistic situation and sharp observable be more idealistic one. It is also shown by one of the authors [6] that the original Bell inequality is inappropriate for unsharp spin observables. Let us consider unsharp spin-$\frac{1}{2}$ observables in the two-dimensional complex Hilbert space

$$E(n_i) = \frac{1}{2}(I + \lambda \hat{n}_i.\sigma_j);\ i = 1,2,3,4,$$

where λ is a real number with $0 \geq \lambda \leq 1$, the $\hat{n}_i$ are unit vectors and the $\vec{\sigma}$ are Pauli matrices. When $\lambda = 1, E(n_i)$ is a projection operator representing a sharp observable. Let us designate the probability and joint probability distributions of the $E(n_i)$ in the state ψ by P_i and P_{ij} respectively. Then

$$P_i = \frac{1}{2} \tag{6}$$

and

$$P_{ij} = \frac{1}{4}(1 - \lambda^2 \hat{n}_i.\hat{n}_j) = (1 - 2\epsilon)\frac{1}{2}\sin^2\left(\frac{1}{2}\theta_{ij}\right) + \frac{1}{2}\epsilon, \tag{7}$$

where $\epsilon = \frac{1}{2}(1 - \lambda^2)$ is the measure of the degree of unsharpness and θ_{ij} is the angle between $\hat{n}_i$ and $\hat{n}_j$. It is now evident that when $E(n_i)$ and $E(n_j)$, each belonging to one component of a single state system, are in the same direction, then in the case of sharp spin observables $P_{ij} = 0$. Hence for this case the CHSH inequalities reduce to the Bell inequalities in the usual form. But for unsharp spin observables

$$P_{ij}(\vec{n}_i = \vec{n}_j) = \frac{1}{4}(1 - \lambda^2) = \frac{1}{2} \neq 0. \tag{8}$$

So, for sharp observables it is possible to have strict (anti-)correlation between two observables, each belonging to one component of a single state system, this no longer happens in the case of unsharp spin observables. In this latter case the reduction of the CHSH inequalities to a situation where two spin observables have coincident directions does no longer yield the original Bell inequalities but instead a set of inequalities containing an additional term accounting for imperfect or degraded correlation. This situation might be tested in the experiment proposed by [10].

3. Proposed experiment of Fry, Walther and Li

We have already mentioned that Clauser–Horne inequality (CH) requires no auxiliary assumptions. It is known as strong Bell inequality (BCH). This is important since it is in terms of physically realizable experiment. Essentially, the strong Bell inequality is based on the ratios of the rates of coincidence between the two detectors to the singles count rates at each detector. By contrast the weak Bell inequalities tested in all previous experiments were based on the ratios of coincidence rates only. Several proposals have been put forward [12,2] to test strong Bell inequalities using high efficiency detectors but none have yet been realized experimentally. The system proposed by Lo and Shimony [13] was the first to use atoms and was a precursor to Fry *et al* proposal [10]. They used ^{199}Hg isotope for an experimental realization of Bohm's spin-$\frac{1}{2}$ EPR-Gedanken experiment. The dissociation of dimers of the ^{199}Hg isotopomer using a spectroscopically selective stimulated Raman process leads to the generation of an entangled state between the two spatially separated atoms. The measurement of nuclear spin correlations between these atoms is achieved by using a spin state selective two photon excitation-ionization scheme that also provides for detection of the atoms. This experiment not only closes the detector efficiency loophole, but also permit enforcement of the locality condition. Using the following notations of Fry, Walther and Li [9]

$$R_{1+}(\theta_1) = R_{2+}(\theta_2) = R_{1-}(\theta_1) = R_{2-}(\theta_2) = \frac{\eta f N}{2}, \tag{9}$$

$$R_{++}(\theta_1, \theta_2) = R_{--}(\theta_1, \theta_2) = \eta^2 f g N \frac{1}{4}[1 - \cos(\theta_1 - \theta_2)], \tag{10}$$

$$R_{+-}(\theta_1, \theta_2) = R_{-+}(\theta_1, \theta_2) = \eta^2 f g N \frac{1}{4}[1 + \cos(\theta_1 - \theta_2)], \tag{11}$$

the strong BCH inequality can be written as

$$S(a, b, a', b') = \frac{R_{++}(a, b) - R_{++}(a, b') + R_{++}(a', b) + R_{++}(a', b')}{R_{1+} + R_{2+}(b)} \leq 1, \tag{12}$$

where a, a' are two values of angle θ_1, and b, b' are two values of θ_2.

Here,

$$R_{++}(\theta_1, \theta_2) = R_{--}(\theta_1, \theta_2) = \eta^2 f g N \frac{1}{4}[1 - \cos(\theta_1 - \theta_2)] \tag{13}$$

and

$$R_{1+}(\theta_1) = R_{2+}(\theta_2) = R_{1-}(\theta_1) = R_{1-}(\theta_1) = \frac{\eta f N}{2}. \tag{14}$$

Subsequent to this analysis Fry and Walther [10] rewrote the above mentioned coincidence and single rates in more realistic term as

$$R_{++}(\theta_1, \theta_2) = \eta^2 f g N \frac{1}{4}[\epsilon_+^2 - \epsilon_-^2 \cos(\theta_1 - \theta_2)] \tag{15}$$

and

$$R_{1+}(\theta_1) = R_{2+}(\theta_2) = \frac{\eta f N}{2} \epsilon_+, \tag{16}$$

where η is the detector efficiency of Hg atoms arriving at the corresponding detectors; ϵ_+ and ϵ_- are defined as $\epsilon_+ = \epsilon_M + \epsilon_m$ and $\epsilon_- = \epsilon_M - \epsilon_m$, respectively, where ϵ_M is the transmission of the analyzers for one spin component and ϵ_m the leakage through the analyzer for the other component. The transmission ϵ_M and the leakage ϵ_m can be determined from the value of $g\eta$ and other ratios of coincidence rates of $R_{++}(\theta_1, \theta_2)$ and $R_{+-}(\theta_1, \theta_2)$.

Now if we consider $R_{++}(\theta_1, \theta_2)$ for sharp spin observable it vanishes for $\theta_1 = \theta_2$ in the first case eq. (13) i.e. without considering transmission and leakage. But if we consider the generalised case eq. (15) i.e. with non-zero transmission and non-zero leakage we will get the value as

$$R_{++}(\theta_1 = \theta_2) = \eta^2 fgN\epsilon_M\epsilon_m. \tag{17}$$

This will vanish only if there exists no leakage at all i.e. $\epsilon_m = 0$. In real physical experiment this is not possible.

It is very important to note that this term vanishes for sharp observable which is also clear from the framework of Fry *et al* [9]. But in the framework of unsharp observable this does not vanish as is evident from eq. (8). Now if one considers the realistic experiment i.e. with transmission and leakage as non-zero, we will R_{++} as non-zero. Now once this term will be non-zero one can not reduce the CHSH inequality to Bell inequality. So the claim made by Fry *et al* [10] that quantum mechanical prediction will be valid and Bell inequality will be violated. On the other hand one of the authors [6] has shown that with this term we can get weaker Bell inequality which will be satisfied for $\epsilon = 0.1$ along with quantum mechanical validity. Now if we put this restriction for the degree of unsharpness we can get the following restriction for the following parameters for the experiment suggested by Fry *et al* [10].

$$R_{++}(\theta_1 = \theta_2) = \eta^2 fgN\epsilon_M\epsilon_m = 0.1.$$

This will put restriction on the detector efficiency as well as the transmission and leakage parameters.

If the nuclear spins of the two ^{199}Hg atoms are in a singlet state, and the detectors are highly efficient, the measurements of the nuclear spins become unsharp due to real physical situation (not the idealised one) and characterized by the 'unsharpness' parameter ϵ, then the probability p of measuring both spins pointing in the same direction can be estimated from the above non-vanishing value of R_{++} eq. (15). If the nuclear spins of the two ^{199}Hg atoms are truly in a singlet state, the detectors are 100% efficient, and the measurements of the nuclear spins are completely sharp, i.e. $\epsilon = 0$, then $p(\theta_1, \theta_1) = 0$. In this case there is zero probability that the measured spins will point in the same direction. But once again this may happnen only in case of idealised measurement.

Although the nature of quantum decoherence is still somewhat unclear and subject to debate, it has been suggested that electromagnetic vacuum state fluctuations may be a cause [16] of decoherence. Davidovich *et al* [17] calculated for a very different physical system a model of quantum decoherence assuming dissipation with a bath of thermal oscillators at zero temperature, and recent experiments [18] seem to be in excellent agreement with the theoretical model. In the present case the unsharpness can be introduced in anti-correlated atoms during the preparation of the singlet state. This might be due to the existence of electromagnetic vacuum fluctuation.

4. Discussion

Einstein, Podolsky and Rosen [19] argued in a famous article more than sixty years ago that quantum mechanics is incomplete. Bohr [20] countered that quantum mechanics permits nonlocal correlations between pairs of particles in an entangled state. Bohr's viewpoint has remained the most widely held position among physicists. A central element in this debate, which often has been left unstated, is the assumption that the spins of two spin 1/2 particles in a singlet state are perfectly anticorrelated. In other words, if a measurement of one electron in a singlet state yields the value of spin up, then the other electron necessarily must be measured as having spin down. Since the 1980s mathematical analysis by

Busch, Grabowski, Lahti and others has shown that actual physical measurements in Stern–Gerlach experiments are not perfectly sharp as described in the previous sentence. They have reanalysed the Stern–Gerlach experiment and shown that if the particles originally had spin up, after leaving the magnetic field the spin state will be a mixture of spin up and spin down components due to presence of B_1 in (2). So even if there exists a perfect anti-correlation, there is a non-zero probability of getting two particles in spin up or spin down states. But as the preparation of the singlet state is also a kind of measurement, some kind of unsharpness can be introduced which can degrade the anticorrelation. It should be noted that for sufficiently unsharp observable, the Bell type experiment need not be explained by non-local correlations between the particles in a pair in an entangled state. The proposed experiment of Fry and Walther [10] can place upper limits on the unsharpness of spin observable. So for idealised experiment one might need quantum non-local correlations (i.e. for sharp observable) to explain EPR-like experiments. In the proposed experiment of Fry and Walther [10] as the efficiency of the detectors are very high, the pairs which would be detected should be considered as the representative of the entire ensemble. The unsharpness which causes the degradation of the anti-correlation in ^{199}Hg dimers, might be due to the electromagnetic state fluctuation or due to the quantum indeterminacy itself. Busch *et al* [5] found that the possible cause of unsharpness introduced by physically analysable experiment can be traced back to its origin to the quantum indeterminacy inherent to the system to be observed. So the existence of non-local correlations between pair of particles in an entangled state in EPR-like experiments (as widely believed among physicists) should be revisited in this new perspective.

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