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Effect of radiative cooling on collapsing charged grains

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Abstract. The effect of the radiative cooling of electrons on the gravitational collapse of cold dust grains with fluctuating electric charge is investigated. We find that the radiative cooling as well as the charge fluctuations, both, enhance the growth rate of the Jeans instability. However, the Jeans length, which is zero for cold grains and nonradiative plasma, becomes finite in the presence of radiative cooling of electrons and is further enhanced due to charge fluctuations of grains resulting in an increased threshold of the spatial scale for the Jeans instability.

Keywords. Jeans instability; radiative cooling; dusty plasma.

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1. Introduction

The gravitational instability of a large cloud complex in connection with the large scale structure formation in the universe is a well researched topic [1]. We know that if the mass of a cloud clump M exceeds Jeans mass $M_{\rm J} = (2\pi\lambda_{\rm J})^3\rho_0$ associated with the average condition of cloud clumps of density ρ_0 (where $\lambda_{\rm J}$ is the Jeans length) the cloud collapses onto itself and a star or a galaxy or a cluster of galaxies forms depending upon the nature of initial and boundary conditions we choose. For example, a recent work by Nakano *et al* [2] discusses in detail the star formation in the core of the dark interstellar cloud. Observations of molecular clouds [3] suggest that their dense core have embedded infrared sources implying that such clouds are main sites of star formation in galaxies.

However, there are problems with both, upper and lower bounds of Jeans mass. On the one hand, there are dark molecular clouds whose mass is well above the critical Jeans mass, suggesting a rate of star formation far in excess of the observed rate. On the other, clouds of sub critical Jeans mass may exhibit condensational features e.g solar prominences, interstellar clouds and planetary nebulae. Whereas, the observed rate of star formation for the clouds above Jeans mass may be caused by the electrostatic interaction of the plasma clouds [4,5], the condensation of the sub critical Jeans matter can be achieved via radiative cooling of its plasma particles [6–8] which along with the charged and neutral grains form its other constituents. The precise nature of cooling function depends upon the prevailing radiative surroundings in the interstellar medium [9]. It may appear that the grains may not survive the hot plasma environment (e.g around new born stars in the HII region or circumstellar shells). However, grains embedded in a hot plasma with temperatures greater than 10^6 K will not evaporate as the equilibrium grain temperature will be less than 100 K [9].

Dark molecular clouds are one of the main ionization sources in the interstellar medium as they are the sites of new born stars. The shell around the star can have a temperature between $\sim (10^4\text{–}10^6)^\circ$ K [10] down to 10° K implying that the stellar envelop will be differentially ionized with the innermost shell, completely ionized. As one will move away from the inner to the outer part of the shell, there will be more of neutrals. Finally, outer part of the stellar shell will merge with the interstellar medium called HI region. Depending upon the temperature, the charge on the grain will vary. Therefore, the model calculation of Jeans instability can be done for purely ionized dusty plasmas ([5] and references therein) as well as for partially ionized plasmas [11]. The calculations for both the variety of plasmas are necessary to describe the gravitaional dynamics of the stellar envelops, circumstellar shells etc. In the present investigation we shall examine the effect of the radiative cooling of electrons on the gravitational collapse of the cold charged dust grains, embedded in the radiative plasma background. The plasma will be assumed to be optically thin towards the emitted photons to avoid the complication of radiative transfer effects.

The dynamics of the charged dusty plasma has been investigated in considerable detail in the recent past [12]. The novel collective feature of the dusty plasma dynamics emerges from the charge fluctuations of the grains [13–16]. The mass of grains m_d may vary between 10^{-5} g [17,18] in the interplanetary medium to $10^{-12}\text{–}10^{-14}$ g in the interstellar medium [10,19]. These are the typical values though heavier (up to 10^{-2} g, [18]) and lighter (down to molecular size [19]) grains have also been inferred. The corresponding sizes of such grains may vary between a few centimeters to a few microns. The charges Q of these grains may vary from $(10^3\text{–}10^4)$ [12] to 1 electronic charge [20]. The large range in the mass permits a situation where gravitational and electromagnetic forces may become comparable. Assuming the 'underlying unity' of the planetary and interstellar grains [21], it is quite possible that the joint action of the two forces might have played a decisive role in the formation of stars and galaxies. Such an investigation with $R = Gm_d^2/Q^2 \approx O(1)$ has been recently carried out by several authors [4,5,22–27,29]. The present analysis will focus on a scenario where the dust grains collapse in the presence of radiating electrons. Such an analysis has recently been carried out by Dwivedi *et al* [29]. However their analysis fails on several counts: (i) the thermal equilibrium of the electrons is obtained by balancing the cooling function with the thermal diffusion term; this implies a zeroth order pressure force term in the momentum transfer equation of electrons which gives rise to inconsistencies when combined with the quasineutrality condition. (ii) In the first order thermal balance equation for the electrons, they drop the term $P_{e0}\nabla \cdot \vec{v_{e1}}$ which then will leave $\partial n_{e1}/\partial t$ term in the electron continuity equation unbalanced. (iii) The condition $R \sim O(1)$ has been imposed but not used anywhere. (iv) Lastly, though they derive the limiting cases from their dispersion relation, the dispersion relation itself remains unanalysed.

In the light of the above remarks, it becomes clear that a correct formulation of the problem is called for. We shall also include the charge dynamics of the grains. The grains are immersed in a non-neutral plasma background [31] and thus, it is more natural to assume an equilibrium electric field balancing the equilibrium gravitational field of the grain. As has been shown by Pandey *et al* [4], one can still assume an asymptotic homogeneous background.

2. Basic equations

The dusty plasma consists of electrons, ions and charged grains. The size, charge and mass distribution of the grains is ignored for simplicity. It is known that grains can be charged via several competing processes viz. electron and ion collisions, photoemission, secondary emission due to electron or ion impact and electric field emission etc. We assume that the grain charging is primarily collisional. Owing to the large mass difference between the plasma particles and the dust grains, we assume that the gravitational potential is solely due to the grains. Then the basic set of equations are given by:

$$\frac{\partial n_\alpha}{\partial t} + \nabla \cdot (n_\alpha \vec{v}_\alpha) = 0, \tag{1}$$

$$m_\alpha n_\alpha \left[\frac{\partial \vec{v}_\alpha}{\partial t} + \vec{v}_\alpha \cdot \nabla \vec{v}_\alpha\right] = -\nabla P_\alpha - e_\alpha n_\alpha \nabla \phi - m_\alpha n_\alpha \nabla \psi, \tag{2}$$

$$\frac{3}{2} n_\alpha \frac{\partial T_\alpha}{\partial t} + P_\alpha \nabla \cdot \vec{v}_\alpha = \hat{\chi}_\alpha \nabla^2 T_\alpha - L(n_\alpha, T_\alpha), \tag{3}$$

$$P_\alpha = n_\alpha T_\alpha, \tag{4}$$

where $\alpha =$ electron, ion. $L(n_\alpha, T_\alpha)$ is the heating–cooling function [6], that is the rate of radiative cooling and the rate of heating per unit mass. The electron thermal conductivity is given by [28]

$$\chi_e = \begin{cases} 2.5 \times 10^3 \sqrt{T_e}, & T_e \leq 4.47 \times 10^4 \text{ K} \\ 1.24 \times 10^{-6} T_e^{5/2}, & T_e > 4.47 \times 10^4 \text{ K} \end{cases} \tag{5}$$

where $\chi_e = \hat{\chi}_e / n_{e0}$. Similar expression can be written for the ion thermal conductivity. The heat loss function is described by L. The quantities n, v, P, ϕ, ψ stand for number density, velocity, thermal pressure, electrostatic and gravitational potential respectively. The dynamics of the cold dust grains is described by

$$\frac{\partial n_d}{\partial t} + \nabla \cdot (n_d \vec{v}_d) = 0, \tag{6}$$

$$m_d n_d \left[\frac{\partial \vec{v}_d}{\partial t} + \vec{v}_d \cdot \nabla \vec{v}_d\right] = -Q n_d \nabla \phi - m_d n_d \nabla \psi. \tag{7}$$

The potential fields are defined by the following Poission's equations:

$$\nabla^2 \phi = -4\pi \left[e(n_i - n_e) + Q n_d\right], \tag{8}$$

$$\nabla^2\psi = 4\pi G m_d n_d. \tag{9}$$

Along with the charge dynamic equation

$$\frac{\partial Q}{\partial t} + \vec{v}_d \cdot \nabla Q = I_e(Q,\phi) + I_i(Q,\phi). \tag{10}$$

Equations (1–10) define the dynamics of a self gravitating dusty plasma including radiative effects. Here, I_e and I_i are the electron and ion currents to the grain surface.

3. Equilibrium

We assume that there is no equilibrium flow in the plasma. Then eqs (1) and (6) give the constant equilibrium densities $n_{\alpha 0}$ and n_{d0}. The equilibrium charge Q_0 on the grain depends on a number of properties of both the grains and the ambient plasma medium. In a steady state without flow $\partial Q/\partial t + v_d \nabla Q = 0$ (eq. (10)) and $Q_0/C = \phi_{\mathrm{f}} - \phi$ where C and ϕ_{f} are the capacitance and the floating potential on the grain surface. Generally, $\phi_{\mathrm{f}} - \phi \neq 0$ [12] and therefore, an equilibrium nonzero electric field exists which can balance the self gravitational field of the grain. Thus from (7) one finds $\psi_0 + (Q_0/m_d)\phi_0 = 0$ and using eqs (8) and (9), one gets the following equation for grain density

$$n_{d0}(x) = -\frac{e[n_{i0}(x) - n_{e0}(x)]}{Q_0(1-R)}. \tag{11}$$

The above levitating equilibrium determines the maximum charge ($Z = Q_0/e$) on the grain. Thus, solving the above equation (11) in terms of Z we get the following equation

$$Z_{\pm} = \frac{1}{2}\left[-\frac{n_{i0}}{n_{d0}}\frac{\delta n}{n_{i0}} \pm \left(\left(\frac{n_{i0}}{n_{d0}}\frac{\delta n}{n_{i0}}\right)^2 + \frac{4Gm_d^2}{e^2}\right)\right], \tag{12}$$

where $\delta n = (n_{i0} - n_{e0})$. We have plotted $Z_{\pm}$ against $\delta n/n_{i0}$ in figure 1 for $m_d = 10^{-6}$ g. As the electron number density increases the grain charge also increases (Z_+). For the positively charged grain, the grain charge is related to the ion number density in the plasma (Z_-) curve of figure 1. Next, the equilibrium solutions of electron and ion equations of motion give

$$n_{\alpha 0}(x) = n_{\alpha 0}\exp\left[-e_\alpha + \frac{Q_0 m_\alpha}{m_d}\right]\frac{\phi_0}{T_{\alpha 0}}, \tag{13}$$

where equilibrium ion and electron temperatures have been assumed to be homogeneous. It is clear from the above expressions then that for $[-e_\alpha + (Q_0 m_\alpha/m_d)](\phi_0/T_{\alpha 0}) \ll 1$ the equlibrium can be assumed spatially homogeneous in this limiting sense. Also, the equilibrium thermal balance equation $L(n_{\alpha 0}, T_{\alpha 0}) = 0$ is consistent with the spatially homogeneous equilibria. In the plane of variable $(n_{\alpha 0}, T_{\alpha 0})$, these equilibrium represents curves on which there is an exact balance between heating and radiative cooling while thermal conduction is ineffective [8].

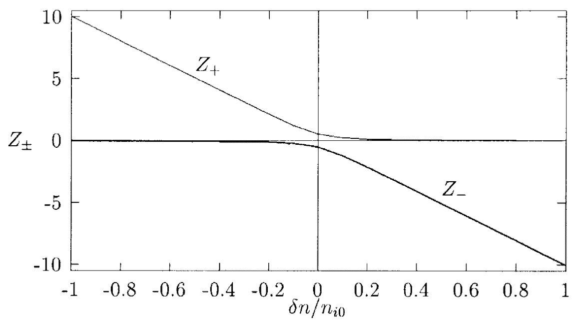

Figure 1. Curve $Z_{\pm}$ is the positive (negative) root of eq. (10) with $n_{i0}/n_{d0} = 10$ and $m_d = 10^{-6}$ g.

4. Stability analysis

The linearized set of equations are:

$$\frac{\partial n_{\alpha 1}}{\partial t} + \nabla \cdot (n_{\alpha 0} \vec{v}_{\alpha 1}) = 0, \tag{14}$$

$$0 = -\nabla P_{\alpha 1} - e_\alpha n_{\alpha 0} \nabla \phi_1, \tag{15}$$

$$\frac{3}{2} n_{\alpha 0} \frac{\partial T_{\alpha 1}}{\partial t} + P_{\alpha 0} \nabla \cdot \vec{v}_{\alpha 1} = \hat{\chi}_\alpha \nabla^2 T_{\alpha 1} - L_1(n_{\alpha 1}, T_{\alpha 1}), \tag{16}$$

where ion and electron inertia have been neglected. We assume that on the slow time scale of the dust dynamics, ions follow the Boltzmann distribution equation (13). This assumption of ion thermalization is valid provided phase velocity of the fluctuations are much smaller than the ion thermal velocity along with infinitely large thermal conductivity ($T_{i1} = 0$) i.e any fluctuation in the ion temperature is immediately short circuited. Then the ion density fluctuation is given by (from eq. (15)):

$$\frac{n_{i1}}{n_{i0}} = -\frac{e\phi_1}{T_i}. \tag{17}$$

Unlike ions, the electrons are considered to have finite thermal conductivity (i.e $T_{e1} \neq 0$) and the radiative cooling of the electrons takes place. Such a scenario can be visualized in a latter phase of the gravitational collapse when after undergoing extreme compression, the radiative cooling time scale of the electrons becomes comparable with the free fall time scale of the grains leading to a new coupled radiative Jeans mode. Therefore, the thermal loss of the electrons prevents them from attaining thermalization whereas thermal loss of ions being unimportant, it easily attains the Boltzmann distribution. Then the equations for electron dynamics are (from eqs (14)–(16)):

$$\frac{\partial n_{e1}}{\partial t} + \nabla \cdot (n_{e0} \vec{v}_{e1}) = 0, \tag{18}$$

$$\frac{n_{e1}}{n_{e0}} = \left(\frac{e\phi_1}{T_{e0}} - \frac{T_{e1}}{T_{e0}}\right), \tag{19}$$

where the isothermal equation of state $P_e = n_e T_e$ has been used. The first order energy equation becomes:

$$\frac{3}{2} n_{e0} \frac{\partial T_{e1}}{\partial t} + P_{e0} \nabla \cdot \vec{v}_{e1} = \hat{\chi}_e \nabla^2 T_{e1} - \frac{\partial L}{\partial n_{e0}} n_{e1} - \frac{\partial L}{\partial T_{e0}} T_{e1}, \tag{20}$$

where $L(n_{e1}, T_{e1}) = (\partial L/\partial n_{e0})\, n_{e1} + (\partial L/\partial T_{e0})\, T_{e1}$. The first order equation for the dust grains are

$$\frac{\partial n_{d1}}{\partial t} + \nabla \cdot (n_{d0} \vec{v}_{d1}) = 0, \tag{21}$$

$$\frac{\partial v_{d1}}{\partial t} = -(Q/m_d) \nabla \phi_1 - \nabla \psi_1. \tag{22}$$

Finally the pair of Poisson's equation are:

$$\nabla^2 \phi_1 = -4\pi \left[e(n_{i1} - n_{e1}) + Q_1 n_{d0} + Q_0 n_{d1}\right], \tag{23}$$

$$\nabla^2 \psi_1 = \omega_{\mathrm{J}}^2 \frac{n_{d1}}{n_{d0}}, \tag{24}$$

where $\omega_{\mathrm{J}}^2 = 4\pi G m_d n_{d0}$ is the Jeans frequency. The first order charge dynamic equation is given by [15]:

$$\left[\frac{\partial}{\partial t} + \eta\right] Q_1 = |I_{e0}| \left[\frac{n_{i1}}{n_{i0}} - \frac{n_{e1}}{n_{e0}}\right]. \tag{25}$$

Fourier analysing the perturbed quantities as $\exp(-i\omega t + i\vec{k} \cdot \vec{r})$, we get from equations (18)–(20), the following expression for the electron density fluctuation

$$\frac{n_{e1}}{n_{e0}} = \frac{e\phi_1}{T_e} \left(1 + \frac{\omega - iL_n}{\Omega}\right)^{-1}, \tag{26}$$

where $\Omega = (3/2)\omega + i(\chi k^2 - L_T), L_n = T_{e0}^{-1}\, (\partial L/\partial n_{e0})\, , L_T = -n_{e0}^{-1}\, (\partial L/\partial T_{e0})$. For dust density fluctuation, we solve equations (21) and (22):

$$\frac{n_{d1}}{n_{d0}} = \frac{Q_0 k^2 \phi_1}{m_d(\omega^2 + \omega_{\mathrm{J}}^2)}. \tag{27}$$

The charge fluctuation equation (25) is

$$Q_1 = \frac{i|I_{e0}|}{\omega + i\eta} \left[\frac{n_{i1}}{n_{i0}} - \frac{n_{e1}}{n_{e0}}\right], \tag{28}$$

where $\eta = (e|I_{e0}|/C)(1/T_{e0} + 1/w_0), w_0 = T_i - e\phi_{\mathrm{f0}}$ with C, ϕ_{f0} as capacitance and floating potential of the dust grain respectively. Now, using eqs (17) and (26)–(28) in Poissons equation (22), one gets the following dispersion relation:

$$0 = (\omega^2 + \omega_{\rm J}^2)\left[1 - \frac{\omega - iL_n}{A(\Omega + \omega - iL_n)}\right] - k^2 C_s^2 + \frac{i\beta(\omega^2 + \omega_{\rm J}^2)}{A(\omega + i\eta)}\left[S - \frac{\omega - iL_n}{A(\Omega + \omega - iL_n)}\right], \tag{29}$$

where

$$A = 1 + k^2\lambda_{De}^2 + \frac{T_{e0}n_{i0}}{T_{i0}n_{e0}}, \quad S = 1 + \frac{\epsilon\lambda_{De}^2}{\lambda_{Di}^2},$$

$$\epsilon = \frac{n_{e0}}{n_{i0}}, \quad \beta = \beta_e = \frac{|I_{e0}|n_{d0}}{en_{e0}}, \quad C_s^2 = \frac{\omega_{pd}^2\lambda_{De}^2}{A},$$

$$\lambda_{D\alpha}^2 = \frac{T_{\alpha 0}}{4\pi n_\alpha e^2}, \quad \omega_{pd}^2 = \frac{4\pi n_{d0}Q_0^2}{m_d}.$$

The dispersion relation equation (29) does not reduce to the dispersion relation of Dwivedi *et al* [29] in the absence of charge fluctuation ($\beta = 0$). As has already been mentioned, the source of this discrepency emanates from their inconsistent assumption that $P_{e0}\nabla\cdot v_{e1} = 0$. In the limiting case, when $\omega \ll L_n, L_T, \chi k^2$, the dispersion relation reduces to

$$\omega^2 + \omega_{\rm J}^2 - k^2 C_s^2 \approx \frac{k^2\lambda_D^2\omega_{pd}^2}{1 + k^2\lambda_D^2 + \frac{i\beta}{\omega + i\eta}}. \tag{30}$$

The above equation can be solved perturbatively by writing $\omega = \omega_0 + \omega_1$ with $\omega_1/\omega_0 \ll 1$. Then assuming $\eta \sim \beta \sim \omega_1$, one can study the effect of charge fluctuation on the wave in the first order. Ignoring $O(\omega_1^2)$ terms, the root of the above equation is

$$\omega = \sqrt{-\omega_{\rm J}^2 + k^2 C_s^2 + \frac{k^2\lambda_D^2\omega_{pd}^2}{1 + k^2\lambda_D^2}} + \left(\frac{i\beta}{2}\right)\frac{k^2\lambda_D^2}{(1 + k^2\lambda_D^2)^2}\frac{\omega_{pd}^2}{\omega_{\rm J}^2 - \left[k^2 C_s^2 + \frac{k^2\lambda_D^2\omega_{pd}^2}{1 + k^2\lambda_D^2}\right]} \tag{31}$$

from where it is clear that the charge fluctuation damps the dust acoustic mode in the absence of gravity ($\omega_{\rm J} = 0$). The self gravitating mode ($\omega_{\rm J} \neq 0$) will also exhibit similar behaviour so long as $\omega_{\rm J}^2$ is less than the terms in the square bracket of the denominator of the second term in (30). In most of the interstellar medium where $R \ll 1$, this scenario prevails. However, in the interplanetary medium $R \geq 1$ and $\omega_{\rm J}^2$ may become larger than the terms in the square bracket of the denominator of equation (31) and thus, charge fluctuation will assist the gravitational collapse of the grains. The numerical roots of (29) supports the conclusion of perturbative analysis [27]. Further, in the absence of the dust grains, one recovers the usual radiative condensational instability [8]

$$\omega = \frac{2i}{3}\left[\frac{L_n}{2} + L_T - \chi k^2\right]. \tag{32}$$

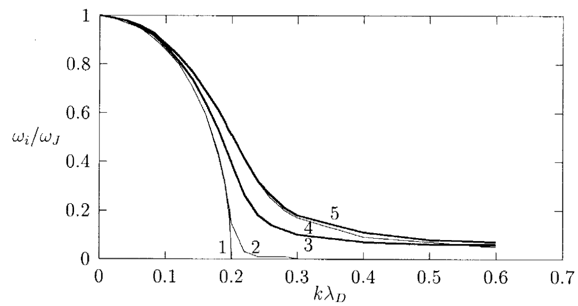

Figure 2. The unstable root ($\omega_i/\omega_{\rm J}$) of eq. (28) is plotted against $k\lambda_D$. Curve 1 corresponds to a purely gravitational mode when $a1 = \beta/\omega_{\rm J} = 0, a2 = \eta/\omega_{\rm J} = 0, a3 = L_n/\omega_{\rm J} = 0, a4 = \chi/(\omega_{\rm J}\lambda_D^2) = 0$ and $a5 = (L_n + L_T)/\omega_{\rm J} = 0$. Curve 2 corresponds to $a4 = 0.1, a1 = \ldots = a5 = 0$. Curve 3 represents $a1 = a2 = a4 = 0$ and $a3 = a5 = 0.1$. Curve 4 and 5 correspnds to $a1 = a3 = a4 = a5 = 0.1$ and $a2 = 0$ and $a1 = \ldots a5 = 0.1$.

It is well-known that if the plasma is isochorically stable i.e $L_T < 0$, then the necessary condition for the isobaric instability is [6]

$$L_T + L_n > 0 \tag{33}$$

for $n_e = n_{e0}$ and $T_e = T_{e0}$. Since the radiative loss rate in plasma grows with n_{e0}, the term L_n is destabilizing in dispersion relation (32). The growth rate is almost independent of the wave number. In the short wavelength regime, thermal conductivity starts to act and has stabilizing effect supressing perturbations of wavelength shorter than the conductive wavelength $\lambda_c \approx \lambda^2/\chi$.

Next, we solve (29) numerically in its totality and plot the normalized growth rate (with respect to the Jeans frequency $\omega_{\rm J}$) in figure 2. Curve 1 in figure 2 is Jeans mode without radiative losses and charge fluctuation on the grain. Curve 2 is Jeans mode in the presence of the thermal conductivity term which we expect to be important at large k. The growth rate does not change except at very large k. On the other hand, when the cooling time is comparable to the free fall time (curve 3) then the growth rate increases at large k. The inclusion of charge fluctuations further increases the growth rate (curve 4). When $\eta = 0$ the growth rate is maximum (curve 5). Thus, numerical picture suggests that most of the radiative effects dominate at large k implying condensation and structure formation down to much shorter scales (nearly by a factor of 2) than otherwise possible by purely Jeans mode (curve 1). Physically, gravitational condensation is inhibited by the 'thermal pressure' (set up in the plasma by the unshielded electrostatic field). However, 'switching on' of the electron cooling leads to the reduction in the 'pressure' of the ionized gas. Therefore, radiative cooling of the electrons will in general facilitate the condensation at larger k. Next, role of charge fluctuations enters through β and η. Here, the increase in growth rate at large k (curve 5, $\eta = 0$, $\beta \neq 0$) is indicative of the contribution of electrons and ions to the reduction of the repulsive field through screening effects. When $\eta, \beta \neq 0$, the growth rate is slightly reduced (curve 4). Lastly, we calculate the Jeans length from the marginal stability condition $\omega = 0$ in (29).

$$\lambda_{\mathrm{J}}^2 = \frac{2\chi(R-1)}{((R-1)(L_T+L_n) - R\chi\lambda_D^{-2}(1+\frac{\beta}{\eta}) + \Delta}, \tag{34}$$

where

$$\Delta = \left[\left((R-1)(L_T+L_n) - R\chi\lambda_D^{-2}\left(1+\frac{\beta}{\eta}\right) \right)^2 + 4\chi(R-1)\lambda_D^{-2}\left(1+\frac{\beta}{\eta}\right)(L_T+L_n) \right]^{1/2},$$

$\lambda_D^{-2} = \lambda_{De}^{-2} + \lambda_{Di}^{-2}$. While writing the above expression, we have assumed that $\beta_e \approx \beta_i \approx \beta$ and $L_n\lambda_{Di}^{-2}(1+\beta_i/\eta) \approx L_n\lambda_D^{-2}(1+\beta/\eta)$. We can delineate two cases of interest: (a) $R \ll 1$, a case relevant to the interstellar medium. Then, assuming radiative cooling to be dominant as compared to the diffusion losses $i, (L \gg \chi\nabla^2 T)$, we find thermal conductivity much smaller than the cooling and one gets

$$\lambda_{\mathrm{J}}^2 \approx \frac{\chi}{(L_T+L_n) - \chi(1+\frac{\beta}{\eta})}. \tag{35}$$

When (b) $R \leq O(1), Q_0/m_d \sim 10^{-4}$ a case relevant to the interplanetary medium, we see that

$$\lambda_{\mathrm{J}}^2 \approx \frac{2\chi}{(L_T+L_n)} \tag{36}$$

charge fluctuation plays no role at all. It would appear then that for radiative Jeans mode, charge dynamics is important for the micron and submicron sized grains.

5. Summary

We have investigated the radiative Jeans mode where the radiative cooling of electron takes place via several collisional processes. In the case when the plasma is completely ionized, cooling of the electrons may take place via bremsstrahlung. As the electrons are coupled to the large charged dust grains, the Jeans collapse of the grains gets affected by the cooling electrons via the electrostatic coupling. We have assumed cold grains. However, inclusion of finite grain temperature will modify the acoustic speed C_s. The kinetic energy lost in the form of radiation amplify the fluctuations and facilitates the condensation. The growth rate of Jeans collapse enhances at large k indicating that the radiative condensation of the electron leads to the condensation to begin at much shorter scales than in the case with pure Jeans mode.

How important is the coupling between Jeans collapse of the grains and radiative condensation of the electrons? The radiative condensational instability operates on all scales. However, at large scale length, gravitational collapse will be the dominant mode. At short wavelength, radiative cooling of electron will help reduce the ‘plasma pressure’ resulting in the reduction of the Jeans length. Therefore, self collapse of a plasma cloud results due to radiative instability at short wavelength.

Let us compare charge fluctuation time scale $t_c = \beta^{-1}$, Jeans time scale $t_{\rm J} = \omega_{\rm J}^{-1}$ and acoustic time scale $t_a = (k\, c_s)^{-1}$. In order to ignore the charge dynamics, we should have $t_c \ll t_{\rm J}$ and $t_c \ll t_a$. From $t_c \ll t_{\rm J}$ one gets $\omega_{pi}/\omega_{\rm J} \ll a/\lambda_D$ and from $t_c \ll t_a$ one gets $a/\lambda_D \gg k\,\lambda_D$. For a dusty plasma $a/\lambda_D \ll 1$ and hence

$$\omega_{pi}/\omega_{\rm J} \ll a/\lambda_D \ll 1. \tag{37}$$

Also, as $a/\lambda_D \gg k\,\lambda_D$, this will imply $k\,\lambda_D \to 0$ i.e one can consider only long wavelength regime. As we have seen, in the long wavelength regime, gravitational mode dominates. Therefore, it is important to retain charge dynamics while considering the effect of radiative instability on the Jeans mode.

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