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# Measurement of $\sigma\tau$ product of solid state laser materials by an alternative method: Application to $Nd^{3+}$ doped $YVO_4$ crystal for ${}^4F_{3/2} \rightarrow {}^4I_{11/2}$ transition

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**Abstract.** In this paper an alternative approach for measurement of $\sigma\tau$ product for ${}^4F_{3/2} \rightarrow {}^4I_{11/2}$ transition of $Nd^{3+}$ doped $YVO_4$ crystal is reported. In this method a microchip laser is formed by keeping a small piece of the sample in plane–plane resonator and a diode laser (808 nm) is used for pumping. The pump power induced thermal lensing effect is used to make the cavity stable. The cavity mode area is estimated by measuring the thermal lens focal length at the threshold and the average pump area is measured by Gaussian fit to the intensity profiles of the pump beam. The value of $\sigma\tau$ product of Nd:$YVO_4$ crystal obtained by this method is within 10% of the reported values. The advantage of this method is that it is a simple method for direct measurement of $\sigma\tau$ product of laser crystals.

**Keywords.** Solid state laser; thermal lens; diode pumping; plane–plane cavity; $\sigma\tau$ product.

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## 1. Introduction

The product of the stimulated emission cross-section ($\sigma$) and the radiative lifetime ($\tau$) of the upper laser level is an important parameter for the assessment of a laser crystal. Knowledge of the $\sigma\tau$ product is essential in evaluating laser system parameters such as maximum gain, saturation power and optimum output mirror reflectivity [1,2]. In case of end-pumped solid state 4-level laser systems, the threshold pump power is inversely proportional to the $\sigma\tau$ product of the lasing crystal [3]. There are several experimental methods reported in the literature [1–5] for finding the $\sigma\tau$ product value of a crystal at the lasing wavelength. The spectroscopic technique [4] for measurement of stimulated emission cross-section require determination of fluorescent branching ratios and the fluorescent quantum efficiency of the ${}^4F_{3/2}$ level which are often difficult to determine [2]. The laser pumped laser technique [1,2] is an easier method for determination of $\sigma$ and is free from the complications involved in spectroscopic method. However, this is an indirect method, which can only provide a relative measurement of $\sigma$. In this method, the $\sigma$ value of the reference material should be known very accurately and the measurement set-up must be identical for both the sample and the reference material. Moreover, if the reference material is not optically good due to the growing process or due to some other reasons it may lead to erroneous result. In addition, the pump power and mechanical stress induced lensing effects are also not taken into account, which may not guarantee the identical mode volume for the sample and reference materials. Mermilloid *et al* [5] reported direct measurement of $\sigma\tau$ value of several $Nd^{3+}$ doped materials. In this method, plano-concave resonator was used for laser oscillation and the cavity mode area was calculated from the resonator parameters. However, this method is also not accounting for the pump power induced thermal lensing effect in the crystal and due to this the value of the cavity waist radius calculated from the resonator parameters may be different from the actual waist radius. Moreover, in their method, the pump area was not averaged over the cavity length and the beam quality factor of the pump beam was not taken into account.

In this paper, we are reporting a new approach of finding the direct and accurate value of the $\sigma\tau$ product of a lasing crystal. In our method the focussed beam from an AlGaAs diode laser array (~808 nm) is used to pump a small sample of $Nd^{3+}$ doped laser crystal on which the laser mirrors are directly deposited to form a plane–plane cavity. The pump power induced thermal lensing effect is used to make the cavity stable [6]. Recently we have developed a technique of finding the focal length of the thermal lens induced by the pump beam and its variation with the pump power in plane–plane resonator [7]. Using this method, we measured the cavity mode area at the threshold. The pump spot size and the beam quality factor ($M^2$-value) of the pump beam for its propagation inside the resonator is measured by the Gaussian fit method [8]. The $\sigma\tau$ value was estimated from the threshold pump power which is a function of the above mentioned parameters. We applied this method on a $c$-axis cut 3 atm% doped $Nd^{3+}$:$YVO_4$ crystal. Nd:$YVO_4$ is a commercially available crystal and its lasing properties are well characterized. We have measured its $\sigma\tau$ product for ${}^4F_{3/2} \rightarrow {}^4I_{11/2}$ transition by our method and found the value is matching within 10% with the reported values [10]. The advantage of this method is that it is a simple technique of finding the $\sigma\tau$ value of a laser crystal directly and accurately.

## 2. Theoretical basis for measurements

The $\sigma\tau$ product can be obtained from measurements of laser thresholds as a function of the output mirror transmission, weighted average of the pump and mode area and the internal losses of the crystal. The absorbed pump power at threshold in an end pumped solid state laser device is given by [3]

$$P_{a,\mathrm{th}} = \frac{\pi h\nu_p}{4\sigma_e\eta_p\tau}\left(\varpi_c^2 + \varpi_p^2\right)(L+T)\,, \tag{1}$$

where $P_{a,\mathrm{th}}$ is the absorbed pump power at threshold, $h$ is the Planck constant, $\nu_p$ is the frequency of the pump beam, $\sigma_e$ is the effective stimulated emission cross section of the gain medium at the lasing wavelength, $\tau$ is the upper manifold life time, $\eta_p$ is the pump quantum efficiency which is the number of ions in the upper manifold created by one absorbed photon and in case of $Nd^{3+}$ lasers the value of $\eta_p$ can be assumed to be equal to 1. $L$ is the total intrinsic round trip loss in the gain medium at the lasing wavelength and $T$ is the transmission of the output-coupling mirror. The internal loss includes absorption by the crystal at the lasing wavelength, scattering at the end faces of the crystal and reflection losses due to slight deviation from the exact parallelism of the crystal. $\varpi_P$ and $\varpi_c$ represent the Gaussian beam radii of the pump and signal (circulating intensity inside the laser resonator), respectively, averaged with proper weight factor along the length of the crystal. However, (1) can be applied for end pump devices using diode laser whose beam quality is not $TEM_{00}$ because the end pumped devices typically operate in a $TEM_{00}$ mode and the threshold is not highly dependent on the exact form of the pump beam as long as the power can be described as being within some radius equal to $\omega_p$ [3]. To account for diffraction, the average values of $\omega_c^2$ and $\omega_p^2$ in the gain medium can be used. Rewriting (1) with the average values of pump area and cavity mode area we get

$$\sigma_e\tau = \frac{h\nu_p}{2\eta_p P_{\mathrm{a,th}}} \left(\langle A_c\rangle + \langle A_p\rangle\right)(L+T)\,, \tag{2}$$

where $\langle A_c\rangle$ and $\langle A_p\rangle$ are the average area of the Gaussian cavity and pump mode respectively. In our set up, we have used plane–plane resonator and the pump power induced thermal lensing effect is responsible for defining the laser mode profile. The thermal lensing effect in plane–plane resonator in end-pumped configuration and its variation with the pump power is investigated by us in our earlier works and is reported in ref. [7]. In that work we have shown by simple physical argument that the effective focal length of the thermal lens can be correlated with the pump power absorbed by the crystal by the following equation:

$$f_e = \frac{1}{\alpha P_{\mathrm{abs}} + \beta}, \tag{3}$$

where $f_e$ is the effective focal length of the thermal lens and any lensing effect induced by the mechanical stress due to mounting of the crystal, $\alpha$ and $\beta$ are two constants and their values depend on the properties of the crystal and configuration of the laser setup. Thus, from (3), thermal lens focal length at the threshold pump power can be obtained if the values of $\alpha$ and $\beta$ are known. One requirement in end-pumped microchip lasers is that the crystal should have high absorption at the pump wavelength so that small gain length can be used. The high value of average absorption coefficient of the gain medium keeps the active region confined to the back mirror of the crystal. Therefore, we can assume that the thermal lens is located at the back mirror plane and the plane–plane resonator can be treated equivalent to a plano-concave resonator. Thus, the cavity mode area at the waist can be obtained as [13]

$$A_{c0} = \frac{\lambda d}{2n}\sqrt{\frac{2nf_e}{d} - 1}, \tag{4}$$

where $A_{c0}$ is the cavity mode area at the waist, $\lambda$ is the lasing wavelength, $d$ is the cavity length, $n$ is the refractive index and $f_e$ is the thermal lens focal length at the absorbed pump power $p_{\mathrm{abs}}$ given by (3). In order to account for the diffraction effect the average mode area is taken. The propagation of the mode area inside the crystal and hence inside the cavity is given by $A_c(z) = A_{c0}[1 + (z\lambda/2nA_{c0})^2]$ and the average mode area is given by

$$\langle A_c\rangle = \frac{1}{d}\int_0^d A_c\,(z)\,\mathrm{d}z, \tag{5}$$

where $d$ is the length of the cavity. The diode laser beam used for pumping the solid state laser, is highly multi-mode in shape. Earlier, we developed a method to determine the spot size of a multi-mode diode laser beam [8]. In this method we have shown that the width of an arbitrary multi-mode laser beam can be measured by least square fitting of a Gaussian profile and the paraxial $ABCD$ method for beam propagation can be applied when the beam parameters are computed with this method. We have measured the pump beam spot sizes at several locations around the focal region of the focusing lens in horizontal and vertical directions by Gaussian fitting. The beam quality factor and the waist radii are obtained by least square fitting of the following paraxial beam propagation equation to the measured spot sizes:

$$\omega_{x,y} = \omega_{0x,y}\sqrt{1+\left(\frac{M_{x,y}^2 z\lambda}{\pi\omega_{0x,y}^2}\right)^2}, \tag{6}$$

where $\omega_{0x,y}$ is the beam waist, $\omega_{x,y}(z)$ is the spot size at the distance $z$ in the horizontal and vertical directions respectively and $M_{x,y}^2$ is the corresponding beam quality factor. Thus, the pump area $A_p$, at any plane $z$ can be obtained as, $A_p(z) = (1/2)\pi\omega_x(z)\omega_y(z)$ and the average pump area is given by:

$$\langle A_p\rangle = \frac{\int_l^{l+d} \mathrm{e}^{-\alpha_p z} A_p\,(z)\,\mathrm{d}z}{\int_l^{l+d} \mathrm{e}^{-\alpha_p z}\mathrm{d}z}, \tag{7}$$

where $l$ is the distance of the resonator input plane from the focussing lens, $d$ is the length of the resonator and $\alpha_p$ is the absorption coefficient of the crystal at the pump wavelength. The optical losses of the crystal and the absorbed pump power at the threshold can be found by the conventional methods [9,11]. In brief, this method for finding $\sigma\tau$ value requires the measurements of (1) absorbed pump power at threshold in micro-chip laser configuration of the sample, (2) focal length of the thermal lens at threshold in order to find average cavity mode area, (3) beam quality parameter of the pump beam for finding average pump area and (4) internal losses of the sample.

## 3. Experimental

The experimental setup for threshold measurement in microchip laser is shown in figure 1. An FTI make (Russian) single array (1 $\mu$m $\times$ 100 $\mu$m) semiconductor diode laser was used as an axial pump source. The diode laser was operated at 808 nm. With a built-in fast axis collimator, the far field $1/e^2$ divergence angles are $\theta_\perp = 0.9^\circ$ and $\theta_\parallel = 8^\circ$. A collimating lens system $f_1$ of effective focal length 8.0 mm is placed at the output of the diode. The distance of $f_1$ from exit plane was optimized in such a way that the dimension of pump beam remains smaller than 1.0 cm aperture in front of the focussing lens. The aperture was present due to the lens holder. The optimized distance of $f_1$ from the exit plane was about 7.7 mm. A plano-convex focussing lens $f_2$ of 25.0 mm focal length was placed 20 cm away from the collimating lens. The laser crystal was placed at the focal plane of $f_2$. The resonator for the microchip laser was formed by directly depositing dielectric mirrors on to the crystal surfaces. The output mirror had a reflectivity of 95.0% and the opposite

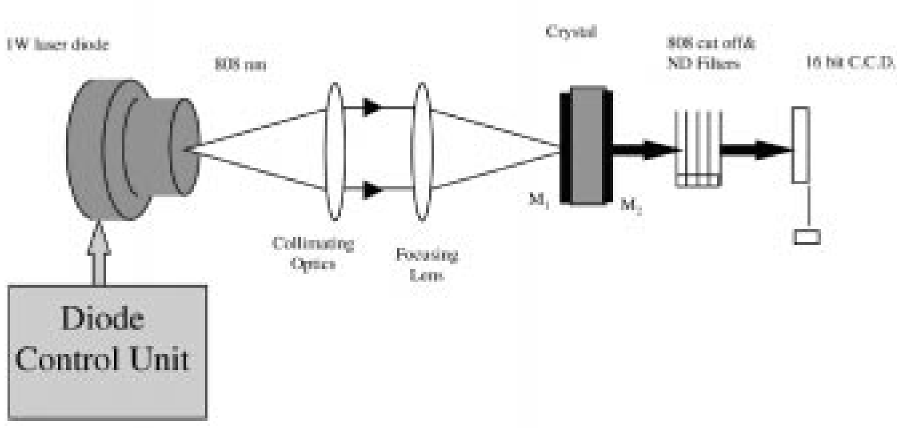


**Figure 1.** Experimental setup for threshold and focal length of thermal lens measurement of diode pumped Nd:$YVO_4$ microchip laser.

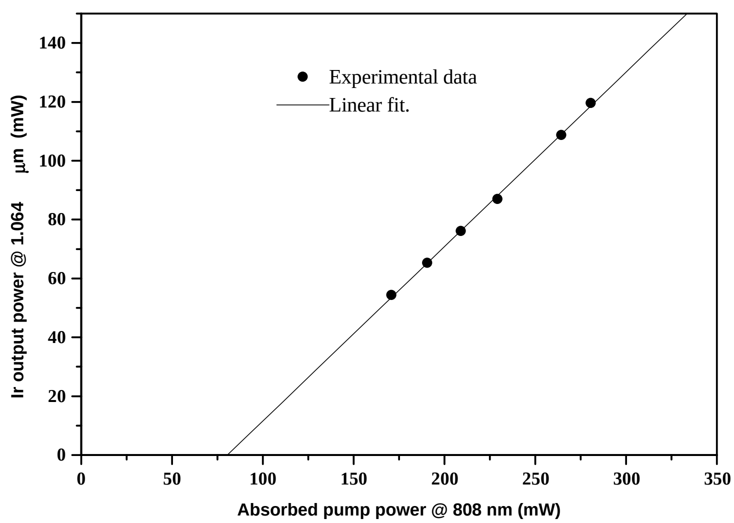


**Figure 2.** Slope efficiency curve for 3 atm% doped 0.5 mm thick Nd:$YVO_4$ crystal in plane–plane resonator configuration. Nd:$YVO_4$ laser shows a slope efficiency of 59% and absorbed pump power at threshold is 80 mW.

mirror facing the diode laser beam had a reflectivity of 99.9% at the lasing wavelength and transmission more than 95% at the pump wavelength. As the laser output was a mixture of pump and laser beams, an edge filter was used to block the pump beam. Figure 2 shows the slope efficiency curves for a $c$-axis cut 0.5 mm thick 3 atm% doped Nd:$YVO_4$ crystal. The incident pump power was varied with the help of neutral density filters instead

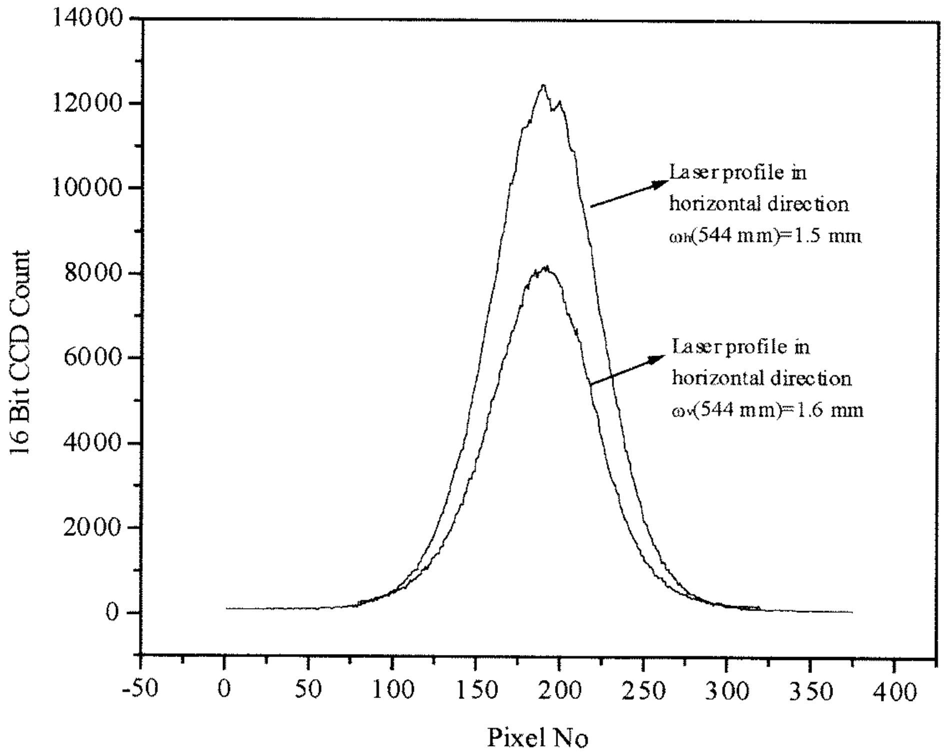


**Figure 3.** Recorded profile of the laser beam at 170 mW pump power. Size of the beam was estimated by fitting zeroth order Hermite–Gaussian intensity profile.

of changing the laser diode current in order to avoid the shift in wavelength and spot size of the pump beam as they are a function of the diode current. The variation of the output power with absorbed pump power was approximated as linear variation and extrapolated linearly to find out the threshold and slope efficiency. Nd:$YVO_4$ microchip laser shows a threshold of 80 mW and slope efficiency of 59%.

The experimental setup for thermal lens focal length measurement is same as figure 1. Intensity profile of 1064 nm laser beam from the microchip laser was recorded at a distance of 544 mm from the output mirror. The profile recorder was an SBIG CCD camera with a 16-bit dynamic range. The pixel sizes are 23 $\mu$m and 27 $\mu$m in the horizontal and vertical directions respectively. The pump power delivered by the diode laser was varied from 50 mW to 400 mW by attenuating it with neutral density filters. The IR laser profile was recorded at different pump powers. For each profile recording, the thermal lens was allowed to stabilize for 10 minutes. All recorded profiles were perfectly Gaussian both in the horizontal and vertical directions but the beam profile itself is elliptical in shape. The beam radius at this distance was obtained by fitting a Gaussian profile to both the horizontal and vertical directions. A typical fit to a laser beam profile in horizontal and vertical directions is shown in figure 3, which shows the profiles are elliptic Gaussian in shape. The size of the beam waist radius in horizontal

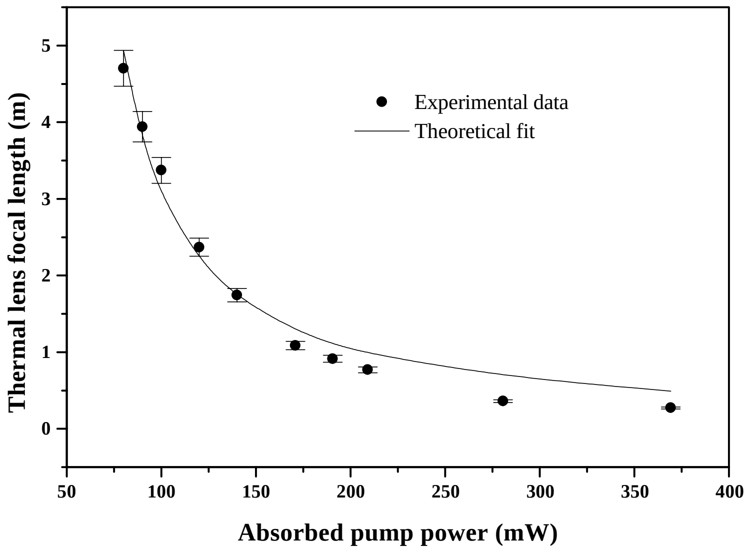


**Figure 4.** Variation of thermal lens focal length with pump power in end-pumped Nd:$YVO_4$ laser in plane–plane configuration. Focal lengths are estimated by computing mode area at the waist. Dotted curve is the theoretical fit of the function depicted in eq. (11).

and vertical directions is estimated from these computed far field beam radius values by knowing the far field beam divergence angle, $\theta_{x,y} = \omega_{x,y}(z)/z$, where $z$ is the distance of the CCD camera from the crystal and propagating back towards the crystal surface ($\omega_{0,x,y} = \lambda/\pi\theta_{x,y}$), where the notation $x, y$ are to represent the horizontal and vertical directions respectively. Once the waist radii in the horizontal and vertical directions are known the waist area $A_{c0} = (\pi/2)\omega_{x0}\omega_{y0}$ is computed and the focal lengths of thin lenses were estimated from the computed waist areas with the help of (4). The measurement error in the thermal focal length was estimated from the maximum possible measurement errors in the far field spot size, crystal thickness and the camera position from the beam waist. The estimated error in the thermal focal length measurement is less than 8%.

The scattered point in figure 4 is the plotting of the thermal lens focal length computed from (4) against the absorbed pump power and the dotted curve is the theoretical fit of the function given in (3). From figure 4 it is clear that the fitting is good in the lower absorbed pump power than in the higher pump power region. This discrepancy is due to the fact that at the high pump power the higher order terms in the refractive index profile in the crystal induced by the pump beam become appreciable which makes the laser to oscillate at the higher order modes rather than the $TEM_{00}$ mode. Since the higher order mode has larger far field divergence, focal length estimated by this way becomes smaller than what it should be if the laser mode was perfectly $TEM_{00}$. From this fitting we can find the value of the constants $\alpha$ and $\beta$ and using the values of these constants we can find the focal length at the threshold pump power using (3). Once the focal length at the threshold is known, the average cavity mode area at threshold can be obtained from eqs (4) and (5).

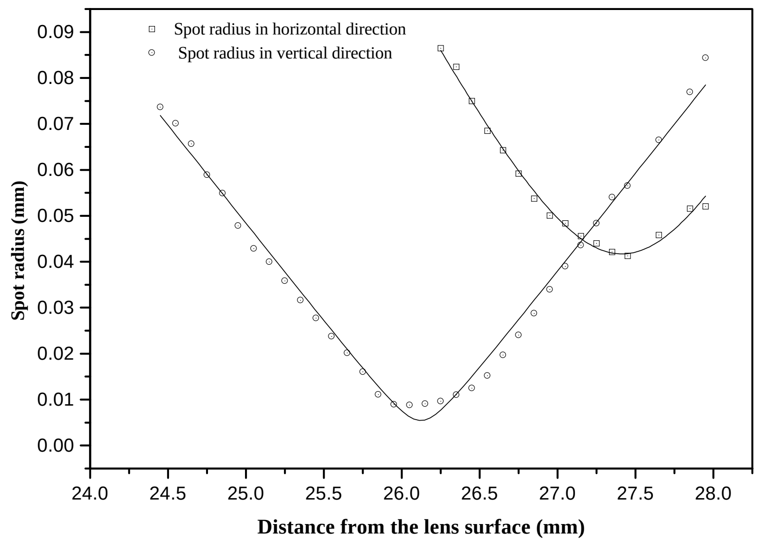


**Figure 5.** Variation of the spot radius of diode laser beam with distance from the secondary principal plane of the focussing lens in horizontal and vertical directions. The continuous line is the theoretical fit of the beam propagation equation in order to find out waist radius, waist location and beam quality factors in the respective directions.

For experimental determination of the pump beam waist, the laser crystal is removed and the focussed beam was imaged with the help of a 50 mm focal length lens on the CCD camera with a image magnification factor of 8. Images of spot size were recorded at several planes before and after the focussed spot size. The camera was oriented in such a way that the axes of the laser beams were almost parallel to the CCD arrays. For reducing the random noise present, all the rows were added to get the horizontal profile and all the columns were added to get the vertical profile. The radii of these spot sizes along both the axes were measured with Gaussian fit method [8]. The measured values of the radii were reduced to actual values by dividing them with the magnification factor. Measured variation of spot size with propagation distance is shown in figure 5. The values of $M^2$ factor and $W_0$ were obtained for both the axes by least square fitting of beam propagation equation (6) to the experimental data. The $M^2$ values along both the axes are $M_x^2 = 10.48 \pm 0.18, M_y^2 = 1.0 \pm 0.21$ and waist size values along both the axes are $W_{0x} = 0.0417 \pm 0.01$ mm and $W_{0y} = 0.00543 \pm 0.0013$ mm. From figure 5 it is clear that the diode beam is astigmatic in nature. After knowing the propagation parameters of the pump beam we computed the pump spot area inside the resonator, considering it as an elliptical profile, as $(\pi/2)w_x(z)w_y(z)$. The average pump spot area was then calculated from (7).

## 4. Results and conclusion

To verify the accuracy of this method of finding $\sigma\tau$ product of laser crystals we applied this technique to a $c$-axis cut 3 atm% doped 0.5 mm thick Nd:$YVO_4$ crystal. In our measurement the output mirror transmission for this laser is 5%, absorbed pump power at threshold is 80 mW and the internal optical loss of this crystal is taken from its supplier's data as 0.2% [10]. This value of internal loss is also verified with the help of Findlay–Clay method [9]. From figure 4 we get the least square fitted value of the $\alpha$ and $\beta$ parameters as $0.00651 \pm 0.00064\ \mathrm{mW^{-1}m^{-1}}$ and $-0.31906 \pm 0.05508\ \mathrm{m^{-1}}$ respectively. Thus the focal length of the thermal lens at the threshold is 4.95 m and the average cavity mode area at threshold is computed from (5) is $2.54 \times 10^{-4}\mathrm{cm^2}$ . The cavity length is 0.5 mm. The average pump area computed from (7) for this laser is $8.13 \times 10^{-6}\mathrm{cm^2}$. Knowing the values of these parameters we found the $\sigma\tau$ value of 3 atm% Nd:$YVO_4$ from (2) as $2.1 \times 10^{-23}$ $\mathrm{cm^2}$ sec. Nd:$YVO_4$ is a widely used solid state laser material for diode pumping. Its lasing parameters are extensively measured by several researchers. The reported value of stimulated emission cross-section ($\sigma$) for 3 atm% Nd doped $c$-axis cut $YVO_4$ crystal is $7.0 \times 10^{-19}$ $\mathrm{cm^2}$ and the radiative lifetime ($\tau$) of the upper laser level is as 33 $\mu$sec [10] resulting $2.31 \times 10^{-23}$ $\mathrm{cm^2}$sec $\sigma\tau$ value. Our measured $\sigma\tau$ value is within 10% of the reported value. Thus, our method can measure the $\sigma\tau$ value of any crystal with fair accuracy. In conclusion, we have demonstrated a simple method for finding $\sigma\tau$ product value of a solid state laser crystal. This method is free from the complications of the previously reported methods and also takes care for the pump power induced thermal lensing effect and the quality of the pump beam.

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