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Electrostatic sheath at the boundary of a collisional dusty plasma

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Abstract. Considering the Boltzmann response of the ions and electrons in plasma dynamics and inertial dynamics of the dust charged grains in a highly collisional dusty plasma, the nature of the electrostatic potential near a boundary is investigated. Based on the fluid approximation, the formation as well as the characteristic behaviours of the sheath is studied. It is expected that the presence of dust charged grains will lead to a very different behaviour of the sheath as compared to that of electron-ion plasma. Moreover, the collisions of the dust charged grains with the neutrals are expected to exhibit novel features.

Keywords. Dusty plasmas; sheath; dust grains.

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1. Introduction

Dusty plasma is an unique multicomponent plasma and is composed of dispersed macroscopic dust charged grains that forms a colloidal type suspension [1] in any given parent plasma background. Dusty plasma represents the most common form of astrophysical [2,3], laboratory [4–6] and industrial plasma. The typically micron-sized dust grains normally acquire negative charges to high order of magnitude with respect to normal electronic charge ($q_d \sim 10^5$–$10^6 e$). The mass of the dust grains can have very high value too, up to 10^6–10^8 times the proton mass. Thus the dust charge to mass ratios are different than the normal ionic species in multicomponent plasmas. Furthermore, the dust charge, mass and size, in general behave as dynamic variables [7–9] and produce novel effects on collective degrees of plasma dynamics. Thus, the theoretical modeling of a dusty plasma requires the consideration of dust charge fluctuation dynamics. However, under certain limits, a constant dust charge model could be justified [3, 13–15] for selective dust-plasma parameter domain.

Presently, among the plasma configurations dusty plasma is perhaps the fastest growing field of study. Most of the earlier investigations on dusty plasma were confined around the dust charging model development [10–12] and the effects of dust charged grains on high frequency collective plasma properties [16–18] under isolated approximation of dust grains. Subsequently, the study of the dynamics of the dust charged grains within collective approximation picked up momentum [2,19,20]. The study extended to many of the ideal model of dusty plasmas based on the assumption that the dust charged grains remain constant in their dynamical system [21–25]. Rao *et al* [21] presented a novel mode of acoustic wave in dusty plasma which is similar to the dynamics of the multi-component plasma with negative ions [26,27] and named it as dust acoustic wave (DAW). Interestingly, their observations encouraged the experimental works carried out by Barkan *et al* [28] which led to new observations on the dust acoustic waves. However, the consideration of dust charge fluctuation in plasma was a milestone on the study of dusty plasmas.

The study on the formation of sheath in dusty plasmas at electron absorbing walls is of current interest [29,30]. Many authors have considered the influence of collisions on the plasma sheath transition [31–34] in the normal electron ion plasma and investigated the Bohm criterion. But the collisional sheath does not evaluate the usual approximated relation similar to Bohm criterion [35]. This observation is significant in the context of dusty plasma wherein the sheath, as will be seen, can be both positive as well as be negative [29,36] unlike the electron-ion plasma sheath which is always positive.

In this paper, the wall region of a dusty plasma model is considered wherein the plasma is contaminated by constant dust charged grains. It is further assumed that the dust charge fluctuations are sufficiently small and cannot be distinguished on the time scale of the fluid approximation. Both electrons and ions are very light in comparison with the massive micron sized dust charged grains and so they are assumed to be in their respective thermal equilibrium. The dust charged grains suffer frequent collisions with the neutrals. The choice of such configuration is of the current interest and, as expected it will be seen that the characteristic behaviour of the sheath is significantly influenced by the presence of the dust charged grains.

2. Mathematical formulation and basic equations

We have considered a plasma consisting of electrons, ions as well as negative dust charged grains. The basic equations governing the plasma, in fluid approximation, are the equations of continuity and motion. On the inertial time scale of the cold but heavier dust charged grains, the dynamics of the electrons and ions are treated as a neutral background in plasma and are assumed to follow respectively the following Boltzmann relations:

$$n_e = n_{eo} \exp\left(\frac{e\phi}{T_e}\right), \tag{1}$$

$$n_i = n_{io} \exp\left(\frac{-e\phi}{T_i}\right). \tag{2}$$

ϕ is the self consistent electrostatic potential. T_i and T_e are respectively ion and electron temperatures.

The dust charged grains are governed by the following equations:

$$\frac{\partial n_d}{\partial t} + \frac{\partial}{\partial x}(n_d v_d) = 0, \tag{3}$$

$$n_d m_d \frac{\partial v_d}{\partial t} + n_d m_d v_d \frac{\partial v_d}{\partial x} = eZn_d \frac{\partial \phi}{\partial x} - F_c, \tag{4}$$

$$\frac{\partial^2 \phi}{\partial x^2} = -4\pi e(n_i - n_e - Zn_d), \tag{5}$$

where m_d is the mass of the dust charged grains moving with the velocity v_d and n_d is the dust number density, n_e and n_i are respectively the number densities of the electrons and ions. Ze represents the charge on each dust grain. F_c is the drag force on the dust charged grains due to their collisions with the neutrals. It is given by

$$F_c = m_d(n_n \sigma v_d)v_d,$$

where n_n is the neutral gas density and σ is the momentum transfer cross section for the collisions between the dust charged grains and neutrals.

The basic equations are supplemented by the charge neutrality condition at equilibrium and is expressed through the relation:

$$n_{io} = Zn_{do} + n_{eo}. \tag{6}$$

The subscript o represents the respective value of the plasma parameters at equilibrium.

In steady state eqs (3) and (4) become

$$\frac{\partial}{\partial x}(n_d v_d) = 0, \tag{7}$$

$$m_d v_d \frac{\partial v_d}{\partial x} = eZ\frac{\partial \phi}{\partial x} - m_d(n_n \sigma v_d)v_d. \tag{8}$$

Now, in order to simplify the basic equations, the following non-dimensional parameters are defined

$$\Phi = \frac{e\phi}{T_i}, \quad \xi = \frac{x}{\lambda_{Di}}, \quad u = \frac{v_d}{c_d}, \quad \alpha = \frac{\lambda_{Di}}{\lambda} \tag{9}$$

with $\lambda_{Di} = \sqrt{T_i/4\pi m_d e^2}$ is the ion Debye length, $c_d = Z\sqrt{(T_i/m_d)\ (n_{do}/n_{io})}$ is the dust acoustic speed for $\lambda_{De}^2 \gg \lambda_{Di}^2$ and $\lambda = 1/n_n\sigma$ is the collision mean free path of the dust charged grains. σ is considered a constant over the energy range of interest.

Based on these non-dimensional parameters the basic equations reduce to

$$n_e = n_{eo}\exp(\beta\Phi), \tag{10}$$

$$n_i = n_{io}\exp(-\Phi), \tag{11}$$

$$n_d u = n_{do} M, \tag{12}$$

$$uu' = \frac{\Phi'}{\varepsilon z} - \alpha u^2, \tag{13}$$

$$\frac{\partial^2 \Phi}{\partial \xi^2} = -\left(\exp(-\Phi) - \varepsilon_- \exp(\beta\Phi) - \frac{\varepsilon Z M}{u}\right) \tag{14}$$

along with the parameters

$$M = \frac{v_{do}}{c_d} = u_0, \quad \varepsilon_- = \frac{n_{eo}}{n_{io}}, \quad \varepsilon = \frac{n_{do}}{n_{io}}, \quad u' = \frac{\partial u}{\partial \xi}, \quad \beta = \frac{T_i}{T_e}.$$

For quasineutral presheath there is no space charge in that region i.e. $n_{io} = n_{eo} + Zn_{do}$. The transition from the presheath to the sheath should be smooth. Thus

$$n'_{io} = n'_{eo} + Zn'_{do}. \tag{15}$$

The prime represents the derivative with respect to ξ. With eqs (10) and (11), eq. (15) transforms as

$$-n_{io}\exp(-\Phi_0)x\Phi'_0 = n'_{eo}\exp(\beta\Phi_0)x\beta\Phi'_0 + Zn'_{do}, \tag{16}$$

where Φ_0 is the potential at the presheath.

Equation (7) leads to

$$n'_{do} = -\frac{n_{do}u'}{M}. \tag{17}$$

Combining eqs (16) and (17)

$$\frac{M}{\varepsilon Z}(\varepsilon_{-}\beta\Phi'_0 + \Phi'_0) = u'_0. \tag{18}$$

Equation (18) when inserted in eq. (13) yields

$$\Phi'_0 = \frac{\alpha M^2 \varepsilon Z}{[1 - M^2(\varepsilon_{-}\beta + 1)]}. \tag{19}$$

Unfortunately, because of the collisions of the dust charged grains with the neutrals $(\alpha \neq 0)$, u cannot be expressed analytically as a function of Φ. Thus the numerical approach has been employed to the eqs (13) and (14) for the new findings, in contrast to the earlier observations of the potential variation in the sheath region.

3. Numerical results

In figures 1 and 2, the variation of the normalized electrostatic potential Φ with ξ is plotted for a plasma with a typical value for $\varepsilon Z = 0.9$ (i.e. dust charged grains are present in copious amount) and with an arbitrary but reasonable mach number $M = 1.3$. The ratio of ion to electron temperature, β is 0.1; a value similar to those encountered in the laboratory plasmas. The emphasis is focused on the affect of collisions of the dust charged grains with the neutrals which is controlled through the variation of α.

When α is small (the collision mean free path is large compared to the ion Debye length) the sheath is found to be a negative sheath. However, for $\alpha \geq 0.6$ the sheath becomes a positive one. Thus, because of the collision of dust charged grains with the neutrals, there is a transition of the negative sheath to a positive sheath. It is also observed that the width of a negative sheath is substantially large compared to ion Debye length, λ_{Di} whereas in the case of a positive sheath it is of the order of λ_{Di}. The negative and positive sheaths differ in the order of the wall potential as well. The absolute value of wall potential for a negative sheath is about an order higher than that for a positive sheath.

The observations from figures 1 and 2 may be compared with those from figures 3 and 4 in which $M = 1.1$ but rest of the parameters are identical. In this plasma configuration

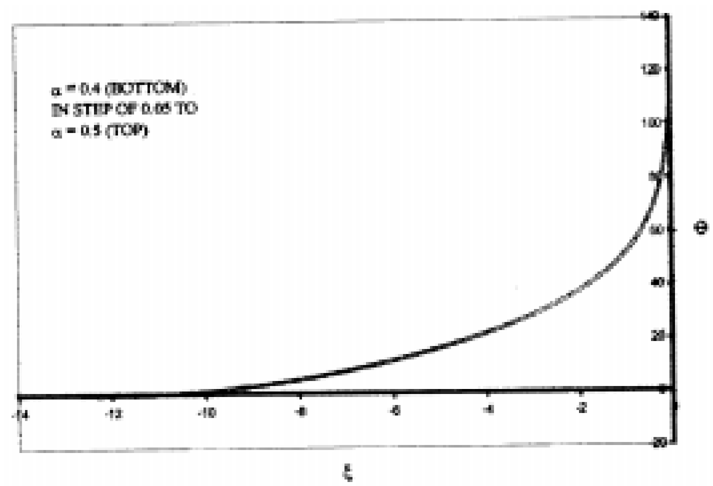

Figure 1. The sheath for $\alpha = 0.5$ (top) in step of 0.05 to $\alpha = 0.4$ (bottom). The parameters are $\varepsilon Z = 0.9$, $\beta = 0.1$ and $M = 1.3$.

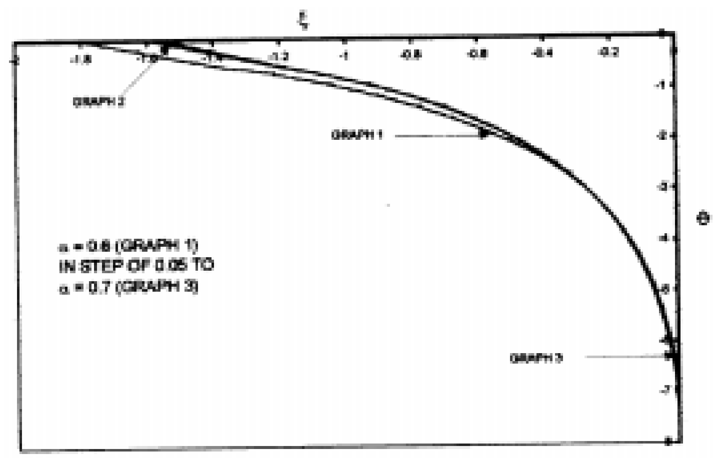

Figure 2. The sheath for $\alpha = 0.7$ (graph 3) in step of 0.05 to $\alpha = 0.6$ (graph 1). The parameters are $\varepsilon Z = 0.9$, $\beta = 0.1$ and $M = 1.3$.

the sheath character changes from negative to positive for $\alpha \geq 0.25$, a comparatively smaller value compared to the case $M = 1.3$.

Figures 5–8 reexamine the sheath characteristic for a plasma in which $\varepsilon Z = 0.1$ and as before, β is 0.1. It is seen that the sheath character transforms from negative to positive nature for values of α which are larger as compared to the plasma configuration in which $\varepsilon Z = 0.9$. In a dusty plasma in which $\varepsilon Z = 0.1$, the transition takes place at $\alpha \geq 0.65$ for $M = 1.3$ (figures 5 and 6) and at $\alpha \geq 0.35$ for $M = 1.1$ (figures 7 and 8).

In a dusty plasma containing negatively charged dust charged grains, the wall potential is generally positive. The electrons and the dust charged grains shield the positive wall from the bulk plasma forming a negative sheath. It may be speculated that for large mean

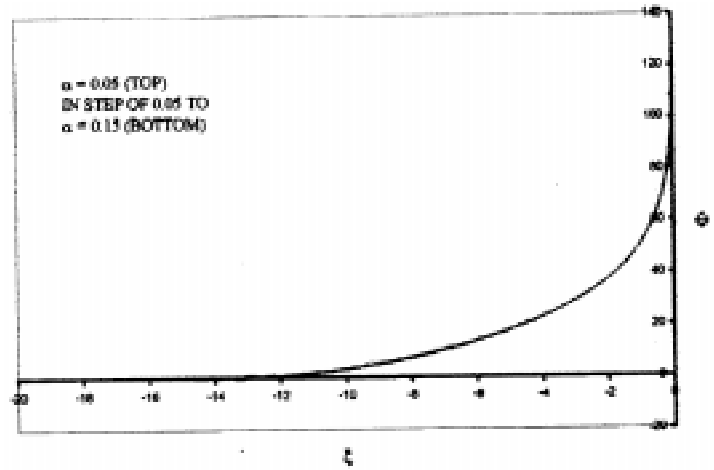

Figure 3. The sheath for $\alpha = 0.15$ (bottom) in step of 0.05 to $\alpha = 0.05$ (top). The parameters are $\varepsilon Z = 0.9$, $\beta = 0.1$ and $M = 1.1$.

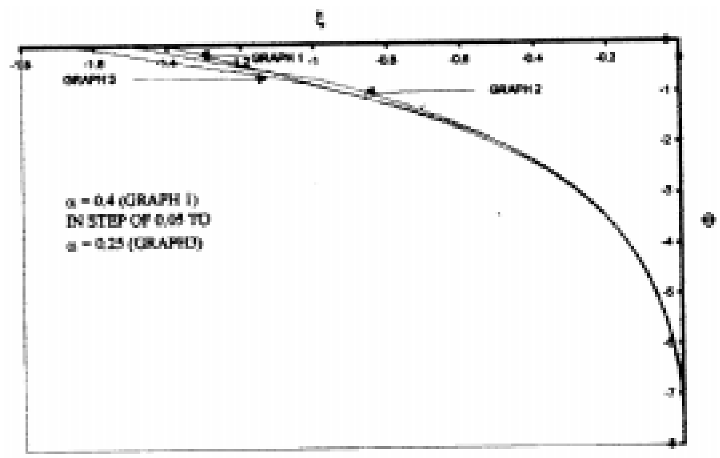

Figure 4. The sheath for $\alpha = 0.4$ (graph 1), $\alpha = 0.3$ (graph 2) and $\alpha = 0.25$ (graph 3). The parameters are $\varepsilon Z = 0.9$, $\beta = 0.1$ and $M = 1.1$.

free path, the collisions of the dust charged grains with the neutrals are not too significant. However, in a highly collisional atmosphere the drag force on the dust charged grains due to collisions becomes sufficiently large and retards them significantly. Since the dust flux is conserved, the population of dust charged grains increases in the sheath. Consequently the sheath may transform to a positive one as the number density of the electrons, ions and the dust charged grain collectively determine the curvature of the sheath through the Poisson's equation.

If the Mach number is large, a stronger drag force on the dust charged grains is necessary to obtain a positive sheath. Therefore, in such situations one observes that the negative

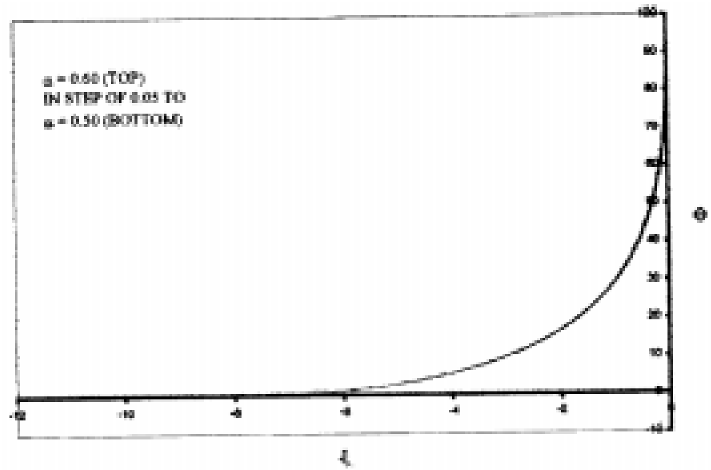

Figure 5. The sheath for $\alpha = 0.6$ (top) in step of 0.05 to $\alpha = 0.5$ (bottom). The parameters are $\varepsilon Z = 0.1$, $\beta = 0.1$ and $M = 1.3$.

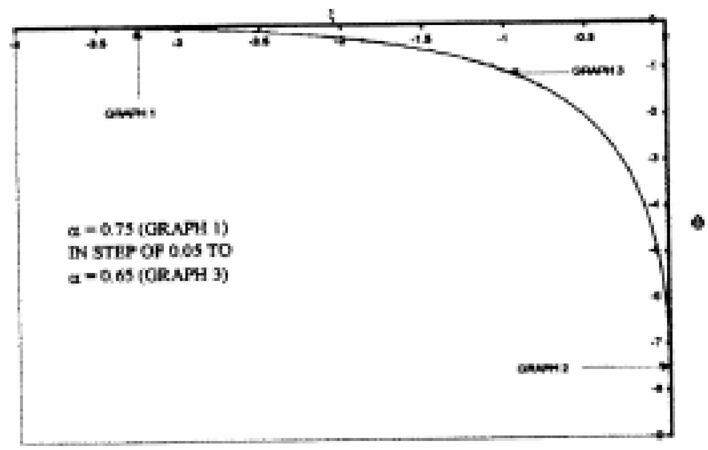

Figure 6. The sheath for $\alpha = 0.75$ (graph 1) in step of 0.05 to $\alpha = 0.65$ (graph 3). The parameters are $\varepsilon Z = 0.1$, $\beta = 0.1$ and $M = 1.3$.

sheath transforms to a positive one only at a larger value of α. On the other hand, if the concentration of dust charged grains is small, an appreciable drag force on the dust charge grains is obtained only at high values of α. Thus, one can explain why the negative sheath transforms to a positive one at higher values of α when εZ is small.

It will be interesting to examine the sheath in a dusty plasma in which the Mach number very large. Such conditions are observed in space plasmas; but the feature of transition of the sheath from negative to positive character does not occur here even for very large values of α. In figure 9, the sheath potential is presented for the parameters $\varepsilon Z = 0.9$, $\beta = 0.1$ and $M = 2.2$. It is seen that the sheath remains negative even for $\alpha = 0.99$.

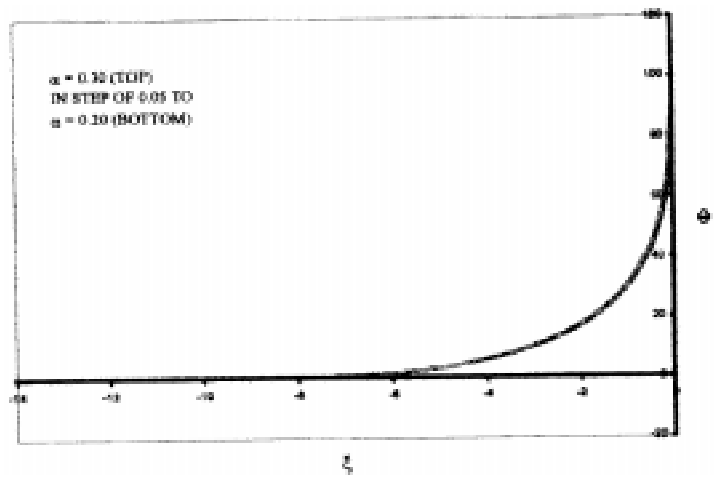

Figure 7. The sheath for $\alpha = 0.3$ (top) in step of 0.05 to $\alpha = 0.2$ (bottom). The parameters are $\varepsilon Z = 0.1$, $\beta = 0.1$ and $M = 1.1$.

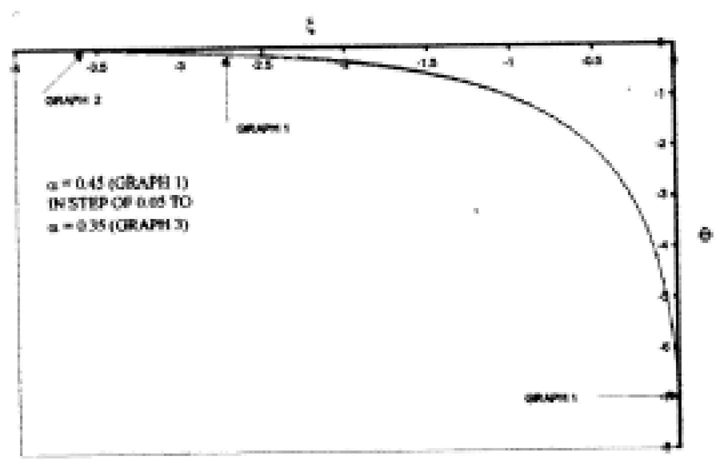

Figure 8. The sheath for $\alpha = 0.45$ (graph 1) in step of 0.05 to $\alpha = 0.35$ (graph 3). The parameters are $\varepsilon Z = 0.1$, $\beta = 0.1$ and $M = 1.1$.

However, for $M < 2.2$ but for identical εZ and β, the transitions do take place. It is likely that for large M, the number density of the dust charged grains does not suffer sufficient decrease to give a positive sheath.

The influence of the temperature ratio, β on the sheath characteristics is also studied and the results are presented in figures 10 and 11. The parameters are $\varepsilon Z = 0.1$ and $M = 1.1$. In figure 10, the observations for $\alpha = 0.3$ are presented. The sheath is negative for $\beta = 0.1$ as well as for $\beta = 0.5$; but in the later case, the wall potential as well as the sheath width is significantly smaller than those could be found for $\beta = 0.1$. In figure 11, the observations for $\alpha = 0.45$ is presented. The sheath changes from negative to positive type when

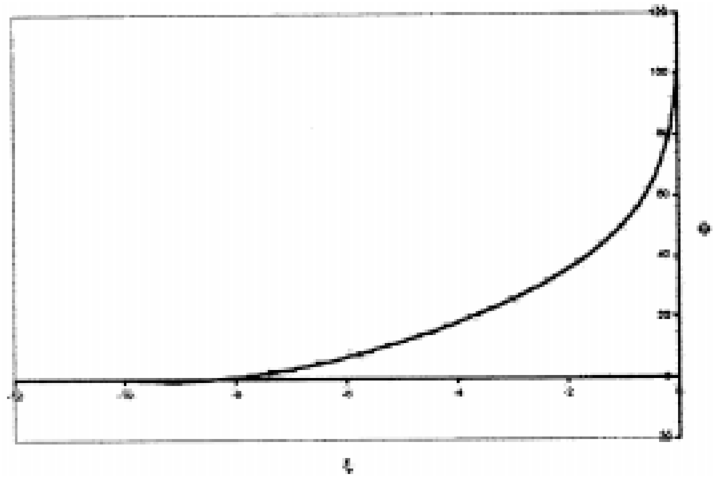

Figure 9. The sheath for $\alpha = 0.99$. The parameters are $\varepsilon Z = 0.9$, $\beta = 0.1$ and $M = 2.2$.

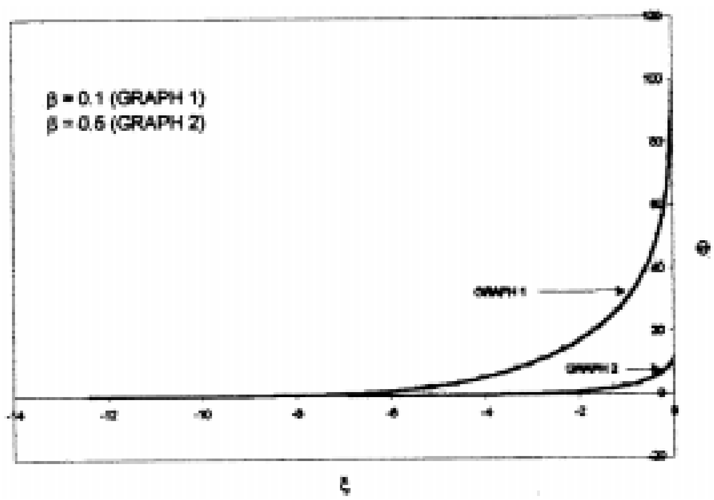

Figure 10. The sheath for $\beta = 0.1$ (graph 1) and $\beta = 0.5$ (graph 2). The parameters are $\varepsilon Z = 0.1$, $\alpha = 0.3$ and $M = 1.1$.

β increases from 0.1 to 0.5. The number density of the electrons increases in the sheath for higher value of β. Consequently, the curvature of the potential profile changes (figure 10) with increase in β and finally one obtains a positive sheath (figure 11).

4. Discussion and conclusion

The present theoretical investigation deals with the interaction between a dusty plasma and a solid boundary. Electrons and ions remain in thermal equilibrium while the dust

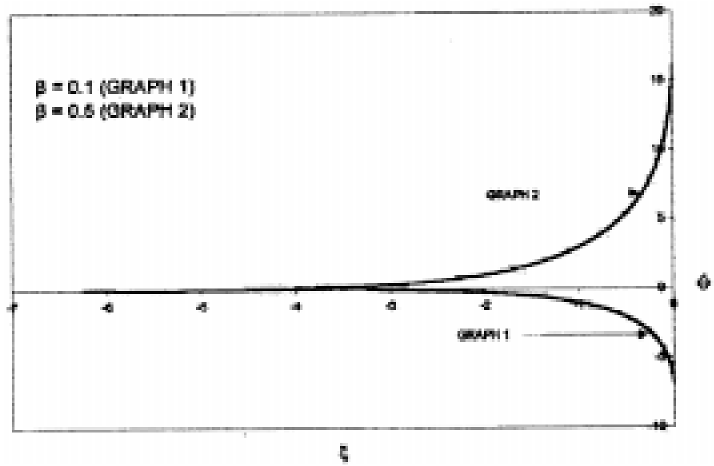

Figure 11. The sheath for $\beta = 0.1$ (graph 1) and $\beta = 0.5$ (graph 2). The parameters are $\varepsilon Z = 0.1$, $\alpha = 0.45$ and $M = 1.1$.

charged grains are the inertial species. The dust charged grains suffer frequent collisions with the neutrals. It is found that, within the plasma fluid approximation, the presence of dust charged grains in plasma significantly alter the characteristic behaviour of the plasma sheath near an absorbing wall in contrast to the usual observations in an ordinary electron-ion plasma.

It is observed that in a dusty plasma the wall potential does not have to be negative always respect to the main body of the plasma. It is because the positive ions and the electrons are both mobile and can escape to the wall together. If the wall potential is positive, it is shielded from the main body of the plasma by the electrons and the dust charged grains and the sheath is negative in character. On the other hand, if the wall potential is negative, the sheath is an ion rich one and it is positive in nature. Since $\lambda_{De} \gg \lambda_{Di}$, it is only natural that the negative sheaths will be wider than the positive ones.

The nature of the sheath is determined by the relative concentrations of the plasma species through the Poisson's equation. The collision frequency of the dust charged grains with the neutrals, velocity of the dust charged grains at the sheath edge and the ion to electron temperature ratio influence the relative concentration of the plasma species in the sheath and hence they play crucial roles in the transition of the sheath from the negative to the positive nature.

However, in this paper we have ignored the size, shape and charge fluctuation among the individual dust charged grains. But the dust charge, mass and size, in general, behave as dynamic variables and produce novel effects on collective degrees of plasma dynamics. Thus the model under consideration is, though having the limitations may be regarded as the beginning of the process of understanding the sheath in a dusty plasma where collisional affects are important. Overall observations could be of interest in laboratory plasmas as well as relate to the observations in astrophysical problems.

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