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Optical guiding of laser beam in nonuniform plasma

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Abstract. A plasma channel produced by a short ionising laser pulse is axially nonuniform resulting from the self-defocusing. Through such preformed plasma channel, when a delayed pulse propagates, the phenomena of diffraction, refraction and self-phase modulation come into play. We have solved the nonlinear parabolic partial differential equation governing the propagation characteristics for an approximate analytical solution using variational approach. Results are compared with the theoretical model of Liu and Tripathi (*Phys. Plasmas* **1**, 3100 (1994)) based on paraxial ray approximation. Particular emphasis is on both beam width and longitudinal phase delay which are crucial to many applications.

Keywords. Laser guiding; preformed plasma; self-defocusing; diffraction; refraction.

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1. Introduction

Extreme focused laser intensities can now be obtained with a new generation of ultra-short high energy solid state lasers including chirped pulse amplification [1,2]. Such a development has led to a number of proposed applications such as plasma based accelerators [3–6], harmonic generation [7–10], x-ray lasers [11–14] and advanced laser-fusion schemes [15,16]. Practically these applications require long laser propagation distances at high intensity in a plasma. However in vacuum, laser propagation is limited by diffraction process, the characteristic distance of which is Rayleigh length, $Z_R = \pi a_0/\lambda$ where λ is the laser wavelength and a_0 is the spot size. Thus it is highly desirable that the laser propagation be extended over sufficient number of Rayleigh lengths if the above mentioned applications are to be realised in practice. Plasma channels [17–20] have been proposed as a means of guiding the laser pulses. The process is quite similar to the one used in optical fibre and light can be guided in plasmas if the refractive index at the beam centre can be increased sufficiently with respect to the beam edge to balance the effect of diffraction. This can be attained with plasma channel profile that has local minimum on the axis . When this occurs, the phase velocity is smaller on the axis $(r = 0)$ than it is off axis and the phase front of the optical field can become curved such that the optical field focuses towards the axis.

Several experimental methods have been proposed to create plasma channels e.g (i) passing a long pulse through an optic to create a line focus in gas, which ionises and heats the gas, creating radially expanding hydrodynamic shocks [22], (ii) using a slow capillary discharge to control the plasma profile [23], and (iii) using the ponderomotive force of an intense relativistically self-guided laser pulse in a plasma, which creates a channel in its own way [20]. The physics of scenario emerging from this situation is given below.

If we create the channel by focusing an intense prepulse, then channeling pulse must propagate through a large plasma region without being absorbed or sufficient beam breakup due to filamentation. The prepulse with Gaussian radial profile will modify plasma density and subsequent dynamics of the delayed pulse. The code developed by Durfee and Milchberg [17] predicts the formation of plasma channel with density minimum on the axis, a few nanoseconds after the ionising pulse is gone. Since the pulse has a Gaussian intensity radial profile, the density profile that evolves is peaked on the axis and falls off rapidly with r before the diffusion ensues. Index of refraction of the plasma thus created will have a minimum on the the axis and increase radially outwards. Such a medium will behave as a defocusing lens. The dynamics of the second pulse sent through such a channel will be governed by competing processes of diffraction and refraction. Liu and Tripathi have studied such phenomenon using paraxial ray approximation [24]. This most popularly used theory has been quite useful because of its mathematical simplicity and furnishes qualitatively good results when low power laser beams are used [25]. In laser plasma experiments, since high power laser beams are usually involved, it is pertinent to use more suitable alternative approach. Two such methods based on invariants of nonlinear Schrödinger wave equation are: (a) moment theory and (b) variational approach. We have used here variational technique [26] which not only predicts correctly the width of the beam but also the self-induced longitudinal phase delay in the plasma channel. Section 2 deals with the the channel formation and self-defocusing of ionising prepulse. In §3, we have investigated the guidance of the delayed pulse in the preformed plasma. Equations governing the beam width and longitudinal phase delay are also derived. Last section is devoted to a brief discussion of the important results.

2. Self-defocusing of ionising laser pulse

Consider the propagation of a Gaussian laser beam of frequency ω_0 through a gas along z-axis. The laser ionises the gas via tunnel ionisation in a time shorter than the pulse duration. Leemans *et al* [21] have given an expression for the rate of increase of plasma density. It has been found that the plasma density builds up rapidly in about 20 ps once the laser intensity crosses 6.0×10^{13} W/cm^2. In the experiment performed by Durfee and Milchberg, the laser power density is within this range [17]. For a Gaussian beam of radius a_0, we expect a high-density plasma within a few picoseconds in the axial region. Such a sharp plasma density profile is modelled by Liu and Tripathi [24].

$$\omega_p^2 = \omega_{p0}^2 \exp(-E_a'/E), \tag{1}$$

where

$$E_a' = \frac{2}{3} E_a \left(\frac{E_i}{E_a}\right)^{1/2}$$

and ω_{p0}^2 is a constant depending on particle density. The dielectric function of the plasma may be written as [24]

$$\epsilon = \epsilon_0 + \epsilon_2 r^2, \tag{2}$$

where

$$\epsilon_0 = \epsilon|_{r=0} = 1 - \left(\frac{\omega_{p0}^2}{\omega^2}\right) \exp(-E'_a/|E|) \tag{3}$$

and

$$\epsilon_2 = \frac{\partial \epsilon}{\partial |E|} \frac{\partial |E|}{\partial r^2}|_{r=0} = \frac{\omega_{p0}^2}{2\omega^2} \frac{E'_a}{E_{00} a^2(z)} \exp(-E'_a/A). \tag{4}$$

In the slowly varying approximation, we get the folowing nonlinear Schrödinger equation (NLSE):

$$2\iota k \frac{\partial E}{\partial z} + \nabla_\perp^2 E + + \frac{\omega^2}{c^2} \epsilon_2 r^2 E = 0. \tag{5}$$

Equation (5) has been solved by Liu and Tripathi [24] using paraxial ray approximation (PRA). As mentioned earlier, when high power laser beams are involved in laser plasma experiments, one must follow methods based on invariants of nonlinear Schrödinger equation (5) and the moments of electromagnetic field. The latter are used to determine the conditions of self-focusing/defocusing as well as beam width of laser beam in a nonlinear medium. However, in spite of its elegance, the description of phase by moment theory is still an open question. Another approach which also involves the invariants of nonlinear Schrödinger equation is the variational approach used by Anderson *et al* to describe self-focusing in plasmas [27] and optical fibres [26]. Variational method which though approximately analytical but fairly general in nature, has been used recently in a number of investigations. Besides that, it gives correct regularised phase description [28]. We have used variational approach to solve eq. (5). Despite its limitation like lack of finer details of phase description, inability to explain singularity and collapse, the method is fairly useful in investigation of wave propagation problems even when high power laser beams are involved. We reformulate this equation into a variational problem corresponding to Lagrangian L so that vanishing of functional derivative gives eq. (5) viz.

$$L = r\left|\frac{\partial E}{\partial r}\right|^2 + \iota k r \left(E \frac{\partial E^*}{\partial z} - E^* \frac{\partial E}{\partial z}\right) - \frac{\omega_2}{c^2} \epsilon_2 r^3 |E|^2. \tag{6}$$

Thus, the solution to the variational problem

$$\delta \iiint L \mathrm{d}x \mathrm{d}y \mathrm{d}z = 0 \tag{7}$$

also solves the nonlinear Schrödinger equation. Using

$$E = A(z) \exp\left[\frac{r^2}{2a^2(z)} + \iota q(z) r^2\right] \tag{8}$$

as a trial function in eq. (6) and carrying out the integration, we get the reduced problem with $\langle L \rangle$ as

$$\langle L\rangle = |A|^2\left(\frac{1}{2}+2q^2a^4\right)+\iota k\left(A\frac{\partial A^*}{\partial z}-A^*\frac{\partial A}{\partial z}\right)\frac{a^2}{2}$$
$$+k|A|^2\frac{dq}{dz}a^4-\frac{\omega^2}{c^2}\epsilon_2|A|^2\frac{a^4}{2}. \quad (9)$$

Variation with respect to A, A^*, a, q etc. and following the procedure of Anderson [26], and using the normalisation $\xi = zc/w_0{a_0}^2$, we arrive at the following equations:

$$\frac{\mathrm{d}}{\mathrm{d}\xi}(A^2{a_n}^2)=0, \quad (10)$$

$$q=\frac{1}{a_n}\frac{\mathrm{d}a_n}{\mathrm{d}\xi}, \quad (11)$$

$$\frac{\mathrm{d}^2a_n}{\mathrm{d}\xi^2}=\frac{1}{2{a_n}^3}\left[1+\left(\frac{\omega_p a_0}{c}\right)^2{a_n}^2\exp(-E_a'/|E|)\right.$$
$$\left.-2{a_n}^2\left(\frac{\mathrm{d}a_n}{\mathrm{d}\xi}\right)^2\right], \quad (12)$$

$$\frac{\mathrm{d}\phi_1}{\mathrm{d}\xi}=\frac{1}{2{a_n}^2}\left[1+4{a_n}^2\left(\frac{\mathrm{d}a_n}{\mathrm{d}\xi}\right)^2+2{a_0}^2{a_n}^2\frac{\mathrm{d}q}{\mathrm{d}\xi}\right.$$
$$\left.-\frac{1}{2}\left(\frac{w_p a_0}{c}\right)^2\exp(-E_a'/|E|)\right]. \quad (13)$$

Equation (12) is second order nonlinear ordinary differential equation governing the normalised beam width $(a_n = a/a_0)$ of the prepulse. As obvious from the form of this equation, the first two terms on the right hand side respectively representing diffraction and defocusing phenomena are the dominant mechanisms in determining the beam width and also play an important role in plasma channel formation. They cause axial nonuniformity in the index of refraction of the channel. In order to compare the results with those of Liu and Tripathi (eq. (15) of ref. [24]), we find an additional last term on the right hand side of eq. (12). This term though initially zero, evolves with propagation thereby counteracting the diffraction and self-defocusing. Inspite of that, overall result is defocusing of a_n with almost constant slope. This contrasts well with those of Liu and Tripathi where very fast defocusing of ionising prepulse is displayed (figure 1 of ref. [24]). Additional feature of the present investigation is that longitudinal phase modulation of the prepulse is also predicted by eq. (13).

3. Guided propagation of laser pulse 2

After the ionising pulse has passed, plasma formed moves radially away from the axis. Durfee and Milchberg have used the density profile [17]

$$\frac{{\omega_p}^2}{\omega^2}=\alpha_1(z)+\alpha_2(z)r^2 \quad (14)$$

on the arrival of the pulse 2. Here $\alpha_1(z)$ and $\alpha_2(z)$ are monotonically decreasing function of z. If we express the electric field of the second pulse as

$$E = x\left[\frac{\varepsilon(r,z)}{(k/k_0)^{\frac{1}{2}}}\right]\exp\left[-\iota\omega t + \iota\int k\mathrm{d}z\right], \tag{15}$$

where $k = (\omega/c)(1-\alpha_1(z))$, $k_0 = k(z=0)$. Substituting in the wave equation and using the slowly varying envelope approximation, we obtain

$$2\iota k\frac{\partial\varepsilon}{\partial z} + \nabla_\perp^2\varepsilon - \frac{\omega^2}{c^2}\alpha_2(z)r^2\varepsilon = 0. \tag{16}$$

The Lagrangian corresponding to this equation is given by

$$L_2 = r\left|\frac{\partial\varepsilon}{\partial r}\right|^2 + \iota kr\left(\varepsilon\frac{\partial\varepsilon^*}{\partial z} - \varepsilon^*\frac{\partial\varepsilon}{\partial z}\right) + \frac{\omega^2}{c^2}\alpha_2 r^3|\varepsilon|^2. \tag{17}$$

Using the averaging procedure similar to the proceeding section to get the reduced variational problem, we obtain equations for the normalised beam width and longitudinal phase delay as follows:

$$\frac{\mathrm{d}^2 b_n}{\mathrm{d}\xi^2} = \frac{1}{2(1-\alpha_1(\xi))b_n{}^3}\left[1 - \frac{\omega^2}{c^2}\alpha_2(\xi)b_n{}^4 - 2b_n{}^2\left(\frac{\mathrm{d}b_n}{\mathrm{d}\xi}\right)^2 + 2\alpha_1(\xi)b_n{}^2\left(\frac{\mathrm{d}b_n}{\mathrm{d}\xi}\right)^2\right], \tag{18}$$

$$\frac{\mathrm{d}\phi_2}{\mathrm{d}\xi} = \frac{1}{2(1-\alpha_1(\xi))b_n{}^2}\left[1 + 4(1-\alpha_1(\xi))b_n{}^2\left(\frac{\mathrm{d}b_n}{\mathrm{d}\xi}\right)^2 + \frac{4b_n{}^2}{[1-\alpha_1(\xi)][1+\xi^2]^{\frac{1}{2}}}\right], \tag{19}$$

where we used the same $\xi = zc/\omega_0 a_0{}^2$. Further, we have also used the model [24]

$$\alpha_2 = \frac{2}{R_d{}^2(1+z^2/R_d{}^2)^{\frac{1}{2}}}, \tag{20}$$

where $R_d = ka_0{}^2$.

4. Discussion

Equations (12), (13), (18) and (19) are solved numerically and the results are displayed in the form of graphs. Here a_n and b_n are normalised beam widths of ionising and guided pulses respectively. The initial conditions at $\xi = 0$ are $a_n = 1, b_n = 1, \mathrm{d}a_n/\mathrm{d}\xi = 0$ and $\mathrm{d}b_n/\mathrm{d}\xi = 0$. The physics of various terms appearing on the right hand side of eq. (12) and their evolution with distance of propagation is already discussed earlier. As apparent from figure 1, width of the pulse 1 increases monotonically leading to defocusing of the ionising pulse. This is because the first diffracting term on the right hand side of eq. (12) is supplemented by refractive induced defocusing second term. However, as z advances,

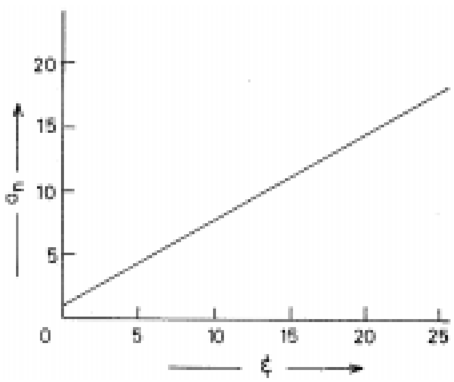

Figure 1. Variation of normalised beam width a_n of the ionising pulse as a function of dimensionless distance of propagation ξ for the following set of parameters: $\left(\frac{\omega_p a_0}{c}\right)^2 = 2.0$, $\frac{E_a{}'}{|E|} = 0.75$, $\frac{\omega_p^2}{\omega_0^2} = 0.25$.

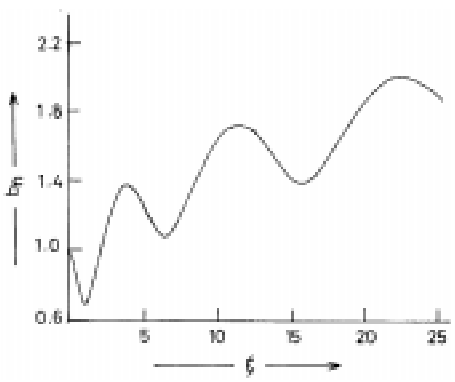

Figure 2. Plot of normalised beam width b_n of the guided laser beam vs ξ. The other parameters are the same as mentioned in figure 1.

there is finite contribution from the last term on the right hand side of eq. (12) which prevents the beam from steep defocusing. This becomes further evident if we compare the results with those of Liu and Tripathi (figure 1 of ref. [24]). Similarly the phase also increases monotonically (figure 3). Equation (18) governs the normalised beam width of guided beam in the preformed plasma. The first term on the right hand side of this equation arising from spatial dispersion, results in diffractional divergence. Second term is due to nonlinearly induced refraction which counteracts the diffraction and leads to the guidance of delayed beam. There are two additional terms, third and fourth respectively in this equation (cf. eq. (22) of ref. [24]). As a matter of fact, these terms containing the derivatives of beam width owe their origin to the form of trial function and the averaging process used in variational approach. The initial contribution of these terms is zero as $\mathrm{d}b_n/\mathrm{d}\xi = 0$ at $\xi = 0$. However, as beam propagates, these two terms contribute significantly due to guidance of laser pulse even though they counterbalance each other and their variation with ξ is disproportionate. Laser spot size varies between $0.5b_0$ and $1.7b_0$ for 12 Rayleigh lengths and also shows oscillatory behaviour up to a distance of 25 Rayleigh lengths. In the course of its guided propagation two minima and two maxima are observed as shown in figure 2. On comparing our results with those of Liu and Tripathi (figure 2, ref. [24]), f_1 variation is $0.75 < f_1 < 1.3$ as ξ grows from 0 to 6 Rayleigh lengths. Model of Liu and Tripathi [24] introduces α_2 (adopted by us as well), is based on modification of model based on the code of Durfee and Milchberg, takes the fast variation of α_2 on z. However, the last term in eq. (18) contributes significantly by supporting the diffraction divergence and leads to defocusing of the guided pulse over long distance of propagation. A model of α_2 showing slower dependence on z may lead to long laser guided propagation. With the availability of pulse shape technology, it is possible to tailor the plasma profile by the prepulse and extend the propagation of the delayed pulse over hundreds of Rayleigh lengths. However, theoretical model to account for such observation is still to be developed. Another important aspect of the present investigation is prediction of longitudinal phase

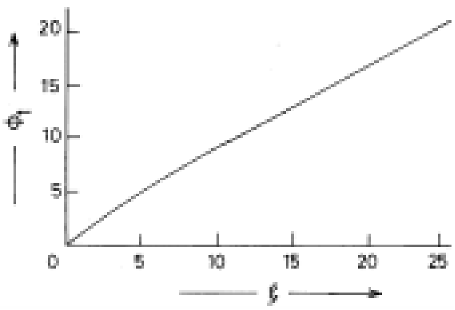

Figure 3. Variation of longitudinal phase delay of the ionising pulse vs ξ. The other parameters are the same as mentioned in figure 1.

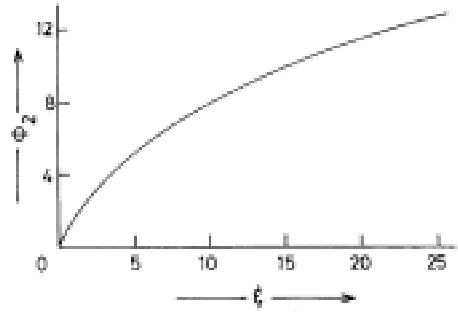

Figure 4. Plot of longitudinal phase delay of the guided beam as function of ξ. The other parameters are the same as those mentioned in figure 1.

delay of guided pulse which is governed by eq. (19). Neither moment theory nor paraxial ray approximation correctly predicts the longitudinal phase modulation. Investigation of such self phase modulation may play an important role since it introduces a wave number shift [29]. Its significance may further be emphasized when chirped laser beam is used for guided pulse, which is our future plan of work. Lastly, figures 3 and 4 respectively display the longitudinal phase delay for ionising and guided pulses. Even though ϕ_1 and ϕ_2 here are positive, it can be shown that regularised phase is always negative [29]. From figure 4, we find that relative competition of evolving and counteracting terms on the right hand side of eq. (19) slows down the self-induced phase delay after the guided beam propagation through a few Rayleigh lengths.

Some additional advantages of the variational approach stems from the fact that it takes into account averaging process over the whole beam cross-section. In the process, some additional terms explicitly showing dependence of beam width on first order dervatives, emanate which hitherto are missing in other theories but play an important role in beam dynamics. On the other hand, praxial ray approximation overemphasize the regions close to be the beam axis. Importance of non-paraxiality in self-foscusing phenomenon has been recently highlighted [30]. However, neither of these methods is suitable to account for singularity formation and collapse associated with self-focusing [31].

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