Cosmological magnetic fields

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Abstract. Magnetic fields are observed not only in stars, but in galaxies, clusters, and even high redshift Lyman-α systems. In principle, these fields could play an important role in structure formation and also affect the anisotropies in the cosmic microwave background radiation (CMB). The study of cosmological magnetic fields aims not only to quantify these effects on large-scale structure and the CMB, but also to answer one of the outstanding puzzles of modern cosmology: when and how do magnetic fields originate? They are either primordial, i.e. created before the onset of structure formation, or they are generated during the process of structure formation itself.

Keywords. Magnetic fields; cosmology.

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1. Introduction

Magnetic fields seem to be everywhere that we can look in the universe, from our own sun out to high-redshift Lyman-α systems. The fields we observe (based on synchrotron radiation and Faraday rotation) in galaxies and clusters have been amplified by gravitational collapse and possibly also by dynamo mechanisms. They are either primordial, i.e. originating in the early universe and already present at the onset of structure formation, or they are protogalactic, i.e. generated by battery mechanisms during the initial stages of structure formation. One way to distinguish these possibilities would be to detect or rule out the presence of fields coherent on cosmological scales during recombination via their imprint on the cosmic microwave background (CMB) radiation. The new generation of CMB observations (especially the MAP and Planck satellites) may be able to achieve this.

The origin, evolution and cosmological impact of magnetic fields represent a fascinating challenge to theorists. I will discuss some aspects of this challenge in the following sections (see [1] for some other recent reviews). Some basic facts from magnetohydrodynamics will be useful for the discussion.

Maxwell's equations are

$$\nabla_{[\mu} F_{\nu\alpha]} = 0 \,, \;\; \nabla_\nu F^{\mu\nu} = J^\mu \,, \tag{1}$$

where $F_{\mu\nu} = \nabla_\mu A_\nu - \nabla_\nu A_\mu$ is the field tensor, A_μ is the four-potential, and J_μ is the four-current. The field tensor is observer-independent, while the electric and magnetic fields depend on the observer's motion:

$$E_\mu = F_{\mu\nu} u^\nu \,, \quad B_\mu = \tfrac{1}{2}\varepsilon_{\mu\nu\alpha} F^{\nu\alpha} \,, \tag{2}$$

where u^μ is the observer's four-velocity, and $\varepsilon_{\mu\nu\alpha}$ is the covariant permutation tensor in the observers' rest space.

Ohm's law is

$$h_{\mu\nu} J^\nu = \sigma F_{\mu\nu} u^\nu \,, \tag{3}$$

where $h_{\mu\nu} = g_{\mu\nu} + u_\mu u_\nu$ projects orthogonal to u^μ (and $g_{\mu\nu}$ is the metric). For most of the history of the universe, the conductivity σ is extremely high. In the magnetohydrodynamic limit, we have $\sigma \to \infty$ while the current remains finite, so that $E_\mu \to 0$. Thus the electric field in the particle frame vanishes: $F_{\mu\nu} u^\nu = 0$. In the observer's frame, with four velocity $\tilde{u}^\mu = u^\mu + v^\mu$, where v^μ is the relative velocity ($v_\mu u^\mu = 0$) and we neglect terms $O(v^2)$, the electric field is of course not zero, but given by

$$\tilde{E}_\mu = -\varepsilon_{\mu\nu\alpha} v^\nu B^\alpha \,. \tag{4}$$

In this limit, Maxwell's equations may be written as [2,3]:

$$\mathrm{D}^\mu B_\mu = 0 \,, \tag{5}$$
$$\omega^\mu B_\mu = -\tfrac{1}{2} J_\mu u^\mu \,, \tag{6}$$
$$\operatorname{curl} B_\mu = h_{\mu\nu} J^\nu + \varepsilon_{\mu\nu\alpha} B^\nu \dot{u}^\alpha \,, \tag{7}$$
$$h_{\mu\nu} \dot{B}^\nu = -\tfrac{2}{3}\Theta B_\mu + \sigma_{\mu\nu} B^\nu + \varepsilon_{\mu\nu\alpha} B^\nu \omega^\alpha \,, \tag{8}$$

where D_μ is the projected covariant derivative, and $\operatorname{curl} B_\mu = \varepsilon_{\mu\nu\alpha} D^\nu B^\alpha$ is the covariant spatial curl. The kinematic quantities are Θ (the volume expansion of u^μ-flowlines), ω_μ (vorticity), $\dot{u}_\mu$ (four-acceleration), and $\sigma_{\mu\nu}$ (shear).

The key equation is (8), which is the induction equation in covariant form. When contracted with B^μ, it leads to the conservation equation for magnetic energy density:

$$\dot{\rho}_{\mathrm{mag}} + \tfrac{4}{3}\Theta \rho_{\mathrm{mag}} = \sigma_{\mu\nu} \pi^{\mu\nu} \,, \tag{9}$$

where

$$\rho_{\mathrm{mag}} = \tfrac{1}{2} B_\mu B^\mu \,, \quad \pi_{\mu\nu} = \tfrac{1}{3} B^\alpha B_\alpha h_{\mu\nu} - B_\mu B_\nu \,, \tag{10}$$

are the energy density and anisotropic stress of the magnetic field. Typically, the term on the right of eq. (9) may be neglected, in which case ρ_{mag} obeys the same evolution equation as isotropic radiation, so that

$$r \equiv \frac{\rho_{\mathrm{mag}}}{\rho_{\mathrm{rad}}} = \text{constant} \,. \tag{11}$$

In a Friedmann universe, where $\Theta = 3H = 3\dot{a}/a$ and a is the scale factor, we have from eq. (9) that

$$a^2 B = \text{constant} \,, \tag{12}$$

where $B = (B_\mu B^\mu)^{1/2}$. If we choose $a = 1$ at the present time, then $a^2 B$ is the comoving magnitude of the magnetic field. Observations show that galactic and cluster fields are at the micro-Gauss level.

Nucleosynthesis imposes limits based on the way in which a magnetic field affects the expansion rate, the reaction rates and the electron phase density [4]:

$$a^2 B \lesssim 10^{-7} \, \text{G} \,, \tag{13}$$

on cosmological scales. We can understand this limit qualitatively by requiring that $\rho_{\text{mag}} < \rho_{\text{rad}}$ at nucleosynthesis, which gives the right order of magnitude.

The upper limit from the CMB on a large-scale field is much tighter [5]:

$$a^2 B \lesssim 10^{-9} \, \text{G} \,. \tag{14}$$

This field strength corresponds to an energy density

$$\Omega_{\text{mag}} \equiv \frac{\rho_{\text{mag}}}{\rho_{\text{crit}}} \sim 10^{-5} \Omega_{\text{rad}} \,, \tag{15}$$

so that, roughly speaking, magnetic fields cannot induce large-angle perturbations in the CMB above the observed level.

2. Magnetogenesis and amplification

Protogalactic magnetogenesis, i.e. the creation of magnetic fields during the process of structure formation, essentially relies upon battery-type mechanisms in which the gradients of electron number density n_e and pressure p_e are not aligned. Ohm's law (3) is modified and leads to the modified induction equation (in Newtonian form) [6]

$$\frac{\partial \vec{B}}{\partial t} = \vec{\nabla} \times (\vec{v} \times \vec{B}) + \alpha \, \vec{n}_e \times \vec{p}_e \,, \tag{16}$$

where α is a constant. It follows that if the gradient terms are non-aligned (as happens for example when shock waves develop in collapsing clouds), then nonzero B can be generated. Very small fields are generated in this way, and typically require strong dynamo-type amplification in order to reach the currently observed levels.

A seed magnetic field, whether generated by battery mechanisms or already present in the form of a primordial field, is amplified adiabatically during gravitational collapse, simply by the fact that field lines are frozen into the plasma, and compression of the plasma results in compression of flux lines. This adiabatic compression is weak, with growth roughly given by

$$B \propto \delta^{2/3} \,, \tag{17}$$

where $\delta = \delta\rho/\rho$ is the fractional over-density of the cloud (this neglects the shear term in eq. (9)). If the observed galactic fields ($\sim 10^{-6}$ G) are the result only of adiabatic compression, then the seed field required could be up to $\sim 10^{-9}$ G (comoving). This is at the level of the CMB limit on large-scales.

If the seed field is much weaker, then a stronger amplification is required – and the prime candidate mechanism for this is the galactic dynamo [7]. This is based on differential rotation and turbulence, whereby small-scale magnetic fields are amplified via parametric resonance. The key issue of how efficient the dynamo is, has not been settled. There

is therefore a large uncertainty in the amount of amplification that can be achieved, and thereby in the size of seed field that is necessary. In general qualitative terms, the seed field will be much less than that required for purely adiabatic compression. In terms of the r-factor in eq. (11), a seed without dynamo amplification requires $r \sim 10^{-14}$, whereas a seed with dynamo amplification could have r as low as $\sim 10^{-34}$ (this may be further reduced in the presence of a cosmological constant [8]).

Primordial magnetogenesis is the creation of magnetic fields in the early universe, before the process of structure formation. Many mechanisms have been proposed, based mainly on phase transitions before recombination, or on inflation. In phase transitions such as the QCD and EW transitions, local charge separation can arise, creating local currents that can generate (hyper-)magnetic fields [9]. Other proposals include bubble-wall collisions, which produce phase gradients that can source gauge fields [10].

These mechanisms produce fields coherent on sub-Hubble scales. In order to generate super-Hubble scale fields, one requires inflationary models [12], or pre-big bang models based on string theory [11], in which vacuum fluctuations of the field are amplified via the dilaton. Inflation stretches perturbations beyond the Hubble horizon and thus can in principle generate magnetic fields on large scales. There is however a problem in that vector perturbations are extremely small in standard models, essentially because the vector gauge field does not couple gravitationally to a conformally flat metric. One needs to break conformal invariance by new high-energy couplings of the photon (or to break gauge invariance). An example of such a coupling is provided by the Lagrangian for scalar electrodynamics:

$$\mathcal{L} = -\tfrac{1}{4} F_{\mu\nu} F^{\mu\nu} - (\mathcal{D}_\mu \phi)^* \mathcal{D}^\mu \phi - V(\phi\phi^*) \,, \tag{18}$$

where ϕ is the charged scalar field, and $\mathcal{D}_\mu = \nabla_\mu - ieA_\mu$ is the gauge-covariant derivative, with e the coupling constant.

Inflation is often followed by a preheating period in which coherent oscillations of the inflaton produce parametric resonant amplification of perturbations. Since the inflaton is coherent on super-Hubble scales, this amplification can in principle affect super-Hubble scales, without in any way violating causality [13]. Magnetic fields arising from inflationary fluctuations could thus in principle be amplified via preheating [14].

3. Magnetic fields and the CMB

In the absence of any preferred model of primordial magnetogenesis, and in view of the complexities of magnetohydrodynamics during structure formation (especially the dynamo mechanism), we need cosmological observational tests for deciding whether magnetogenesis is primordial or protogalactic. If magnetic fields could be detected in the voids between galactic clusters, this would be very strong evidence for a primordial origin.

The other key observational test is provided by the CMB. Dynamically significant magnetic fields present during recombination must be primordial. These primordial fields have various effects on the CMB.

In the absence of a magnetic field, the tightly coupled baryon-photon fluid undergoes longitudinal acoustic oscillations in density and velocity perturbations, with

$$\delta \,, v \propto \exp(ikc_s\eta) \,, \tag{19}$$

where c_s is the sound speed and η is conformal time. A magnetic field splits these modes into 3 types:

(a) Fast magnetosonic waves, which are like sound waves, but with increased speed,

$$c_s^2 \;\to\; c_s^2 + c_a^2 \sin^2 \theta \,, \tag{20}$$

where $c_a^2 = \rho_{\mathrm{mag}}/\rho$ is the Alfvén speed squared and θ is the angle between $\vec{B}$ and the propagation direction;

(b) Slow magnetosonic waves, which have speed $c_a \cos\theta$ and are partly transverse in velocity;

(c) Incompressible Alfvén waves, whose speed is the same as the slow magnetosonic waves, and for which $\delta = 0$.

The fast magnetosonic waves have a direct and simple, though small, effect on the acoustic peaks in CMB temperature anisotropies [16], based on the modification of the sound speed. The effect of Alfvén modes on CMB anisotropies has also been calculated [17].

Fast magnetosonic modes suffer diffusion damping just like the non-magnetized acoustic modes. The slow magnetosonic and Alfvén modes by contrast can be overdamped and survive on scales below the Silk scale [15]. This could play an interesting role in structure formation.

In general, the dissipation of magnetized fluctuations injects non-thermal energy into the photon spectrum, which introduces chemical-potential and Compton distortions (μ and y distortions) in the CMB black body. Upper limits on these distortions provided by the FIRAS experiment on COBE then place upper limits on the magnetic field strength [18]:

$$a^2 B \lesssim 10^{-8}\,\mathrm{G} \quad \text{on scales} \quad \sim 0.5 - 600\,\mathrm{kpc}\,. \tag{21}$$

The anisotropic stress $\pi_{\mu\nu}$ induced by a magnetic field can source gravitational wave perturbations during recombination. This can be seen through the wave equation that governs the transverse traceless magnetic part of the Weyl tensor, $H_{\mu\nu}$, which provides a covariant description of gravitational waves [19]:

$$-\ddot{H}_{\mu\nu} + D^2 H_{\mu\nu} = 7 H \dot{H}_{\mu\nu} + 2\rho(1 - w) H_{\mu\nu} + 2 H\,\mathrm{curl}\,\pi_{\mu\nu}\,, \tag{22}$$

where $\mathrm{curl}\,\pi_{\mu\nu} = \varepsilon_{\alpha\beta(\mu} D^\alpha \pi^\beta{}_{\nu)}$ is the covariant spatial tensor curl and $w = p/\rho$. In order to keep the tensor contribution to CMB temperature anisotropies within the observed limits, this places upper limits on the magnetic field [20].

Magnetic fields have an important effect on the polarization of the CMB via Faraday rotation [21]. Linearly polarized radiation with frequency ν and wave vector $\vec{e}$, in a magnetized plasma with free electron density n_e, has its plane of polarization rotated through an angle φ, where

$$\frac{\mathrm{d}\varphi}{\mathrm{d}t} \propto \frac{n_e}{\nu^2}\,\vec{B} \cdot \vec{e}. \tag{23}$$

For a given line of sight $\vec{e}$, the polarization angle φ may be measured at different frequencies, thus providing in principle a measure of the magnetic field strength. The Planck

experiment may be able to detect a field at the $\sim 10^{-9}$ G level. An indirect effect of Faraday rotation is to depolarize the CMB on small angular scales, leading to a reduction in damping and thus a small increase in power in the temperature anisotropies [22].

Perhaps more significant than the small quantitative effects on polarization angle and on small-scale temperature anisotropies is an intriguing correlation introduced by magnetic fields [23]. Scalar perturbations can only generate E-type polarization, while tensor perturbations generate both E- and B-type polarization. A magnetic field also generates both E- and B-type polarization, but in addition, it induces a correlation via the Faraday rotation coupling in the evolution equations for polarization. This means that the B-type polarization will be correlated with the temperature anisotropies. Such a correlation does not arise in the context of statistical isotropy, but a large-scale magnetic field breaks the isotropy and produces a novel signature, which may be more accessible to observation.

Another potentially important (although probably extremely small) effect on CMB temperature anisotropies arises from the general relativistic interaction between gravity and electromagnetism, whereby electromagnetic radiation may be induced from a magnetic field by gravitational waves [24].

4. Magnetized structure formation

The effects of a weak cosmological magnetic field on structure formation in the linear regime are necessarily very small. The pioneering analysis was given in [25] (see also [26]). In the matter era on sub-Hubble scales, a Newtonian approach is justified, based on the magnetized Euler equation

$$\frac{\partial \vec{v}}{\partial \eta} + aH\vec{v} = -c_s^2 \vec{\nabla}\delta - \vec{\nabla}\Phi + \frac{1}{\rho}(\vec{\nabla} \times \vec{B}) \times \vec{B}\,, \tag{24}$$

where Φ is the gravitational potential perturbation. The standard, non-magnetized adiabatic growing mode of density perturbations is slightly damped by magnetism [25,2]:

$$\delta \propto a^n\,, \quad n = \tfrac{1}{4}\left[-1 + 5\sqrt{1 - \alpha_{\mathrm{mag}}k^2}\right]\,, \tag{25}$$

where α_{mag} is a constant determined by c_a^2, and k is the wave number. New non-adiabatic constant and decaying modes are also introduced by the magnetic field. A magnetic field can induce density perturbations in a homogeneous medium, although it cannot on its own reproduce the features of the observed power spectrum [27]. An analysis of the complex dynamics of magnetized damping during recombination [15,17] shows that incompressible and slow magnetosonic modes may survive on scales well below the Silk scale, and this could lead to interesting variations on the non-magnetized scenario of structure formation. These small-scale modes that survive damping could seed early star or galaxy formation and could also precipitate fragmentation of early structures.

The magnetic field also acts as a source of incompressible rotational instabilities, which satisfy the wave equation [3]

$$-\ddot{W}_\mu + \left[\frac{c_a^2}{3(1+w)}\right] D^2 W_\mu = (4-3w)H\dot{W}_\mu$$
$$+\tfrac{1}{2}\rho\left[1 - 7w + 3c_s^2(1+w)\right] W_\mu\,, \tag{26}$$

where $D^\mu W_\mu = 0$. On small scales, these vortices may have some interesting effects on structure formation. Magnetic fields can generate not only vorticity, but also anisotropic distortion in the density distribution [3].

On super-Hubble scales, a fully general relativistic analysis is needed, and this is developed in [2,3] (see [28] for a dynamical-systems analysis of the equations). During the radiation era, the non-magnetized adiabatic growing mode is incorrectly predicted to suffer small magnetic damping via an analysis which does not incorporate all relativistic effects. In fact, there is a crucial *magneto-curvature coupling*, [2,3,29] which arises from the non-commutation of the projected covariant derivatives of the magnetic field:

$$D_{[\mu}D_{\nu]}B_\alpha = \tfrac{1}{2}\mathcal{R}_{\mu\nu\alpha\beta}B^\beta - \varepsilon_{\mu\nu\beta}\omega^\beta \dot{B}_\alpha \,, \tag{27}$$

where the projected curvature tensor is

$$\mathcal{R}_{\mu\nu\alpha\beta} = h_\mu{}^\sigma h_\nu{}^\chi h_\alpha{}^\gamma h_\beta{}^\delta R_{\sigma\chi\gamma\delta} - V_{\mu\alpha}V_{\nu\beta} + V_{\mu\beta}V_{\nu\alpha} \,, \tag{28}$$

with

$$V_{\mu\nu} = \tfrac{1}{3}\Theta h_{\mu\nu} + \sigma_{\mu\nu} + \varepsilon_{\mu\nu\alpha}\omega^\alpha \,. \tag{29}$$

This coupling combines with the tension of the magnetic force-lines to reverse the damping effect and leads to a small enhancement of the growing mode, which satisfies the equation [3]

$$a^2 \frac{\mathrm{d}^2\delta}{\mathrm{d}a^2} - (2 - c_a^2)\delta = c_a^2 \left(C + 2a^2\mathcal{R} \right) \,, \tag{30}$$

where C is a constant and $\mathcal{R} = h^{\mu\nu}h^{\alpha\beta}\mathcal{R}_{\mu\alpha\nu\beta}$ is the projected curvature scalar.

The coupling between magnetism and curvature essentially injects the elastic properties of magnetic field lines into space itself, and can lead to rather unexpected dynamical and kinematical effects [29,30].

5. Conclusion

Cosmic magnetic fields provide a fascinating set of unsolved problems challenging theorists in cosmology. Not only do we need to resolve the key question as to whether these fields are primordial or protogalactic in origin, but we also need to develop a satisfactory theory of magnetogenesis and amplification. Furthermore, there are a number of open issues in calculating the magnetic effects on structure formation and on CMB anisotropies. The required theoretical developments will be driven by advances in observations, both directly of magnetic fields beyond the galactic scale, and indirectly via future advances in CMB observations and large-scale structure surveys.

the site of resistance to a former colonial prison, where many freedom fighters refused to surrender.

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