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Living with lambda

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Abstract. This talk presents a brief overview of recent results pertaining to the cosmological constant 'Λ'. I summarize the observational situation focussing on observations of high redshift Type Ia supernovae which suggest $\Lambda > 0$. Observations of small angular anisotropies in the cosmic microwave background complement Type Ia supernovae observations and both CMB and Sn can be combined to place strong constraints on the value of Λ. The presence of a small Λ-term increases the age of the universe and slows down the formation of large scale structure. I also review recent theoretical attempts to generate a small Λ-term at the current epoch and a *model independent* approach for determining the cosmic equation of state.

Keywords. Cosmology; supernovae; cosmic microwave background; cosmological constant; vacuum fluctuations.

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1. A brief historical overview

The cosmological constant was introduced in 1917 by Einstein who, fascinated by Ernst Mach's ideas on inertia, decided to modify the equations of general relativity to

$$R_{ik} - \frac{1}{2}\, g_{ik}R - \Lambda g_{ik} = \frac{8\pi G}{c^4} T_{ik} \tag{1}$$

in order that they would accomodate a spatially closed and static universe (the static Einstein universe). A few years later the universe was found to be expanding and Einstein abandoned the cosmological constant calling it his biggest mistake. Although discarded by Einstein, the cosmological constant (or Λ-term) lived on in the work of many theorists and even came to occupy the cosmological center stage on several occasions including:

(i) In the same year in which Einstein introduced Λ, de Sitter discovered a static solution to (1) with $T_{ik} = 0$. The de Sitter metric was to prove very influential and formed the basis of many important developments in cosmology including steady state cosmology [1] and inflationary models of the very early universe [2,3].

(ii) The static Einstein universe (SEU) is unstable and in 1927 Lemaitre proposed a quasi-static model which originates from SEU in the past. The Lemaitre model has a long age and for this reason proved useful whenever the age constraint became too restrictive for standard FRW cosmology. The Lemaitre model with a prolonged quasi-static (or loitering) stage at the redshift $z \simeq 2$ was invoked in the early 1960's, when it appeared that an excess of QSO's were being seen at that redshift. A generalized version of the loitering scenario to resolve the *age* and *growth* problems in standard cosmology has also been proposed in [4].

(iii) Current observations based on the use of high redshift Type Ia supernovae as standard candles appear to indicate that our universe is accelerating, with a large fraction of the cosmological energy density in the form of a Λ-term [5–7].

(The reader is referred to [8–11] for reviews of the cosmological constant issue.)

2. FRW models in the presence of Λ

The cosmological Λ-term possesses the somewhat unusual equation of state $P_\Lambda = -\rho_\Lambda \equiv -\Lambda/8\pi G$. Matter with such an equation of state violates the strong energy condition $\rho + 3P \geq 0$ and can lead to interesting departures from so-called 'standard behaviour' characteristic of perfect fluid cosmologies with non-negative pressure. The Einstein equations in the presence of Λ and a perfect fluid with density ρ and pressure P are

$$3\left(\frac{\dot{a}}{a}\right)^2 = 8\pi G\rho + \Lambda c^2 - 3\frac{\kappa c^2}{a^2}, \tag{2}$$

$$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3}(\rho + 3P/c^2) + \frac{\Lambda c^2}{3}. \tag{3}$$

Equation (2) can be written in the suggestive form

$$\frac{1}{2}\dot{a}^2 + V(a) = E, \tag{4}$$

where

$$V(a) = -\left(\frac{4\pi G}{3}\rho a^2 + \frac{\Lambda a^2}{6}\right), \quad E = -\frac{\kappa}{2}. \tag{5}$$

The potential (5) permits the analysis of the Einstein equations using the kindered approach of studying the one dimensional motion of a particle under the influence of the potential $V(a)$; for more details see Sahni and Starobinsky [11].

Besides SEU and the loitering universe, a universe with Λ also admits other novel solutions including the possibility that an open universe with matter and $\Lambda < 0$ will eventually recollapse, while a closed universe with matter and $\Lambda > 0$ can continue to expand forever.

Current observations suggest that the universe may be flat ($\Omega_{\rm total} \simeq 1$) in agreement with predictions made by the inflationary scenario over two decades ago. A flat universe containing pressureless matter (dust) and $\Lambda > 0$ expands as [11]

$$a(t) = A\left(\sinh\frac{3}{2}\sqrt{\frac{\Lambda}{3}}ct\right)^{2/3}. \tag{6}$$

The expansion law (6) smoothly interpolates between a matter dominated regime in the past ($a \propto t^{2/3}$) and an inflationary epoch in the future ($a \propto {\rm e}^{\sqrt{\frac{\Lambda}{3}}t}$).

3. Cosmological parameter estimation from high redshift supernovae

In 1970 Alan Sandage described cosmology as being a 'search for two numbers'. The first of these numbers is the Hubble parameter $H_0 = (\dot{a}/a)_0$, knowing its value we can determine the observable size of the universe and its age. The second number: the deceleration parameter $q_0 = -H_0^{-2}(\ddot{a}/a)_0$, probes the equation of state of matter and is very sensitive to the presence of a cosmological constant since

$$q_0 = \frac{\Omega_m}{2} - \Omega_\Lambda. \tag{7}$$

In a critical density universe $\Omega_m + \Omega_\Lambda = 1$ and

$$q_0 = \frac{3}{2}\Omega_m - 1. \tag{8}$$

A critical density universe will accelerate if $\Omega_m < 2/3$ and decelerate if $\Omega_m > 2/3$.

Current evidence for Λ stems from the observation that high redshift objects such as supernovae are fainter in a Λ-dominated universe than in standard cosmology. The luminosity flux $\mathcal{F}$ reaching us from a supernova of absolute luminosity $\mathcal{L}$ at redshift z is given by

$$\mathcal{F} = \frac{\mathcal{L}}{4\pi d_L^2}. \tag{9}$$

The quantity d_L is the luminosity distance to the object. In a multicomponent universe consisting of matter and a cosmological term d_L has the general form [10,11]

$$d_L(z) = \frac{(1+z)cH_0^{-1}}{|\Omega_{\rm total} - 1|^{\frac{1}{2}}} S(\eta_0 - \eta), \tag{10}$$

where

$$\eta_0 - \eta = |\Omega_{\rm total} - 1|^{\frac{1}{2}} \int_0^z \frac{{\rm d}z'}{h(z')}, \tag{11}$$

and $S(x) = \sin(x)$ if $\kappa = 1$ $(\Omega_{\rm total} = \Omega_m + \Omega_\Lambda > 1)$, $S(x) = \sinh(x)$ if $\kappa = -1$ $(\Omega_{\rm total} < 1)$, $S(x) = x$ if $\kappa = 0$ $(\Omega_{\rm total} = 1)$. The dimensionless Hubble parameter $h(z)$ is defined as

$$h(z) = \frac{H(z)}{H_0} = \left[(1 - \Omega_{\rm total})(1+z)^2 + \Omega_m(1+z)^3 + \Omega_\Lambda\right]^{\frac{1}{2}}. \tag{12}$$

Ω_m is the dimensionless energy density of matter $\Omega_m = 8\pi G\rho_m/3H_0^2$, and Ω_Λ is the dimensionless energy density of a cosmological constant $\Omega_\Lambda = \Lambda/3H_0^2$. The contribution of radiation at the current epoch is assumed to be negligible. (The present value of Λ is given by $\Lambda_0 = 3H_0^2[1 - \Omega_m]$.) It is interesting that for identical values of Ω_m, d_L will be larger in the presence of Λ than in its absence. This fact taken together with the empirical observation that Type Ia supernovae appear to be excellent standard candles (see [12] for a review) makes cosmological parameter determination a reality.

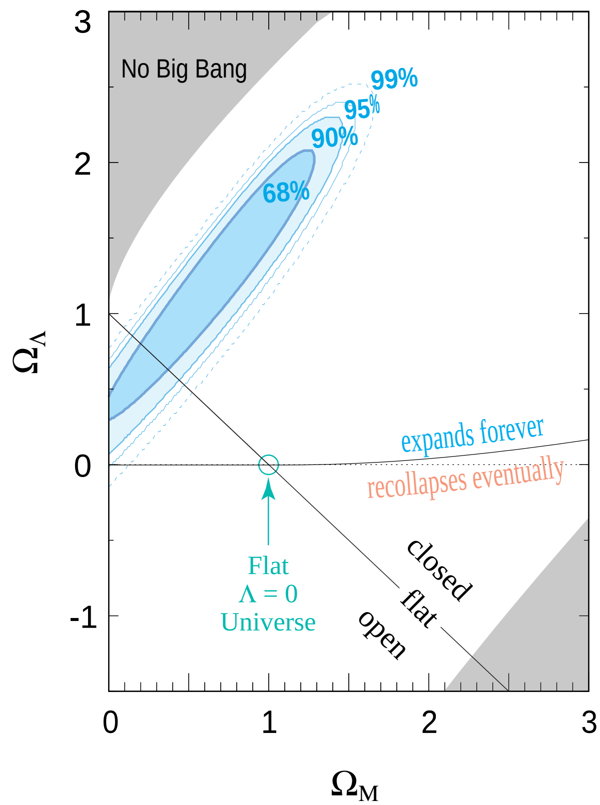

Figure 1. High confidence values of $\{\Omega_m, \Omega_\Lambda\}$ from an analysis of Type Ia high redshift supernovae by Perlmutter *et al* [6].

Currently high redshift supernovae are being used to probe the density and equation of state of cosmic matter by two teams: the supernova cosmology project [5] and the high-z supernova search team [7]. Examining several dozen Type Ia supernovae both teams agree that a positive Λ-term appears to be strongly favoured by the data (see for instance figure 1). From an examination of 42 high z supernovae (with $z \lesssim 0.83$) Perlmutter *et al* [6] find that the joint probability distribution of the parameters Ω_Λ and Ω_m is well approximated by the relationship (valid for $\Omega_m \leq 1.5$)

$$0.8\Omega_m - 0.6\Omega_\Lambda \simeq -0.2 \pm 0.1.$$

4. Constraining 'dark energy' using the cosmic microwave background

The presence of a cosmological Λ-term which could be exactly constant or weakly time varying affects fluctuations in the cosmic microwave background (CMB) and the *combined use* of CMB fluctuations and supernovae observations can be used to place very tight constraints on the value of Λ.

It is widely believed that fluctuations in the CMB originated during an early epoch of cosmological inflation when the universe expanded at a close to exponential rate for an exceedingly brief period of cosmic time. Strong support for the inflationary paradigm came in 1992 when the COBE satellite detected fluctuations in the CMB on large angular scales $\theta > 7^\circ$ considerably greater than the angle $\theta \simeq 2^\circ$ subtended by the horizon at the epoch of the cosmological recombination of hydrogen. The COBE results indicated a spectral index $n \simeq 1$ for primordial density fluctuations $P(k) \equiv \langle |\delta_k|^2 \rangle \propto k^n$ which was in excellent agreement with predictions made by the inflationary scenario and published almost a decade prior to the COBE discovery.

The CMB temperature fluctuations expanded on the celestial sphere acquire the form

$$\frac{\delta T}{T}(\theta, \phi) = \sum_{l=2}^{\infty} \sum_{m=-l}^{l} a_{lm} Y_l^m(\theta, \phi), \tag{13}$$

where the coefficients a_{lm} are statistically independent and distributed in the manner of a Gaussian random field with zero mean. Of considerable interest is the angular power spectrum of fluctuations

$$C_l \equiv \langle |a_{lm}|^2 \rangle \tag{14}$$

which is one of the main diagnostic tools used to place constraints on: (i) the form of the density perturbation spectrum, (ii) the values of $\Omega_m, \Omega_\Lambda, \Omega_{\rm baryon}$, (iii) the Hubble parameter H_0. At low multipole values $l \lesssim 20 (\theta = \pi/l \gtrsim 7^\circ)$ the form of C_l has been used to normalize the primordial spectrum of perturbations and to determine their slope. At large $l \gtrsim 60$ the known form of C_l can be used to place constraints on $\Omega_{\rm total} = \Omega_m + \Omega_\Lambda$. The reason for this has to do with the fact that the main contribution to the value of C_l for $l \gtrsim 60$ comes from coherent oscillations in the photon-baryon plasma which leave their imprint in the CMB at the time of matter-radiation decoupling. Plasma oscillations give rise to wiggles in C_l characterized by peaks and troughs. The location of the first such peak is determined by the angle subtended by the sound horizon at the time of decoupling, which in turn depends upon the values of Ω_m and Ω_Λ. In adiabatic structure formation models with $\Omega_{\rm total}$ close to unity, the location of the first peak is predicted to occur at $l_{\rm peak} \sim 200\, \Omega_{\rm total}^{-1/2}$. Data obtained from the 1997 test flight of the BOOMERANG experiment suggest that the first Doppler peak has been measured near $l \simeq 200$ and the implied value of $\Omega_{\rm total}$ is $0.85 \leq \Omega_{\rm total} \leq 1.25$ at the 68% confidence level [14,15].

As pointed out in a number of papers the degeneracy in the $\Omega_m - \Omega_\Lambda$ plane from supernova observations is almost orthogonal to the degeneracy from CMB measurements [16,17,13,18] – see figure 2. (A degeneracy arises when a result remains unaffected by a specific combination of parameter changes.) Therefore by combining CMB measurements on subdegree scales with supernova observations one can hope to substantially narrow down the values of Ω_m and Ω_Λ. A combined likelyhood analysis from the BOOMERANG experiment and Type Ia supernovae data gives best fit values [14]

$$0.2 \leq \Omega_m \leq 0.45, \; 0.6 \leq \Omega_\Lambda \leq 0.85 \tag{15}$$

which clearly favours a flat universe with $\Omega_m + \Omega_\Lambda \simeq 1$.

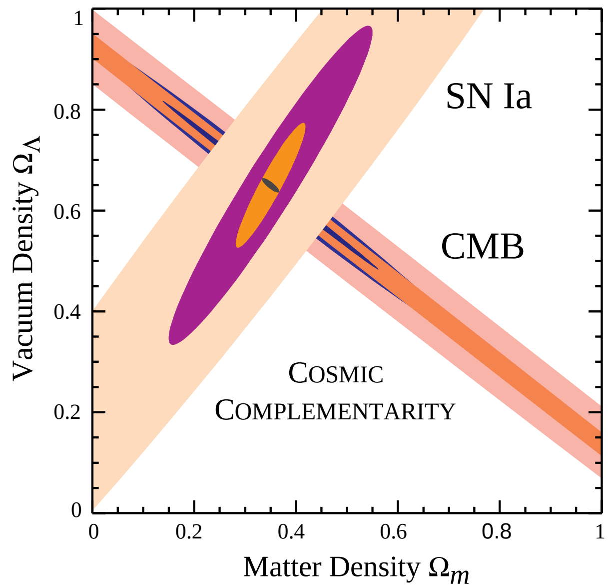

Figure 2. The principle of 'cosmic complementarity' is illustrated by this futuristic plot from Tegmark *et al* [13]. The best-fit contours for 68% confidence regions are shown for three sets of hypothetical supernovae data containing 100, 200 and 400 events providing 'pessimistic' and 'optimistic' prognosis of the number of Type Ia supernovae likely to be recorded in five years time. The CMB analysis refers to upcoming MAP and PLANCK satellite missions. The degeneracy in parameter space from supernovae observations is almost orthogonal to the degeneracy arising from CMB measurements, combining Sn and CMB therefore substantially narrows down the corridor of allowed values of Ω_m, Ω_Λ.

5. Other astrophysical effects of a Λ-term

5.1 *The age of the universe*

The subject of the universe's age has been the focus of considerable debate in cosmology over the past few decades. It would be very embarassing if, for instance, the universe turned out to be younger than the oldest of its members such as stars in globular clusters. This could very well happen in a flat matter dominated universe for large values of the Hubble parameter $H \geq 85$ km/sec/Mpc. In this case $t_0 = 2/3\ H_0 \simeq 8$ Gyr, which is smaller than the typical globular cluster age of $t_0 = 11.5 \pm 1.5$ Gyr (see [11] and references therein). Smaller values $H_0 \leq 65$ km/sec/Mpc alleviate the age problem for globular clusters, however there might still be an age problem for standard cosmology if the discovery of a 3.5 Gyr radio galaxy at the redshift $z = 1.55$ is confirmed [20]. (A flat matter dominated universe can be reconciled with this discovery only if the Hubble parameter is rather small $H_0 \leq 45$ km/sec/Mpc [21].) A general expression for the age of the universe at a given redshift is

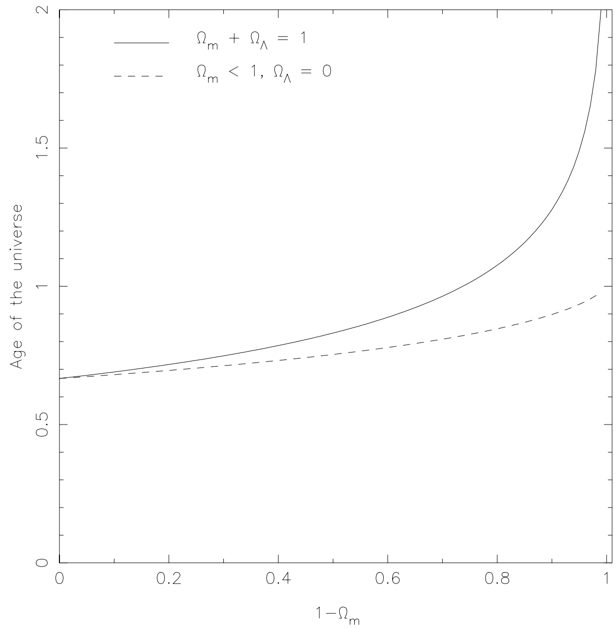

Figure 3. A plot of the age of the universe (in units of H_0^{-1}) as a function of $1 - \Omega_m$ for (i) flat models with a cosmological constant $\Omega_\Lambda = 1 - \Omega_m$ (solid line), and (ii) open cosmological models with $\Omega_m < 1$ (dashed line); from Sahni and Starobinsky [11].

$$t(z) = H_0^{-1} \int_z^\infty \frac{\mathrm{d}z'}{(1+z')h(z')}, \tag{16}$$

where $h(z)$ is given by (12) and one finds the following closed form expression for the present age of a flat universe

$$t_0 = \frac{2}{3H_0}\left[\frac{1}{2\,\Omega_\Lambda^{1/2}} \log \frac{1+\Omega_\Lambda^{1/2}}{1-\Omega_\Lambda^{1/2}}\right]. \tag{17}$$

In figure 3 we plot the age of a flat universe both with and without a cosmological constant, a universe with $\Omega_\Lambda = 1 - \Omega_m$ is clearly older than an open matter dominated universe for identical values of $1 - \Omega_m$.

Recent work using high redshift supernovae to determine the form of $h(z)$ has shown that the existence of old high redshift radio galaxies and QSO's can be easily accommodated within the framework of a Λ-dominated cosmology (see figure 6) [22].

5.2 *Large scale structure*

A cosmological constant influences the growth of large scale structure indirectly by speeding up the rate of expansion of the universe. This slows down the collapse of very massive objects (clusters and superclusters of galaxies) which are expected to have formed recently in gravitational instability scenario's. As a result massive clusters are significantly rare at high redshifts in an $\Omega_m = 1$ universe than they are in a low density universe ($\Omega_m \ll 1$) both with and without Λ. X-ray observations from satellites such as XMM are likely to probe the cluster population at high redshifts thereby placing constraints on the value of Ω_m. However since open models without Λ ($\Omega_m \ll 1,\ \Lambda = 0$) and flat models with Λ ($\Omega_m + \Omega_\Lambda = 1$) give qualitatively similar results for the abundance of high redshift objects, it is not clear whether cluster observations alone will help differentiate a low density matter dominated universe from a flat universe with $\Omega_m + \Omega_\Lambda \simeq 1$.

5.3 *The fluctuation spectrum*

The fluctuation spectrum of clustered matter is influenced by the value of the particle horizon at matter-radiation equality $d_{\rm eq}$ which, in turn, is sensitive to the net matter content of the universe $d_{\rm eq} = 16/\Omega_m h^2$ Mpc. A small value of Ω_m leads to a larger value of $d_{\rm eq}$ and hence also to more long wavelength power in the fluctuation spectrum $P(k) = \langle|\delta_k|^2\rangle$. Low density CDM models (OCDM) and flat models with a cosmological constant (ΛCDM) show better large scale agreement with catalogues of galaxies such as the APM survey [23]. Other astrophysical consequences of a cosmological constant include its effect on gravitational lensing [24,25] and on the angular size redshift relation [11].

5.4 *Fate of a Λ-dominated universe*

The presence of a cosmological constant can change the course of expansion and the geometry of the universe in a very radical manner. The cosmological history of the universe now consists of three main epochs instead of two: (i) radiation domination, $10^4 \lesssim z \lesssim \infty$; (ii) matter domination, $z_\star \leq z \lesssim 10^4$; (iii) the present Λ dominated epoch $z < z_\star$. The value of $z_\star$ marks the commencement of the epoch when the cosmological constant begins to dominate the energy density of the universe, its value is easily determined from the Einstein equations to be

$$(1+z_\star)^3 = \frac{\Omega_\Lambda}{\Omega_m}. \tag{18}$$

The Einstein equations can also be used to determine the redshift z_* when the universe began to accelerate

$$(1+z_*)^3 = 2\frac{\Omega_\Lambda}{\Omega_m}. \tag{19}$$

Substitution of $\Omega_m \simeq 0.28$, $\Omega_\Lambda \simeq 0.72$ leads to $z_\star \simeq 0.37$, $z_* \simeq 0.73$ and $q_0 \simeq -0.58$. The universe begins to accelerate *even before* it becomes Λ dominated ! If Λ is a constant then the universe will soon enter an epoch of exponential expansion $a \propto e^{\sqrt{\frac{\Lambda}{3}}t}$ just as the Hubble parameter freezes to a constant value: $H = H_\infty = \sqrt{\Lambda/3} = H_0\sqrt{1-\Omega_m}$. An intriguing property of a universe dominated by a positive cosmological constant and accelerating over a large enough region is that the volume of space from which an observer is able to receive signals begins to shrink in size and contract. Likewise, the coordinate volume of space which can be directly influenced by our civilization (in the absence of wormholes) is finite. It can be easily shown that for $\Omega_m \simeq 0.3$ and $\Omega_\Lambda \simeq 0.7$, observers located beyond the redshift surface $z_H \simeq 1.8$ will forever remain inaccessible to signals emitted by our civilization and 'comoving observers once visible to us will gradually disappear from view as light emitted by them gets redshifted and declines in intensity [11,26].' These properties are related to the presence of a de Sitter-like (future) event horizon in a universe which begins expanding exponentially in the future. (An analogous process is observed for objects falling through the horizon of a black hole.) More discussion on these issues can be found in [11,27,28,26].

6. The vacuum energy and Λ

Theoretical foundations for the cosmological constant were laid when Zeldovich, intrigued by the 'discovery' of an excess of QSO's at the redshift $z \simeq 2$ and its explanation within the framework of the 'loitering' Lemaitre model decided to take a closer look at Λ. Zeldovich showed that the vacuum energy-momentum tensor generated by one-loop quantum effects in an expanding space-time geometry had exactly the form of a cosmological constant $\langle T_{ik}\rangle_{\rm vac} = \Lambda_{\rm vac} g_{ik}$ [8]. Unfortunately the value of $\Lambda_{\rm vac}$ turns out to be divergent so that $\Lambda_{\rm vac} \propto k_{\rm max}^4$ if an ultraviolet cutoff is set at $k_{\rm max}$. A Planck scale cutoff results in a vacuum energy density which is 123 orders of magnitude larger than its currently observed value $\rho_{\rm vac} \sim 10^{-29}$g/cm^3. There was some hope during the seventies that supersymmetric theories might reduce the value of $\Lambda_{\rm vac}$ since fermions and bosons contribute to $\langle T_{ik}\rangle_{\rm vac}$ with opposite signs [29]. However since supersymmetry is broken on scales $< 10^3$ GeV it is unlikely that one can appeal to this mechanism to generate the small value of Λ which is observed today. (Curiously the SUSY breaking scale in some models is rather low $M_{\rm SUSY} \sim 10^3$ GeV [30] implying $\rho_{\rm SUSY} \sim M_{\rm SUSY}^4 \sim (10^3{\rm GeV})^4$. It is interesting that on the logarithmic scale the value of $\rho_{\rm SUSY}$ lies midway between the Planck value $\rho_{\rm Pl} \sim (10^{18}{\rm GeV})^4$ and the observed value of the vacuum energy $\rho_{\rm vac} \sim (10^{-3}{\rm eV})^4$. This might indicate that the present value of the cosmological constant is provided by a theory in which the effective energy scale of the vacuum was given by $M_{\rm SUSY}^2/M_{\rm Pl} \sim 10^{-3}$ eV.)

A recent study of quantum effects in an expanding universe has shown that vacuum polarization and particle production associated with an extremely light non-minimally coupled scalar field could give rise to a vacuum energy-momentum tensor having the desired form $\langle T_{ik}\rangle_{\rm vac} = \Lambda_{\rm vac} g_{ik}$ [31,32]. The corresponding value of the dimensionless vacuum density is [31] $\Omega_\Lambda = \Lambda_{\rm vac}/3H^2 \simeq 1/(6|\xi|)(m/H)^2$ (where $|\xi| \ll 1$). Thus the observed value of $\Omega_\Lambda \simeq 0.7$ can be explained by this class of models provided ultra-light fields with $m \sim H \sim 10^{-33}$ eV exist. Such fields have been discussed in the context of pseudo-Nambu–Goldstone bosons in [33].

7. A dynamical Λ-term

Since its value is held fixed, a cosmological *constant* runs into the *fine tuning* problem: the ratio $\rho_\Lambda/\rho_{\rm rad}$ must be set to the extraordinary accuracy of one part in 10^{123} at the Planck time in order to ensure that ρ_Λ begins to dominate the energy density of the universe at precisely the present epoch. As we shall see, a time dependent Λ-term can get around this difficulty.

Historically many phenomenological Λ models have been proposed, following [11] we classify them into three main groups:

(1) *Kinematic models*: Λ is assumed to be a function of the cosmic time either explicitly: $\Lambda(t)$ or implicitly: $\Lambda[a(t)]$. (Note $T_{ik} \neq \Lambda g_{ik}$ if Λ is a function of time.)
(2) *Hydrodynamic models*: Λ is described by a barotropic fluid with equation of state $p_\Lambda(\rho_\Lambda)$.
(3) *Field-theoretic models*: Λ is assumed to be a new physical field (Λ-field) whose properties are described by a phenomenological Lagrangian.

In a FRW setting the above descriptions of Λ are related. For instance the simplest class of kinematic models

$$\Lambda \equiv 8\pi G\rho_\Lambda = f(a) \tag{20}$$

is equivalent to hydrodynamic models describing an ideal fluid with the equation of state

$$p_\Lambda(\rho_\Lambda) = -\rho_\Lambda\left(1 + \frac{1}{3}\frac{\mathrm{d}\ln\rho_\Lambda}{\mathrm{d}\ln a}\right). \tag{21}$$

The connection between hydrodynamic models and field-theoretic models will be discussed in the next section.

7.1 *Scalar field models of* Λ

It is well known that inflationary models based on a minimally coupled scalar field can generate a Λ-term that is weakly time dependent. It is therefore conceivable that a mechanism similar to the one which generates a large Λ-term during an early inflationary epoch could generate a small Λ-term at present. The energy density and pressure of a minimally coupled scalar field are, respectively

$$\rho = \frac{1}{2}\dot{\phi}^2 + V(\phi),$$
$$P = \frac{1}{2}\dot{\phi}^2 - V(\phi), \tag{22}$$

where the scalar field evolves according to the equation

$$\ddot{\phi} + 3H\dot{\phi} + \frac{\mathrm{d}V}{\mathrm{d}\phi} = 0, \tag{23}$$

with the value of the Hubble parameter given by

$$H^2 = \frac{8\pi G}{3}\left(\rho_m + \frac{\dot{\phi}^2}{2} + V\right),\ \rho_m = \frac{3\Omega_0 H_0^2}{8\pi G}\left(\frac{a_0}{a}\right)^3. \tag{24}$$

From (22) we find that the inflationary equation of state $P \simeq -\rho = -\Lambda/8\pi G$ arises if $\dot{\phi}^2 << V(\phi)$, where $V(\phi)$ plays the role of a time-dependent cosmological Λ-term. However the simplest cosmological models based on the 'chaotic inflationary' potential $V \propto m^2\phi^2, \lambda\phi^4$ run into a fine tuning problem similar to that encountered by a cosmological constant: the scalar field equation of motion (23) is enormously overdamped during the radiation and matter dominated epochs due to which $V(\phi) \simeq \Lambda/8\pi G$ remains virtually unchanged over a prolonged period of cosmic time. If $V(\phi)$ is to be small today then this implies that its value was *always small* which leads to an enormous imbalance between the energy density in radiation and that in the ϕ-field at early times. Fortunately the fine tuning problem can be substantially reduced in a class of scalar field potentials in which the scalar field does not 'slow-roll' during the radiation and matter dominated epochs. This property is best illustrated by the exponential potential

$$V(\phi) = V_0 \exp\left(-\lambda\phi/M_P\right). \tag{25}$$

In a spatially flat universe a field rolling down such a potential demonstrates a beautiful 'chameleon-like' property by mimicking the equation of state of the dominant matter component so that the ratio of the scalar field density to that of matter/radiation remains fixed as the universe expands [34–36]

$$\frac{\rho_\phi}{\rho_B + \rho_\phi} = \frac{3(1+w_B)}{\lambda^2} \tag{26}$$

($w_B = 0,\ 1/3$ respectively for dust, radiation). Nucleosynthesis constraints require $\rho_\phi/\rho_B \lesssim 0.2$, so that a scalar field rolling down an exponential potential cannot significantly influence the expansion dynamics of the universe and therefore cannot cause the universe to accelerate. It would be ideal if the energy density of the scalar field were to track background matter during most of its evolution and 'emerge from the shadows' at a later time when the value of its potential was small. Exactly this is achieved by a class of potentials studied by Sahni and Wang [37]

$$V(\phi) = V_0(\cosh\lambda\phi - 1)^p. \tag{27}$$

$V(\phi)$ has asymptotic forms:

$$V(\phi) \simeq \tilde{V}_0 \mathrm{e}^{-p\lambda\phi} \ \text{ for } |\lambda\phi| \gg 1 \ (\phi < 0), \tag{28}$$

$$V(\phi) \simeq \tilde{V}_0 (\lambda\phi)^{2p} \ \text{ for } |\lambda\phi| \ll 1, \tag{29}$$

where $\tilde{V}_0 = V_0/2^p$. Thus at early times, as long as the value of ϕ is large and negative, the energy density in ϕ tracks the the radiation/matter component. During later times the potential changes to a power law and the field begins to oscillate about $\phi = 0$. The change in the form of the potential leads to an important change in the scalar field equation of state which, during oscillations, becomes [37]

$$\langle w_\phi \rangle = \left\langle \frac{\frac{1}{2}\dot{\phi}^2 - V(\phi)}{\frac{1}{2}\dot{\phi}^2 + V(\phi)} \right\rangle = \frac{p-1}{p+1}. \tag{30}$$

The equation of state therefore becomes inflationary ($\langle w_\phi \rangle < -1/3$) if $p < 1/2$. For $p = 1$ $\langle w_\phi \rangle = 0$ and the ϕ-field behaves like cold dark matter (CDM). Thus *both* a quintessence-type Λ-field and CDM can be accommodated by the class of potentials (27). From

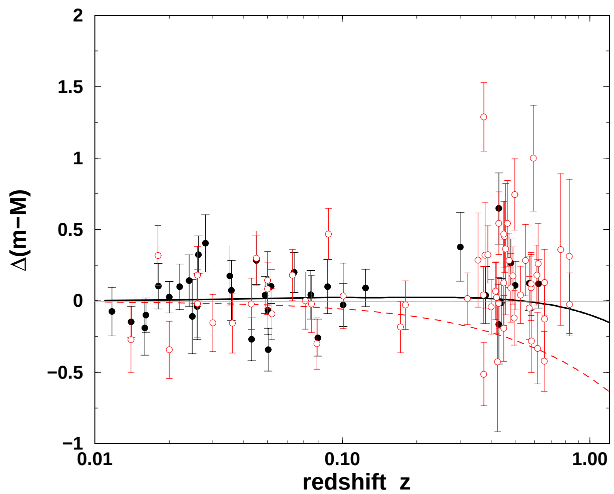

Figure 4. The luminosity distance is shown as a function of redshift for the Λ-field model (27) with $p = 0.2$. For comparison we also show the standard CDM model (dashed line) and the empty Milne universe $\Omega_m \rightarrow 0$ (horizontal line). Data from the high-z supernova search team and the supernova cosmology project are shown as filled circles and opaque circles respectively. The low-z supernovae are from the Calan-Tololo sample. From Sahni and Wang [37].

figure 4 we find that this class of models agrees reasonably well with supernovae data. A time dependent Λ-term also arises for the following potentials: $V(\phi) = k/\phi^\alpha$, $\alpha > 1$, [34]; $V(\phi) = V_0[\mathrm{e}^{M_P/\phi} - 1]$ [38].

Finally we would like to point out that models of Λ in which the equation of state is held fixed so that $p_\Lambda = \left(\frac{\alpha}{3} - 1\right)\rho_\Lambda$ and $\Lambda \propto a^{-\alpha}$ can be determined from a one parameter family of scalar field potentials. This is demonstrated by writing the Einstein equations as

$$\frac{H^2}{H_0^2} = \Omega_m \left(\frac{a_0}{a}\right)^3 + (1 - \Omega_m)\left(\frac{a_0}{a}\right)^\alpha, \tag{31}$$

$$8\pi G V(\phi) = aH\frac{\mathrm{d}H}{\mathrm{d}a} + 3H^2 - \frac{3}{2}\,\Omega_m H_0^2 \left(\frac{a_0}{a}\right)^3, \tag{32}$$

which can be solved exactly to give [11]

$$V(\phi) = \frac{(3 - \frac{\alpha}{2})(1 - \Omega_m)^{1+\alpha} H_0^2}{8\pi\Omega_m^\alpha G} \times \sinh^{-\frac{2\alpha}{3-\alpha}}\left((3 - \alpha)\sqrt{\frac{2\pi G}{\alpha}}\,(\phi - \phi_0 + \phi_1)\right). \tag{33}$$

This last expression describes the correspondence between hydrodynamic models and field-theoretic models alluded to in the previous section.

7.2 *Model independent reconstruction of the* Λ*-field*

It is necessary to point out that although Λ-field (quintessence) models have been discussed within the context of supersymmetric and supergravity theories, string and M-theory, extra dimensions etc. (see [11] for a review) no unique model of a time dependent Λ-term has so far emerged. It therefore becomes quite important to be able to determine properties of the Λ-field such as its potential, equation of state etc. in a *model independent* manner. This is in fact possible since eq. (32) allows us to determine $V(\phi)$ once the Hubble parameter and its derivative are known. Furthermore (10) permits us to reconstruct $H(z)$ if we know the luminosity distance d_L since [39,22]

$$H(z) = c\left[\frac{\mathrm{d}}{\mathrm{d}z}\frac{d_L(z)}{1+z}\right]^{-1}. \tag{34}$$

The resulting form of $V(\phi)$ using high redshift supernovae to determine $d_L \to H(z) \to V(\phi)$ is shown in figure 5. The corresponding 'best fit' equation of state varies slightly with time so that $w_\phi \simeq -1$ at $z=0$ and $w_\phi \simeq -0.7$ at $z = 0.83$. However a cosmological constant with $w_\phi \simeq -1,\ \forall z$ also agrees with the data. Knowing $H(z)$ we can also determine the age of the universe in a model independent manner by combining (34) & (16) as shown in figure 6. We find that a model independent estimate of the age of the universe agrees very well with both low z and high z observations.

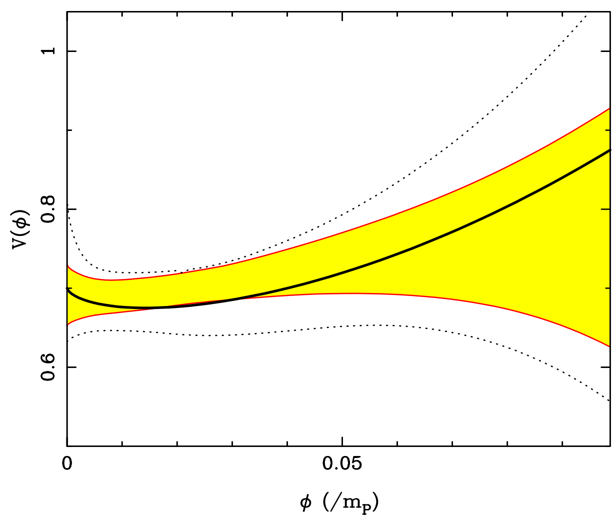

Figure 5. A supernovae based *model independent* reconstruction of the Λ-field potential $V(\phi)$ is shown in units of $\rho_{\rm cr} = 3H_0^2/8\pi G$. The value of ϕ (known up to an additive constant) is plotted in units of the Planck mass m_P. The solid line corresponds to the best-fit values of the parameters while the shaded area covers the range of 68% errors, and the dotted lines the range of 90% errors, calculated by a Monte-Carlo method. From Saini *et al* [22].

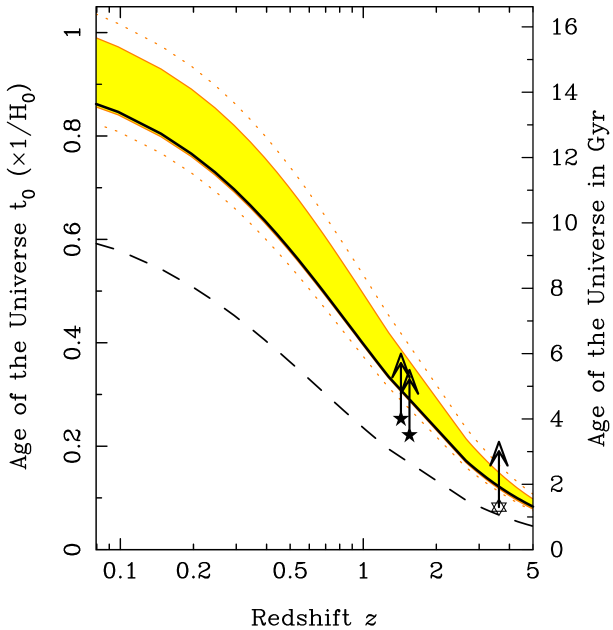

Figure 6. A supernovae based *model independent* estimate of the age of the Universe at a redshift z, is shown in units of H_0^{-1} (left vertical axis) and in Gyr, for the value of $H_0 \sim 60$ km/sec/Mpc (right vertical axis). The shaded region represents the range of 68% errors, and the dotted lines the range of 90% errors. The three high-redshift objects for which age-dating has been published [20] are plotted as lower limits to the age of the Universe at the corresponding redshifts. The dashed curve shows the same relation for an $(\Omega_m, \Omega_\Lambda) = (1, 0)$ Universe for the same H_0. From Saini *et al* [22].

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