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Neutrino masses and mixing in supersymmetric theories

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Abstract. It has been known for sometime that supersymmetric theories with R-parity violation provide a natural framework where small neutrino masses can be generated. We discuss neutrino masses and mixing in these theories in the presence of trilinear lepton number violating couplings. It will be shown that simultaneous solutions to solar and atmospheric neutrino problems can be realized in these models.

Keywords. R-parity violation; neutrino masses.

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1. Introduction

The solar and atmospheric neutrino experiments provide evidence for small neutrino masses. For the atmospheric neutrino anomaly, the recent results from the super-Kamiokande [1] favour a neutrino mass squared difference of $\Delta m^2 \sim 10^{-3}$ eV^2 with large mixing, whereas the solar neutrino [2] experiments indicate a mass squared difference of $\Delta m^2 \sim 10^{-6}$ eV^2 (MSW conversion) with small or large mixing or $\Delta m^2 \sim 10^{-10}$ eV^2 (vacuum oscillations) with large mixing. Thus, the neutrino mass spectrum seems to be characterized by hierarchical masses and one or two large mixings. Various theoretical models have been proposed to realise such a mass spectrum for the neutrinos [3]. Here we consider an alternative framework for generation of neutrino masses namely, supersymmetric standard models with R-parity violation.

Supersymmetric standard models naturally allow for lepton number violation. These couplings are given as

$$W_{\not{L}} = \epsilon_i L_i H_2 + \lambda'_{ijk} L_i Q_j d^c_k + \lambda_{ijk} L_i L_j e^c_k, \tag{1}$$

where L, Q, H_2 stand for leptonic, quark and Higgs doublets ; e^c, d^c stand for leptonic and down quark singlet superfields and i, j, k stand for generation indices. The presence of any of these couplings would naturally lead to neutrino masses [4]. Though this has been known for a long time, there has been a renewed interest in the structure of neutrino masses and mixing in these models after the recent super-Kamiokande results [5]. In particular, it has been shown that bilinear R-parity violating models [5–7] provide a very economical framework where solutions for neutrino anomalies can be realised. We would like to refer to Roy's article in this issue for more details on bilinear R-parity violating models [8].

On the other hand, models with trilinear lepton number violation *a prori* appear to contain large arbitrariness due to the large number of couplings. But, it was shown that these models can be quiet predictive if one assumes all the trilinear couplings to be of the similar magnitude [9,10]. Here, we present the structure of neutrino masses and mixing in presence of trilinear lepton number violating couplings in the superpotential. We show that in the limit where are all the trilinear couplings are taken to be of the same order, these models naturally prefer simultaneous solutions for solar and atmospheric neutrino problems through vacuum oscillations.

2. Neutrino masses and mixing

In the present case, we consider only trilinear λ' couplings to be present in the superpotential. We would comment on the inclusion of λ couplings later on. In the presence of non-zero λ' couplings neutrinos attain masses through two sources.

The key feature of the first source is the generation of the sneutrino vev in the renormalization group (RG) improved low energy effective potential [11–13]. In the conventional supergravity framework with only trilinear lepton number violating interactions, the soft potential does not contain bilinear lepton number violating terms at the high scale. They are however generated at the weak scale due to RG scaling and thus should be retained in the soft potential at the weak scale. The relevant part of the soft potential is now given as

$$\begin{aligned} V_{\text{soft}} = {} & m^2_{\tilde{\nu}_i} \mid \tilde{\nu}_i \mid^2 + m^2_{H_1} \mid H^0_1 \mid^2 + m^2_{H_2} \mid H^0_2 \mid^2 + \big[m^2_{\nu_i H_1} \tilde{\nu}^\star_i H^0_1 \\ & - \mu \, B_\mu H^0_1 H^0_2 - B_{\epsilon_i} \tilde{\nu}_i H^0_2 + \text{h.c} \big] + \cdots , \end{aligned} \tag{2}$$

where B_{ϵ_i} and $m^2_{\nu_i H_1}$ represent the bilinear lepton number violating soft terms and standard notation has been used for the other terms. As mentioned earlier, the parameters B_{ϵ_i} and $m^2_{\nu_i H_1}$ are absent at the high scale but, are generated at the weak scale through RG evolution in the presence of non-zero λ' couplings. The solutions for the relevant RG equations given in [10,11,13] can be represented as [10]

$$\begin{aligned} B_{\epsilon_i} &= \lambda'_{ipp} h^D_p \kappa_{ip}, \\ m^2_{\nu_i H_1} &= \lambda'_{ipp} h^D_p \kappa'_{ip}. \end{aligned} \tag{3}$$

The parameters κ, κ' represent the running of the parameters present in the RGE's from the high scale to the weak scale. The above soft potential (eq. (2)) would now give rise to sneutrino vevs,

$$< \tilde{\nu}_i > \; = \frac{B_{\epsilon_i} v_2 - m^2_{\nu_i H_1} v_1}{m^2_{L_i} + \frac{1}{2} m^2_Z \, \cos 2\beta}. \tag{4}$$

The sneutrino vevs so generated will now mix neutrinos with neutralinos giving rise to a tree level neutrino mass matrix of the form [4,14]:

$$\mathcal{M}^0_{ij} = \frac{\mu (c g^2 + g'^2) \; < \tilde{\nu}_i > \; < \tilde{\nu}_j >}{2(-c\mu M_2 + 2 M^2_w c_\beta s_\beta (c + \tan\theta^2_w))} \;\; , \tag{5}$$

where all the parameters in the above are represented in their weak scale values. Thus, this mass can be called as RG induced tree level mass or simply tree level mass. The second source is due to 1-loop diagrams. A majorana mass term for the neutrinos is also generated by 1-loop diagrams involving squarks and anti-squarks and their ordinary partners in the loops [4]. This mass can be written as

$$\mathcal{M}^l_{ij} = \frac{3}{16\pi^2}\lambda'_{ilk}\lambda'_{jkl}\ v_1\ h^D_k\ \sin\phi_l\ \cos\phi_l\ \ln\frac{M^2_{2l}}{M^2_{1l}} \quad . \tag{6}$$

The total neutrino mass matrix is now the sum of the tree level mass and the 1-loop level mass. This can be written as

$$\begin{aligned} \mathcal{M}^\nu_{ij} &= \mathcal{M}^0_{ij} + \mathcal{M}^l_{ij} \\ &\approx (m_0 + m_{\rm loop})\, a_i a_j + m_{\rm loop}\ h^D_2\ h^D_3\ A_{ij}, \end{aligned} \tag{7}$$

where $a_i \equiv \lambda'_{ikk} h^D_k$ and

$$A_{ij} = \lambda'_{i23}\lambda'_{j32} + \lambda'_{i32}\lambda'_{j23} - \lambda'_{i22}\lambda'_{j33} - \lambda'_{i33}\lambda'_{j22}.$$

We have also neglected $O(h^D_1, h^{D2}_2)$ contributions in writing the above. m_0 which contains rest of the contribution to the tree level mass is determined by solving the RGE and is roughly given as

$$m_0 \sim \left(\frac{3}{4\pi^2}\right)^2 \frac{v^2}{M_{\rm SUSY}} \left(\ln\frac{M^2_X}{M^2_Z}\right)^2, \tag{8}$$

where $M_{\rm SUSY}$ is the typical scale of SUSY breaking. $m_{\rm loop}$ contains rest of the contribution to the loop mass and is typically of the order,

$$m_{\rm loop} \equiv \frac{3\ v_1}{16\pi^2}\ \frac{\sin\phi_l\ \cos\phi_l}{h^D_l}\ \ln\frac{M^2_{2l}}{M^2_{1l}} \sim \frac{3\ v^2_1}{16\pi^2}\ \frac{1}{M_{\rm SUSY}}. \tag{9}$$

From eqs (8) and (9) we see that the tree and loop level contributions approximately differ only by the logarithmic factor in eq. (8). For $M_X = M_{\rm GUT} \sim 3\times 10^{16}$ GeV, this factor is a large number. This leads to the domination of the tree level mass over the loop mass for large regions of the parameter space.

The eigenvalues of the above mass matrix (7) are approximately given as

$$\begin{aligned} m_{\nu_1} &\sim m_{\rm loop} h^D_2 h^D_3 \delta_1, \\ m_{\nu_2} &\sim m_{\rm loop} h^D_2 h^D_3 \delta_2, \\ m_{\nu_3} &\sim (m_0 + m_{\rm loop})\ (a^2_1 + a^2_2 + a^2_3). \end{aligned} \tag{10}$$

Both δ_1 and δ_2 given in [10] are generically of $O(\lambda'^2)$ when all λ'_{ijk} are assumed to be similar in magnitude. As a consequence, neutrino masses follow the hierarchy,

$$m_{\nu_1} \sim m_{\nu_2} \ll m_{\nu_3}. \tag{11}$$

With

$$\frac{m_{\nu_2}}{m_{\nu_3}} \sim \frac{m_s}{m_b}\frac{m_{\rm loop}}{m_0}\left(\frac{\delta_2}{\Sigma_i \lambda'^2_{i33}}\right), \tag{12}$$

where $m_{s(b)}$ stands for strange (bottom) quark mass. The mixing among the neutrinos is governed by

$$U = \begin{pmatrix} c_1c_2 - s_1s_2c_3 & s_1c_2 + c_1s_2c_3 & s_2s_3 \\ -s_2c_1 - s_1c_2c_3 & -s_1s_2 + c_1c_2c_3 & c_2s_3 \\ s_1s_3 & -s_3c_1 & c_3 \end{pmatrix}. \tag{13}$$

The angles s_2, s_3 are determined as

$$s_2 = \frac{a_1}{\sqrt{a_1^2 + a_2^2}} \ ; \ s_3 = \frac{(a_1^2 + a_2^2)^{\frac{1}{2}}}{\sqrt{a_1^2 + a_2^2 + a_3^2}}.$$

s_1 is determined by

$$\tan 2\theta_1 = \frac{2A'_{12}}{A'_{22} - A'_{33}},$$

where A'_{ij} are functions of λ' given in [10]. The mixing among the neutrinos is basically determined by the ratios of the trilinear couplings and hence can be naturally large in the limit when they are all taken to be of the same order. Thus, we have seen that hierarchical and large mixing are the natural features of the neutrino mass spectrum in these models. Below, we show that simultaneous solutions to the solar and the atmospheric neutrino problems can be achieved naturally in these models.

3. Neutrino anomalies

Due to hierarchy in masses, one could simultaneously solve the solar and atmospheric neutrino problems through vacuum solutions provided $m_{\nu_1} \sim m_{\nu_2} \sim 10^{-5}$–$10^{-6}$ and $m_{\nu_3} \sim 10^{-1}$. Large regions in the parameter space can be found where $(m_{\rm loop}/m_0) \sim 10^{-1}$–$10^{-2}$ leading to $(m_{\nu_2}/m_{\nu_3}) \sim 10^{-3}$–$10^{-4}$. In figure 1a we show a typical region of the parameter space where this can be achieved. The region corresponds to $M_2 = 200$ GeV, $A = 0$ and tan $\beta = 2.1$ for positive μ parameter. Thus for $m_{\nu_3} \sim 10^{-1}$–10^{-2} eV, the right range required to solve the solar neutrino problem is achieved. The typical value of $m_0 \sim$ GeV implies that $\lambda' \approx 10^{-4}$. Large mixing angles can also be achieved for example by choosing $c_3 = s_3 = s_1 = c_1 = 1/\sqrt{2}$. The CHOOZ constraint restricts $s_2 = 0.254$ for this choice of parameters. This can be achieved without significant fine tuning by choosing,

$$\frac{\lambda'_{133}}{\lambda'_{233}} \sim \frac{1}{4}.$$

While hierarchy required for the vacuum solution follows more naturally, relevant scales for MSW conversion could also be achieved. This happens for very specific regions of the parameters in which the two contributions to the sneutrino vev cancel. In these regions, $(m_{\rm loop}/m_0) \sim 1$ leading to correct range required for MSW solution. In figure 1b we show a typical region where this cancellation occurs for $M_2 = 400$ GeV, $A = 0$ for tan $\beta = 2.1$. In the limit where all the λ' are of the same order, these

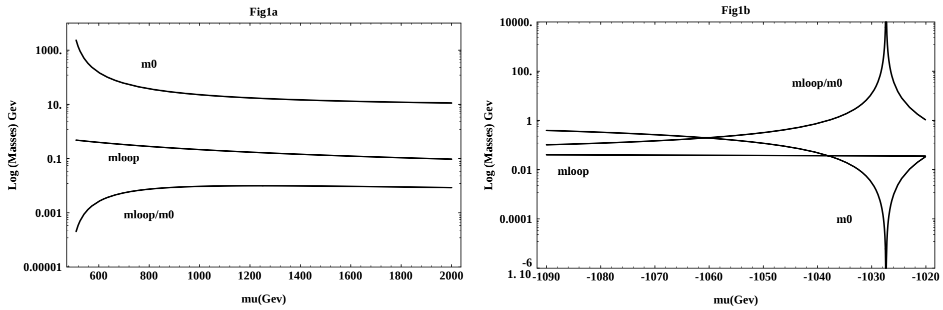

Figure 1. Regions in parameter space are shown where $m_{\rm loop}/m_0 \sim 10^{-2}$ (**a**) and $m_{\rm loop}/m_0 \sim 1$ (**b**).

models naturally predict large angle MSW solution. Additional constraints on the λ' couplings like discrete symmetries [9] or hierarchies have to be imposed on λ' couplings if one would like to have small angle MSW solution. The authors of [15] have concentrated in this region of the parameter space and imposed hierarchical conditions on the λ' to attain the small angle solution.

Unlike the λ' case, the antisymmetric nature of the λ would give rise to a completely different nature of the mixing in these models. It can be shown that two large mixings cannot be simultaneously present in these models in the limit where all the λ couplings are taken to be of the same order.

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