PRAMANA — journal of physics
© Indian Academy of Sciences
Vol. 55, Nos 1 & 2
July & August 2000
pp. 247–251

Analysis of two recent tests of T-invariance

P K KABIR
Beams Physics Laboratory, University of Virginia, Charlottesville, Virginia 22904, USA

Abstract. Inequality of the rates for $K^0 \to \pi^+ e^- \bar{\nu}$ and $\bar{K}^0 \to \pi^- e^+ \nu$ transitions, reported by CPLEAR, and an asymmetry in the distribution of the dihedral angle between the $\pi^+\pi^-$ and e^+e^- planes in $K_L \to \pi^+\pi^- e^+ e^-$ decays, found by KTeV, have been announced as demonstrations of T-noninvariance. These results are critically interpreted and compared as proofs of the failure of reciprocity.

Keywords. Time-reversal; direct tests.

PACS No. 11.30.Er

1. Introduction

In 1964, Fitch, Cronin and collaborators discovered CP-noninvariance in $K^0 \to 2\pi$ decays which, together with the CP-asymmetry found in $K_L \to \pi l \nu$ decays, remained until 1998 the only clear experimental indications that CP is not a universal symmetry. Accordingly, the only assurance of observing T-asymmetric effects, required by the TCP theorem, is from the associated T-asymmetric $K^0 - \bar{K}^0$ mixing. The most obvious consequence is an asymmetry [1] between $K^0 \to \bar{K}^0$ and $\bar{K}^0 \to K^0$ transformations, viz. a failure of reciprocity which would signal simultaneous CP- and T-noninvariance. The CPLEAR collaboration reported [2] detection of a closely related effect in agreement with theoretical prediction, but the identification of the observed CP-asymmetry with the predicted T-asymmetry depends on an additional assumption which should, in principle, be avoided [3]. A second, less direct, method of testing T-invariance, reported [4] by the KTeV group, is through measurement of a CP-asymmetric angular dependence in $K_L^0 \to \pi^+\pi^- e^+ e^-$ decays, predicted [5] by Sehgal and partners, belonging to the class of 'T-odd' correlations [6] long sought for without success in nuclear and particle reactions.

2. Direct test of reciprocity

Annihilations of stopping antiprotons in hydrogen provide a symmetric source of K^0 and $\bar{K}^0$ mesons, which made possible a significant comparison of $K^0 \to \bar{K}^0$ and $\bar{K}^0 \to K^0$ transformation-rates. Any non-zero value of the strangeness-inversion asymmetry [1]:

$$A_T = \frac{P_{K\bar{K}}(\tau) - P_{\bar{K}K}(\tau)}{P_{K\bar{K}}(\tau) + P_{\bar{K}K}(\tau)} \tag{1}$$

would manifestly display a simultaneous failure of CP- and T-invariance. CP-invariance requires that the mirror-image of $K^0 \to \bar{K}^0$ transformations should show $\bar{K}^0 \to K^0$ transmutations, which must therefore proceed at the same rate. T-invariance clearly requires the rate for the first process and its inverse to be equal. In Weisskopf–Wigner approximation,

$$A_T^{\rm th} = 2{\rm Re}(\epsilon_S + \epsilon_L) = 2{\rm Re}\langle K_L|K_S\rangle \tag{2}$$

to lowest order in the parameters $\epsilon_{S,L}$ which determine the structure of the exponentially decaying states K_S and K_L. The right-hand-side (RHS) can be estimated by appeal to unitarity which, in the Weisskopf–Wigner approximation, requires [7]:

$$\langle K_S|K_L\rangle = [1 + \gamma_L/\gamma_S + 2i(m_L - m_S)]/\gamma_S]^{-1} \cdot 2\Sigma_f B_S^f \eta_f, \tag{3}$$

where B_S^f is the fraction of K_S decays into the channel f and $\eta_f = \langle f|T|K_L\rangle/\langle f|T|K_S\rangle$. Improved knowledge about $K^0 \to 3\pi$ decays and leptonic decay modes has reduced the uncertainty about contributions to the RHS from $known$ decay channels, but did not significantly change the size of the effect predicted previously:

$$A_T^{\rm th} = (6.4 \pm 1.2).10^{-3}\,. \tag{4}$$

TCP-invariance imposes $\epsilon_S = \epsilon_L$, and requires $A_T = 2\delta_L$, where

$$\delta_L = \frac{R[K_L \to \{\pi^- e^+ \nu\}] - R[K_L \to \{\pi^+ e^- \bar{\nu}\}]}{R[K_L \to \{\pi^- e^+ \nu\}] + R[K_L \to \{\pi^+ e^- \bar{\nu}\}]} \tag{5}$$

apart from possible corrections arising from $\Delta S = -\Delta Q$ transitions, which are theoretically contra-indicated and have not been detected experimentally. The value $\delta_L = (3.27 \pm 0.12).10^{-3}$ is known [8] considerably better than the uncertainty in eq. (3).

The method adopted by CPLEAR to measure A_T is to assume associated production in $\bar{p}p \to KK^{\pm}\pi^{\mp}$ annihilations to select neutral kaons of known strangeness at birth. For those neutral kaons which subsequently beta-decay, the sign of the pion's charge in $K \to \pi e \nu$ identifies the strangeness at the moment of death by the $\Delta S = \Delta Q$ rule, and thus permits the study of $K^0 \to \bar{K}^0$ and $\bar{K}^0 \to K^0$ transformations. Assuming the validity of the $\Delta S = \Delta Q$ rule, the quantity:

$$A_l = \frac{R[\bar{K}^0(\tau=0) \to \{\pi^- e^+ \nu\}_\tau] - R[K^0(\tau=0) \to \{\pi^+ e^- \bar{\nu}\}_\tau]}{R[\bar{K}^0(\tau=0) \to \{\pi^- e^+ \nu\}_\tau] + R[K^0(\tau=0) \to \{\pi^+ e^- \bar{\nu}\}_\tau]} \tag{6}$$

becomes

$$A_l = \frac{P_{K\bar{K}}(\tau)R[K^0 \to \pi^- e^+ \nu] - P_{\bar{K}K}(\tau)R[\bar{K}^0 \to \pi^+ e^- \bar{\nu}]}{P_{K\bar{K}}(\tau)R[K^0 \to \pi^- e^+ \nu] + P_{\bar{K}K}(\tau)R[\bar{K}^0 \to \pi^+ e^- \bar{\nu}]}\,. \tag{7}$$

TCP-invariance requires the (perturbatively calculated) beta-decay rates to be equal, in which case the common factors in eq. (7) cancel, and the RHS reduces to that of eq. (1), so that a measurement of A_l becomes equivalent to a measurement of A_T. But, if the purpose is to test T-invariance, one should preferably not invoke TCP-invariance because

TCP-invariance *requires* T-noninvariance if CP-noninvariance is taken as established. If TCP-invariance is *not* assumed for neutral kaon beta-decays, measurements of A_l and A_T are no longer equivalent and a non-zero value of A_l, eq. (7), could equally be interpreted, under the hypothesis of exact reciprocity, $P_{K\bar{K}}(\tau) = P_{\bar{K}K}(\tau)$, as arising from a difference between conjugate beta-decay rates! The CPLEAR group sought [9] to exclude this alternative by appealing to the unitarity relation, eq. (3). Since their evaluation of $2\mathrm{Re}(\epsilon_S + \epsilon_L)$ accounts entirely for the asymmetry A_l which they measured, they concluded that possible TCP-noninvariant contributions to K_L beta-decays are limited to a level which could not possibly be the *sole* cause of the observed asymmetry. But, if one is prepared to accept the unitarity sum, eq. (3), as evidence for non-vanishing $\mathrm{Re}\langle K_L|K_S\rangle$ – which signals T-noninvariance, then this was already known in 1970! There is no need to invoke A_l.

The difficulty with the use of eq. (3) is that it requires a summation over *all* channels of neutral kaon decay. Contributions from *known* channels yield a value of order 10^{-3} for the RHS of eq. (3); but the various branching ratios have not been measured with accuracy sufficient to exclude the possibility that one or more unknown (neutral?) decay channels could cancel [10] the contributions of known channels. Possible TCP-noninvariance in neutral kaon beta-decays *can* be limited without recourse to unitarity arguments, but these require [11] measurements beyond the asymptotic asymmetries reported by CPLEAR.

3. Observation of 'T-odd' correlations

True tests of T-invariance necessarily involve some form of detailed balance: comparison of a chosen time-dependent process with its inverse, e.g. the case discussed above of $K^0 \to \bar{K}^0$ and $\bar{K}^0 \to K^0$ transformations. For decay processes describable by perturbation theory, viz. those in which the final particles may be treated as non-interacting, the appearance of 'T-odd' correlations [6] signals T-noninvariance. T-invariance requires reciprocity [12]:

$$\langle f_{\mathrm{in}}|i_{\mathrm{out}}\rangle = \langle \tilde{i}_{\mathrm{in}}|\tilde{f}_{\mathrm{out}}\rangle, \tag{8}$$

where the tilde denotes a motion-reversed channel: $\tilde{c}$ is obtained from c by reversing all momenta and spins.The RHS of eq. (8) is the complex-conjugate of $\langle \tilde{f}_{\mathrm{out}}|\tilde{i}_{\mathrm{in}}\rangle$ and the labels 'in' and 'out' are superfluous for non-interacting particles, thus we obtain

$$R(i \to f) = R(\tilde{i} \to \tilde{f}) \tag{9}$$

if interactions between the particles in the initial and final states, respectively, can be neglected. For $a \to b + c + \cdots$, the process $\tilde{a} \to \tilde{b} + \tilde{c} + \cdots$ must occur with equal frequency if interactions between $b, c, ...$ are negligible. If a is a spinless particle, the overall decay distribution must therefore transform into itself under motion-reversal. The KTeV group found [4] a significant deviation from this requirement in the rare decay mode $K_L \to \pi^+\pi^-e^+e^-$. For given values of the invariant masses, $m^2_{\pi\pi} = (p_+ + p_-)^2$ and $m^2_{ee} = (k_+ + k_-)^2$, of the dipion and the electron-pair, an arbitrary decay configuration is specified, in the K_L rest-frame, by the angle θ_π at which the π^+ is emitted with respect to the relative velocity of the particle-pairs, and the polar angles θ, ϕ of the positron relative to this axis. Motion-reversal does not change θ_π or θ, but reverses the sign of ϕ, therefore T-invariance would require the distribution of $K_L \to \pi^+\pi^-e^+e^-$ decays to be symmetric with respect to the transformation $\phi \to -\phi$ if interactions between the final particles are negligible. Consequently, the positron should not exhibit any preference to be emitted on either side of the $\pi^+\pi^-$ plane. KTeV found a strong ϕ-dependence and a clear preponderance of decays with $\phi > 0$ according to their convention.

Dalitz conversion of the photon from the known process of $K_L \to \pi^+\pi^-\gamma$ suffices to describe all observed features of $K_L \to \pi^+\pi^- e^+e^-$ decays, so we need [10] to retain only this essential part of the model proposed by Sehgal *et al* to correctly predict the branching ratio and nature and magnitude of the azimuthal correlation seen by KTeV. The observed [13] photon spectrum from $K_L \to \pi^+\pi^-\gamma$ clearly shows two components: the theoretically unambiguous 'inner' bremsstrahlung (IB), dominant for low E_γ, which necessarily accompanies $K_L \to \pi^+\pi^-$ decays, is supplemented by 'direct' radiation for photon energies in the middle of the kinematically allowed range. The absence of interference between the two components supports the theoretical expectation that the direct radiation is M1, which is CP-allowed and can thus compete favourably with IB, whose matrix-element is proportional to the CP-forbidden amplitude for $K_L \to \pi^+\pi^-$ decays.

Since IB must be polarized in the $\pi^+\pi^-\gamma$ plane (in the K_L rest-frame), Dalitz pairs from the conversion of such photons cannot favour either side of the $\pi^+\pi^-\gamma$ plane and could not possibly give rise to the ϕ-asymmetry found by KTeV. Addition of a magnetic radiative amplitude contributes a component polarized $perpendicular$ to the $\pi^+\pi^-\gamma$ plane, which could account for the observed asymmetry if its magnitude and phase are appropriate. The excess of the observed photon spectrum above the predictable IB distribution determines the $magnitude$ of the direct radiative amplitude but says nothing about its phase, if it is purely magnetic. Let us denote the radiative amplitude symbolically by

$$A = \mathcal{E} \cdot \hat{x} + \mathcal{M}.\hat{y}, \tag{10}$$

where we explicitly display the polarization directions of the electric and magnetic amplitudes $\mathcal{E}$ and $\mathcal{M}$, respectively, choosing the z-axis along the direction of the virtual photon which materializes into the electron-pair. If the two terms in A, eq. (10), have the $same$ phase, the corresponding photons are linearly polarized in the xy-plane at an angle $\psi = \tan^{-1}|\mathcal{M}/\mathcal{E}|$ relative to the x-axis. Electron-pairs from such linearly-polarized photons have an angular distribution:

$$W(\theta, \phi) = \cos^2\theta + \sin^2\theta \cdot \sin^2(\phi - \psi) \tag{11}$$

which serves as an analyzer for the photon polarization. For purely electric radiation, $\psi = 0$, the distribution is symmetrical about $\phi = 0$ as we stated already for IB. For $\psi \neq 0$, the form of the angular distribution is unchanged, but the symmetry-axis is rotated away from $\phi = 0$ and loses, in general, its symmetry in ϕ. If $\mathcal{M}$ is $\pi/2$ out of phase with $\mathcal{E}$, the two amplitudes contribute incoherently to the decay rate and the distribution again becomes symmetrical in ϕ, because the angular distribution from purely magnetic radiation is also symmetric, as can be seen from the form of eq. (11) for $\psi = \pi/2$.

The phase of the non-radiative amplitude for $K_L \to \pi^+\pi^-$ is required to be $(\phi_{+-} + \delta_0)$ where ϕ_{+-} is the well-measured [8] phase of η_{+-} and δ_0 is the less well-known s-wave phase shift of the presumably dominant $I = 0$ $\pi\pi$ final state at $E = m_K$. TCP-invariance requires [14], apart from corrections of order ϵ, that the phase of $\mathcal{M}$ should be $(\pi/2 + \delta_1)$ where δ_1 is the p-wave $\pi\pi$ phase- shift at the same energy. Within the uncertainties of the $\pi\pi$ phase-shifts, $(\phi_{+-} + \delta_0)$ cannot be distinguished from $\pi/2$; which is almost true also for the phase of $\mathcal{M}$, since δ_1 is a small angle. Therefore, if $\mathcal{E}$ in eq. (10) is approximated by the IB contribution and $\mathcal{M}$ is assumed to have the phase required by TCP-invariance, both terms have the $same$ phase within the accuracy of our present knowledge, so the associated photon is linearly polarized to the same precision. This accounts very satisfactorily for the strong azimuthal variation and asymmetry observed by KTeV. On the other hand, the original estimation [5] by Sehgal and Wanninger was based on the assumption that $\mathcal{M}$ has the phase δ_1, which is small and causes $\mathcal{M}$ to be nearly real, which cannot generate much ϕ-asymmetry since $\mathcal{E}$ is close to being pure-imaginary. Therefore, to the extent that the original estimate does not yield the observed asymmetry, KTeV's measurement rules out the original phase-hypothesis which, although the authors did not mention it, corresponds to the assumption of T-invariance.

Even though the particular mechanism described above is probably the most important contributor, the occurrence of $K_L \to \pi^+\pi^-e^+e^-$ decays and the asymmetry reported by KTeV do not depend specifically on it. Whatever may be the contributing processes, the argument leading to eq. (9) shows that 'T-odd' asymmetries are forbidden by reciprocity, eq. (8), for all decays in which interactions between the final particles can be neglected. For the case of $K_L \to \pi^+\pi^-e^+e^-$, electromagnetic interactions between the pions and the electrons are not expected to generate 'T-odd' asymmetry to account for the effect observed by KTeV. Detailed discussion will be presented elsewhere [11].

In completely different and complementary ways, the two experiments strongly support the theoretical expectation that the long-established CP-noninvariance in neutral K-meson decays is associated with T-noninvariant interactions. Even more conclusive results are awaited.

Acknowledgments

I thank Rahul Sinha for arranging the invitation to Chennai, and the organizers of WHEPP6 for local hospitality.

References

[1] P K Kabir, *Phys. Rev.* **D2**, 540 (1970)
[2] A Angelopoulos *et al*, for CPLEAR, *Phys. Lett.* **B444**, 47 (1998)
[3] P K Kabir, *Phys. Lett.* **B459**, 335 (1999)
[4] A Alavi-Harati *et al*, for KTeV, *Phys. Rev. Lett.* **84**, 408 (2000)
[5] L M Sehgal and M Wanninger, *Phys. Rev.* **D46**, 1035 and 5209(E) (1992)
[6] See E M Henley, *Ann. Rev. Nucl. Sci.* **19**, 367 (1969)
[7] J S Bell, in *High Energy Physics* edited by C DeWitt and M Jacob (Gordon and Breach, New York, 1965)
[8] C Caso *et al*, Particle Data Group, *Eur. J. Phys.* **C3**, 1 (1998)
[9] A Apostolakis *et al*, for CPLEAR, *Phys. Lett.* **B456**, 297 (1999)
[10] P K Kabir, *Proc. Conf. Frontiers of Matter* (Blois, 1999) in press
[11] P K Kabir, to be published
[12] F Coester, *Phys. Rev.* **84**, 1259 (1951)
[13] E Ramberg *et al*, *Phys. Rev. Lett.* **70**, 2525 (1993)
[14] G Costa and P K Kabir, *Nuovo Cimento* **A61**, 564 (1967)