

What's new at small x ?

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Abstract. We discuss some recent developments in small x physics.

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1. Introduction

In the last couple of decades, following the discovery of asymptotic freedom in the early 70's, perturbative QCD has been enormously successful in describing the physics of very high $Q^2 \gg \Lambda_{\text{QCD}}^2$. However, it is sobering to think that these very high Q^2 processes comprise the tails of distributions – their contribution is a small fraction of the total cross-section at high energies. The vast bulk of the cross-section, corresponding to soft and semi-hard processes, is still ill understood in QCD. Note that for momentum transfer square Q^2 , the high energy limit $s \rightarrow \infty$ is also the limit $x \rightarrow 0$ since $x \propto Q^2/s$. The physics of high energy soft and semi-hard processes in QCD is therefore also the physics of small x . Understanding the origin of these small x processes within the framework of QCD is an outstanding challenge to both theory and experiment. In this talk, I will attempt to summarize some recent theoretical studies on the physics of high energy (or small x) processes in QCD. Note: Due to space-time limitations, several important topics will not be treated – vector meson production, nuclear shadowing, etc. Also, given the large amount of activity in the field, it is unavoidable that my survey of the literature will not be as complete as it should be.

Phenomenological ideas motivated by Regge theory have had some success in describing some of the data. For example, the t -channel exchange of an object with vacuum quantum numbers, the notorious Pomeron, provides a reasonable description of total cross-sections at very high energies [1]. It corresponds to a simple pole, with unit intercept, for the amplitude in the (t, j) plane, which through a Sommerfeld–Watson transformation to the (s, t) plane gives cross-sections rising with the energy:

$$A(t, j) \sim \frac{1}{j - \alpha(t)} \xrightarrow{\text{SW}} A(s, t) \sim s^{\alpha(t)}, \quad (1)$$

where $\alpha(t) = \alpha_0 + \alpha' t$. For the Pomeron, phenomenologically, $\alpha_0 \approx 1.08$ and the string tension $\alpha' = 0.25 \text{ GeV}^{-2}$. Invoking the optical theorem, one can easily show that

$$\sigma_{\text{tot}} \sim s^{\alpha(0)-1}. \quad (2)$$

Donnachie and Landshoff [1] have shown that $\sigma(s) \sim s^{0.08}$ provides a good fit to the available data on $\bar{p}p$, pp , πp , Kp and γp collisions at high energies. Strictly speaking, they find that

$$\sigma(s) = As^{0.08} + Bs^{-0.45}, \quad (3)$$

gives a good fit to the above mentioned data. The term with the decreasing contribution as a function of the energy corresponds to the ‘Reggeon’ exchange of ρ , ω , f_2 , and a_2 mesons. The Regge form of the amplitude also explains high energy, small $|t|$, differential cross-sections and the shrinkage of the diffractive peak at high energies.

Despite the apparent phenomenological successes of the Pomeron concept, we still do not have a very good idea of what it is or why it works. A popular conjecture is the one first postulated by Low and Nussinov [2], where Pomeron exchange is taken to be the color singlet component of two gluon exchange in the t -channel. In weak coupling, Lipatov and collaborators have shown that the leading logarithmic $\alpha_S \ln(1/x)$ result in perturbative QCD corresponds to the t -channel exchange of two ‘reggeized’ gluons. In the color singlet channel, they constitute the well known BFKL hard pomeron [3]. Recent developments suggest however that this Pomeron may not be entirely robust at next to leading order, thereby confounding Pomeron enthusiasts. A possible resolution within the Pomeron framework is that multi-Pomeron exchanges become important sooner than one expects them to. I say sooner because one expects these exchanges to become important eventually anyway since they help ensure that unitarity is satisfied at asymptotically high energies. The Onium-model of Mueller [4], where the mass of the Onium pair provides a perturbative scale, provides the framework of several recent multi-Pomeron studies. See for instance ref. [5].

Alternatively, one may eschew the Pomeron language altogether. One such approach describes the physics of high density QCD within a Wilsonian renormalization group improved [6,7] classical effective field theory (EFT) of small x QCD [8]. One may expect that the two approaches are related. Recently, it has been argued that the formalism of multi-Pomeron exchanges can be recovered as a limit of the Wilsonian renormalization group formalism [9].

With the advent of HERA in the early 90’s, one was able to explore the regime in QCD where $x = Q^2/s \ll 1$ but $Q^2 \gg \Lambda_{\text{QCD}}^2$. The advantage of this regime is that while the coupling $\alpha_S(Q^2) \ll 1$, large logarithms $\alpha_S \ln(1/x) \sim 1$ make the physics non-perturbative. From a theoretical point of view, this regime of high parton densities is interesting since it affords one the opportunity to study the interplay between perturbative and non-perturbative physics. In particular, one expects to see the effects of the high parton densities we mentioned above. An especially useful probe of this interplay is hard diffraction, which constitutes a significant part of the cross-section in deeply inelastic scattering (DIS). In this case, there is a color singlet exchange between the hadron and the projectile, the latter fragmenting into a hard final state. Interestingly, one can show that the usual factorization theorems apply to diffractive DIS [10,11]. We will discuss diffraction further later on in this talk.

This talk is organized as follows. In the next section, we briefly discuss the DGLAP double log limit at small x . We will comment on how well it does in describing the HERA data. In §3, we will discuss the BFKL equation, and the next to leading order BFKL

equation. Next, we will discuss different approaches to the physics of high parton densities, which study the regime where the linear evolution equations in x and Q^2 break down. In §5, we discuss recent results on hard diffraction. In §6, we make the connection between small x physics and heavy ion collisions. We stress the importance of understanding of the small x component of the nuclear wavefunction to better understanding the initial conditions and possible thermalization in heavy ion collisions at RHIC and LHC. We end with a brief outlook on future directions in theoretical and experimental studies of high energy QCD.

2. DGLAP evolution in QCD

This topic is of course highly developed and discussed at length in several textbooks. We will discuss particular aspects of it in order to motivate the discussion in the following sections. This section and the next have been influenced in part by the nice lectures of Salam – for more details, we refer the reader to them [12]. Let us begin with the canonical process, DIS of electrons off hadrons or nuclei. The kinematic invariants here are

$$x_{Bj} = \frac{-q^2}{2P \cdot q} ; Q^2 = -q^2 > 0 ; y = \frac{P \cdot q}{P \cdot k} ; s = 2P \cdot k, \quad (4)$$

and these satisfy the relation $xy = Q^2/s$. In the rest frame of the target, the virtual photon fluctuates into a quark anti-quark pair, which subsequently interacts with the target. The $q\bar{q}$ pair undergoes Bremsstrahlung, emits a gluon, which subsequently splits into another (or with differing probability a $q\bar{q}$ pair), and so on until the parton on the lowest rung of the ladder interacts with the target. Each rung of the ladder therefore contributes a Bremsstrahlung phase space integral

$$\alpha_S \int \frac{d^2k_t}{k_t^2} \int \frac{dx}{x} \rightarrow \alpha_S^n \ln^q \left(\frac{x_0}{x} \right) \ln^r \left(\frac{Q^2}{Q_0^2} \right). \quad (5)$$

If we are interested in the kinematic region $x_0 \sim x_{Bj}$, and $Q_0^2 \ll Q^2$, clearly the dominant contribution to the scattering will be $(\alpha_S \ln(Q^2/Q_0^2))^n$ logs which are summed over all n . The DGLAP equations are renormalization group (RG) equations that sum up these large logarithms [13].

In at least a good chunk of the region probed by HERA, the kinematics are such that it is likely that the leading contribution is from logs in both x and Q^2 – one therefore sums large logs in both x and Q^2 : $(\alpha_S \ln(x_0/x) \ln(Q^2/Q_0^2))^n$. The unintegrated gluon distribution $\tilde{G}(x, Q^2) = d(xg(x, Q^2))/dQ^2$ satisfies the RG-equation

$$G(x, Q^2) = G^{(0)} + \int_x^1 \frac{dz}{z} \int dk_t^2 K_{\text{DGLAP}}(Q^2, k_t^2) G\left(\frac{x}{z}, k_t^2\right), \quad (6)$$

where $K_{\text{DGLAP}}(Q^2, k_t^2) = \bar{\alpha}_S \theta(Q^2 - k_t^2)/Q^2$, and $\bar{\alpha}_S = \alpha_S N_c/\pi$. This integral equation is diagonalized by the simultaneous Mellin transform with respect to x and Q^2 . For a particular initial condition,

$$\tilde{G}_{\gamma, \omega} = \frac{1}{\gamma} \frac{1}{(\omega - \frac{\alpha_S}{\gamma})}, \quad (7)$$

where γ is the leading order gluon anomalous dimension.

First performing the inverse Mellin transform with respect to ω , and then performing a saddle point integration over γ , one obtains the well known result [14]

$$Q^2 G(x, Q^2) \approx \frac{1}{2} \left(\frac{1}{\pi^2 \alpha_S \ln(1/x) \ln(Q^2/Q_0^2)} \right)^{\frac{1}{4}} \exp \left(2 \sqrt{\alpha_S \ln(1/x) \ln(Q^2/Q_0^2)} \right). \quad (8)$$

The strong rise in the gluon distribution is mirrored by the structure function,

$$R_F F_2(x, Q^2) = \exp(\kappa), \quad (9)$$

where R_F is a coefficient which also depends on x and Q^2 , and $\kappa = \sqrt{\log(1/x) \log(Q^2/Q_0^2)}$.

This is the so-called double asymptotic scaling, the slope of the structure function is a universal quantity. In this region, the scaling violations are relatively independent of the particular form of input parton distributions. At HERA, the running coupling has been extracted in the double log region: a NLO fit gives $\alpha_S(M_Z) = 0.120 \pm 0.05(\text{exp.}) \pm 0.09$ (theory) [15]. The theoretical uncertainties include estimates of small x corrections to the NLO calculation, and factorization and scale uncertainties [16]. There are also some unresolved issues regarding the normalization R_F of double asymptotic scaling in the HERA kinematic region [17]. A potential problem with this nice picture is the possibility that the contribution of higher order corrections to the DGLAP gluon anomalous dimensions (in the same kinematic region) induce an even faster rise – opening a Pandora’s box of problems [16]. These will be directly related to the issues discussed in the following sections.

3. The BFKL and NLO BFKL summations

The BFKL summation corresponding to summing only leading logarithms in x – $(\alpha_S \ln(1/x))^n$, is applicable when there are two large scales in the problem $Q^2 \sim Q_0^2 \gg \Lambda_{\text{QCD}}^2$. Consider again the integral equation discussed in eq. (6). The BFKL equation is obtained by replacing $K_{\text{DGLAP}} \rightarrow K_{\text{BFKL}}$, where the BFKL kernel is [3]

$$K_{\text{BFKL}} = \bar{\alpha}_S \left(\frac{1}{|\vec{Q} - \vec{k}_t|^2} - \delta(Q^2 - k_t^2) \int^{k_t} \frac{d^2 p_t}{\pi p_t^2} \right). \quad (10)$$

Again, as discussed in the previous section, the integral equation can be solved by performing the Mellin transform, which reads

$$\tilde{G}_{\gamma, \omega} = \frac{\omega \tilde{G}_{\gamma, \omega}^{(0)}}{(\omega - \bar{\alpha}_S \chi(\gamma))}. \quad (11)$$

The function $\chi(\gamma)$, defined to be $\chi(\gamma) = \psi(1) - 0.5 \cdot (\psi(\gamma) + \psi(1 - \gamma))$, (where ψ is the logarithmic derivative of the gamma function) is known as the characteristic function.

Taking the inverse Mellin transform with respect to ω , and performing a saddle point expansion of the γ integral around $\gamma = 1/2$, one obtains the result

$$G(x, Q^2) \approx \frac{x^{-\bar{\alpha}_S \chi(1/2)}}{\sqrt{2\pi\bar{\alpha}_S \chi''(1/2) \log(1/x)}} \frac{1}{QQ_0}, \quad (12)$$

where χ'' is the second derivative of χ with respect to γ . For $\alpha_S = 0.2$, the power of x is $-\bar{\alpha}_S \chi(1/2) = -0.5$, a rise that is too rapid to be compatible with the HERA data. Furthermore, the scaling violations are also incompatible with the HERA data. Arguably, since the BFKL equation is derived under the assumption that $Q^2 \sim Q_0^2 \gg \Lambda_{\text{QCD}}^2$, one should not expect it to explain the HERA DIS data. BFKL-like effects have been studied, for instance, in $\gamma^* - \gamma^*$ scattering at LEP, and in jet production at HERA and Fermilab.

Of greater concern, conceptually, is the fact that the solution to the BFKL equation exhibits k_t diffusion [18]. Although the typical momenta in the BFKL ladder are hard, the solution 'diffuses' to the infrared at small x – the solution is therefore sensitive to momenta in the non-perturbative region.

Until recently, it was believed that next-to-leading-log (NLL) resummation of the form $\alpha_S (\alpha_S \ln(1/x))^n$ might clarify the theoretical picture. In the Mellin transform language, the characteristic function χ can be expanded as

$$\bar{\alpha}_S \chi(\gamma) = \bar{\alpha}_S \chi_0(\gamma) + \bar{\alpha}_S^2 \chi_1(\gamma) + O(\bar{\alpha}_S^3), \quad (13)$$

where χ_0 is the usual BFKL characteristic function we discussed above, and χ_1 is the NLL-term. The computation of this term took about 10 years (!) and was done independently by two groups [19]. The result is

$$\chi\left(\frac{1}{2}\right) = \chi_0\left(\frac{1}{2}\right) [1 - 6.47\bar{\alpha}_S]. \quad (14)$$

The power of the gluon distribution then for $\bar{\alpha}_S = 0.2$ is -0.15 . The correction is thus not only large, it also switches sign! Also, the structure of $\chi(\gamma)$ is now very different. One now has complex saddle points which give rise to cross-sections which, albeit real, oscillate with $\ln(Q^2/Q_0^2)$ [22].

Clearly, the resummation procedure, as developed thus far, is flawed. There have been several suggestions recently on how one may 'cure' this result. One detailed proposal [23] suggests that even though the full NLL characteristic function χ_1 has many contributions, a few collinear contributions give the bulk of the contribution. There are collinear corrections arising from (a) running coupling effects, (b) the non-singular part of the splitting functions, and (c) the choice of energy scale. Collecting these, one obtains the relatively simple collinear contribution to the NLL characteristic function

$$\chi_1^{\text{coll}}(\gamma) = \frac{A_1}{\gamma^2} + \frac{(A_1 - b)}{(1 - \gamma)^2} - \frac{1}{2\gamma^3} - \frac{1}{2(1 - \gamma)^3}, \quad (15)$$

where $A_1 = -11/12$, and $b = 11/12 - n_f/6$. The authors of ref. [23] have shown that χ_1^{coll} is in very good numerical agreement with the full NLL result. These collinear contributions can now be summed to all orders, and give rise to stable results (as a function of α_S) for the gluon anomalous dimensions, and for the exponent of the gluon Green's function at high energies.

The specific proposal we discussed briefly is very elegant and clever. It relies heavily though on the idea of ‘collinear dominance’ to all orders. Whether this is indeed the case is not entirely clear at present. For instance, multi-Pomeron (or high parton density) effects become important [24] at rapidities $y_{\text{mult}} \sim \frac{1}{(\alpha_P - 1)} \ln(1/\alpha_S^2)$, where $\alpha_P - 1 = 4\bar{\alpha}_S \ln(2)$. However, certain running coupling effects become important [25] at rapidities $y_{\text{NLO}} \sim 1/\alpha_S^{5/3}$. Thus, parametrically, multi-Pomeron effects appear sooner than some running coupling effects. This fact is not taken into account in BFKL-related proposals.

4. Classical EFT and ‘Onium’ approaches to high parton densities

As one goes to higher energies, smaller x ’s, one might ask whether there is a simpler organizing principle than computing an endless number of diagrams. For example, the properties of condensed matter systems in the vicinity of a critical point can be formulated in terms of effective theories which capture much of the physics. Attempting to compute critical behavior in the full theory would be an impossibly difficult task.

In small x physics, our quest for the right effective theory is helped by the following. Firstly, since the density of partons is growing with energy, occupation numbers become large. This makes it likely that classical methods are applicable. Secondly, at high energies, in the infinite momentum frame, partons have large field strengths on the transverse sheet corresponding to a large parton density per unit area. This provides a scale, which at sufficiently small x , is large enough to make weak coupling methods feasible. Finally, since small x partons are short-lived relative to partons at large x , the latter act as static sources whose dynamics can be ignored, a la Born–Oppenheimer, on the time scales of interest.

Rather ironically, the problem is simpler to formulate for a large nucleus [8], where there are $A^{1/3}$ more partons per unit area on the transverse sheet than in a hadron. Since the sources are confined in different nucleons, they are uncorrelated. Classical parton distributions can then be computed as correlation functions of a 2-dimensional Euclidean field theory with random, Gaussian sources. The problem is then formally just like that of computing the infra-red properties of a spin glass [26]. In practice, path ordering of the space-time rapidity is necessary, and leads to an analytical solution for classical parton distributions in the ‘colored glass condensate’ [6,27]. At transverse momenta $k_t \gg Q_s$, where $Q_s \gg \Lambda_{\text{QCD}}$ is a saturation scale, parton distributions have the usual Weizsäcker–Williams $1/k_t^2$ behavior. However, for $k_t \leq Q_s$, their behavior saturates, growing only logarithmically at small k_t .

Quantum corrections to the classical EFT give large logs in $\alpha_S \ln(1/x)$ [28]. A Wilson renormalization group procedure was devised which sums up these large logs [6]. The form of the effective action remains the same as one goes to small x – the only thing that changes is the weight function for the sources. For Gaussian sources, this gives $Q_s \rightarrow Q_s(x, Q^2)$. In general, the weight function obeys a non-linear Wilson renormalization group equation [7]. In the limit of low parton densities, it is just the BFKL equation we discussed previously [29]. At large Q^2 , an all twist result is obtained [30], whose leading term is the small x DGLAP equation, and the next-to-leading term is the higher twist correction previously computed by Gribov, Levin and Ryskin [31], and by Mueller and Qiu [32]. The outstanding question is whether this approach, by incorporating high parton density (multi-ladder) effects already at leading order through the non-linearities of the

classical field, provides a more stable expansion than the BFKL-motivated approach.

One can also compute the structure function F_2 to all orders in the classical background field [33]. For Gaussian sources, one recovers the Glauber formula [34] originally derived in the nuclear rest frame

$$F_2 = \frac{Q^2 N_c}{(2\pi)^3} \int d^2b \int_0^1 dz \int_0^{\frac{1}{\Lambda_{\text{QCD}}}} dx_t x_t \left(1 - \exp \left(-\frac{\alpha_S \pi^2}{2\sigma(b) N_c} x_t^2 xG \left(x, \frac{1}{x_t^2} \right) \right) \right) \times [\Phi_L(x_t, z, Q^2) + \Phi_T(x_t, z, Q^2)], \quad (16)$$

where $\Phi_{L(T)}$ is the probability of a longitudinally (transversely) polarized virtual photon to split into a quark–anti-quark pair, and the rest is the probability of that pair to scatter off the hadron/nucleus. Similar expressions have been used by several authors to reproduce the HERA data in the $Q^2 = 1\text{--}10 \text{ GeV}^2$ region [35]. These fits, however, are not conclusive evidence for screening corrections since a QCD fit with appropriately adjusted parton distributions also reproduces this data.

A very interesting approach to small x physics is through the study of ‘Onium’ scattering [4]. The large mass of the quarkonium state provides the large scale at which the coupling constant is evaluated. At high energies, the Onium state contains a large number of soft gluons in addition to the quark–anti-quark pair. In the large N_c limit, these gluons can be viewed as color dipoles. The cross-section for Onium scattering is then given by the product of the number of color dipoles in each Onium state times the elementary dipole-dipole scattering cross-section. The dipole density in the Onium state obeys an integral equation whose kernel is none other than the BFKL kernel. In this Onium picture, the scattering cross-section grows rapidly because the number of dipoles in the wavefunction multiplies rapidly at high energies. The Onium formalism thus gives us a way to quantify when multi-Pomeron effects, due to overlapping dipoles, overtakes BFKL multiplication.

These multi-Pomeron effects are again easier to quantify in DIS off a very large nucleus. In this generalized case, the dipole density in the $q\bar{q}$ -pair wavefunction obeys a non-linear integral equation [5,36], which sums up Pomeron ‘fan’ diagrams. Pomeron loop contributions are suppressed if $\alpha_S^2 A^{1/3}$ is large. This non-linear equation has been solved perturbatively outside the saturation region $k_t > Q_s$ [37].

The reader might wonder how the two approaches to high parton densities discussed in this section are related. It has been argued recently, that the non-linear integral equation for the dipole density in the Onium state can be obtained as a particular limit of the non-linear Wilson RG equation [9].

5. Hard diffraction

With the discovery of hard diffraction by UA8 [38], and subsequent experiments at Fermilab and HERA, diffraction is again a hot topic. For nice recent reviews of accompanying theoretical developments, see refs [39,40]. In QCD, naively, the struck quark forms a colored string with the rest of the hadron – the probability of a gap decreases exponentially with the size of the gap. One can define hard diffraction as events with hard final states accompanied by large rapidity gaps that are not exponentially suppressed. Monte Carlo event generators such as POMPYT [39] which allow color singlet exchanges do a better job of describing the data than do event generators which contain only colored strings.

Diffraction has traditionally been interpreted in terms of Pomeron exchange. Hard diffraction is especially interesting because it lets us probe the parton content of the exchanged color singlet object. In a phenomenological picture [41], the cross-section for single hard diffraction (to pick one of several topologies) is

$$d\sigma(\bar{p} + p \longrightarrow p + 2 \text{ jets}) = f_{\underline{p}}(x_{\mathcal{P}}, t) d\sigma(\mathcal{P} + \bar{p} \longrightarrow 2 \text{ jets}), \quad (17)$$

where the Pomeron flux factor $f_{\mathcal{P}/\bar{p}}(x, t) \propto (1/x_{\mathcal{P}})^{2\alpha(t)-1}$. Here $\alpha(t)$ is the same function as that defined below eq. (1). One can then write the factorized expression

$$d\sigma(\mathcal{P} + \bar{p} \longrightarrow 2 \text{ jets}) = \int dx_1 dx_2 dt \sum_{ji} f_{\bar{p}}^i(x_1, Q^2) f_{\bar{p}}^j(x_2, Q^2) \frac{d\sigma_{ij \rightarrow 2\text{jets}}}{dt}, \quad (18)$$

where $f_{i/\mathcal{P}}$ is the probability of finding a parton i in the Pomeron. The CERN and Fermilab collider data seem to suggest that there is a larger $q\bar{q}$ than a glue component in the Pomeron, though both data do not seem to agree with model predictions. One problem lies with the difficulty in defining absolute normalizations for the Pomeron flux [42]. A more serious problem may however be the breakdown of the factorization hypothesis for hadron-hadron scattering.

Hard diffraction has also been studied extensively at HERA where it comprises $\sim 10\%$ of the cross-section! In analogy to F_2 , one can define an experimental observable – the diffractive structure function $F_2^{D(4)}(x, Q^2, x_{\mathcal{P}}, t)$, in terms of the differential cross-section for the process $ep \longrightarrow ep + X$. The more inclusive variable $F_2^{D(3)}(x, Q^2, x_{\mathcal{P}})$ is easier to measure. Following Ingelman and Schlein, this can be factorized as $F_2^{D(3)} = f_{\mathcal{P}/p}(x_{\mathcal{P}}, t) F_2^{\mathcal{P}}(\beta, Q^2)$, where $\beta = x/x_{\mathcal{P}}$ is the fraction of the Pomeron momentum carried by the parton. Recent HERA data [43] show deviations from universal factorization i.e., the flux factor shows a β dependence. Fits which include sub-leading Reggeon exchange show agreement with the Pomeron intercept $\alpha(0) \sim 1.08$ only at the 3σ level. The diffractive structure function shows a very weak dependence on Q^2 -hard diffraction in a leading twist phenomenon.

There has been considerable theoretical work recently suggesting that, in exact analogy to the usual structure functions, one may define universal diffractive structure functions in diffractive DIS [20]. They may be identified as the matrix elements of bi-local field operators, and shown to obey leading twist RG-equations. This factorization breaks down when there is more than one hadron in the final state (unlike DIS). The reason why it breaks down is that gluons from the color singlet exchange may coherently scatter off gluons in the other hadron – unlike inclusive structure functions, these processes do not cancel. This breakdown of factorization has been shown empirically – diffractive structure functions from HERA, used to compute diffractive cross-sections at the Tevatron, vastly overpredict the experimental data [21].

The HERA data have been analysed within the framework of diffractive parton distributions [44]. It is found that the gluon component, predictably, dominates the quark component of the diffractive distribution. Also, the data are consistent with the presence of a semi-hard saturation scale. Is this scale the same as the scale Q_s discussed earlier? It may be so since phenomenological models that explicitly include saturation are quite successful

in fitting the data [45]. Such a result also arises in an approach where the scattering off the hadron is modelled by scattering off semi-classical color fields of the target using the eikonal approximation [46]. In the classical EFT approach, the difference between inclusive and quasi-elastic diffractive cross-sections is simply the following [47]. In the former case, one squares the amplitude before averaging over the random color sources; in the latter, one averages over the amplitude with the color sources before squaring the result. The energy dependence of rapidity gaps has recently been studied in the multi-Pomeron fan diagram approach [48]. (For earlier related work, see for instance ref. [49].) It will be interesting to see how it arises in the classical EFT approach.

6. Small x physics and heavy ion collisions

Much of the interest in heavy ion collisions have to do with the possibility of forming a quark gluon plasma at RHIC and LHC energies. At these energies, whether a plasma is formed, and how it formed, depends strongly on the initial conditions in the collision [50]. These in turn strongly depend on the small x parton distributions in the nuclei. For momenta $k_t \sim Q_s$, coherence effects are significant, and the factorization picture of mini-jet production may break down. At what energies that happens is a quantitative question that has no clear answer thus far [51].

The nice thing about the classical fields approach is that it provides a consistent space-time picture of the collision [52]. The initial conditions are obtained by matching the Yang–Mills equations, in the forward and backward light cones, along the lightcone. We remind the reader that analytic solutions are known for the classical fields in the nuclei before the collision [6,27]. In QCD, at small x , the classical $2 \rightarrow 1$ process dominates. Naively, in collinear factorization, this process would be suppressed in favor of the $2 \rightarrow 2$ process.

Gluon production in nuclear collisions is computed perturbatively, and is found to be infra-red divergent [52]. Recently, the Yang–Mills equations have been solved non-perturbatively to all orders in the classical background field, and the energy and number distributions computed [53]. One finds, self-consistently, a ‘formation time’ beyond which it is meaningful to define these objects as partons – as opposed to field amplitudes and energies. The most relevant results are the following. The energy distribution of gluons produced per unit area per unit rapidity is

$$\frac{1}{\pi R^2} \left(\frac{dE}{d\eta} \right)_{\Delta\eta=1} = \frac{(N_c^2 - 1)}{N_c} \frac{c(Q_s^2 R^2)}{4\pi^2 \alpha_S} Q_s^3, \quad (19)$$

where $c(Q_s^2 R^2) \approx 4.5$, is approximately constant in the regime of interest for RHIC and LHC. It has been estimated that $Q_s \sim 1$ GeV for RHIC and $Q_s \sim 2\text{--}3$ GeV at LHC [54]. Similarly, one can compute the number per unit area per unit rapidity, and one finds $dN/(\pi R^2)/d\eta = (\tilde{c}/4\pi^2 \alpha_S) ((N_c^2 - 1)/N_c) Q_s^2$, where $\tilde{c} \sim 1.08$. For $k_t \gg Q_s$, number distributions fall as $1/k_t^4$, but saturate at smaller values of k_t – the distributions are infrared finite.

How does a nuclear collision proceed from its very earliest moments? When produced, the gluons are on a transverse sheet, with typical momenta $k_t \sim Q_s$. They begin to scatter slowly (small angle scattering dominates) off the sheet, acquiring longitudinal momenta.

This process is described by solving the Boltzmann equation for the single particle distributions [54]. The approach to equilibrium can be studied numerically, and the initial temperature and chemical potential of the equilibrated quark-gluon plasma can be extracted as a function of the only scale the problem – the saturation scale Q_s [55]. Equivalently, in principle, the scale Q_s can be extracted from studying final states in heavy-ion collisions. Besides energy and multiplicity distributions, rapidity correlations in event by event fluctuations, would also be sensitive to the saturation scale [56]. If successful, heavy ion collisions will provide important information not only about a hot gluon plasma, but also about a cool color glass condensate.

7. Outlook

Our current understanding of small x physics is that pQCD works at HERA and the Tevatron, but perhaps better than we expect it to. There is much flexibility in parton distributions to hide interesting new effects. Indeed, there are strong hints from HERA that we are on the threshold of a new regime of truly high parton densities, where one may expect qualitative changes in the behavior of distributions.

Exciting times lie ahead. The relativistic heavy ion collider (RHIC) will start collecting data soon, hopefully providing us with a window to study the initial strong field strength regime in QCD and the possible subsequent phase transition in hot and dense parton matter. Proposals are afoot to study electron DIS off nuclei at HERA energies both at DESY and at BNL. The latter project is now known by the acronym eRHIC. *The nuclear advantage is that parton densities that would be probed only at c.m energies comparable to LHC c.m. energies with an ep collider, are accessible at RHIC c.m energies with an eA collider!* Multi-particle production is still one of the least understood aspects of QCD. Hopefully, the next generation of experiments will help us reveal its mysteries.

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