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Coherent structures in presence of dust charge fluctuations

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Abstract. This paper shows the formation of nonlinear coherent structures in a dusty plasma in presence of dust charge fluctuations. Using the typical plasma parameters the potential of the nonlinear coherent structures is derived.

Keywords. Nonlinear waves; coherent structures; dust charge fluctuation.

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1. Introduction

Recently there has been much interest in studying the dusty plasmas which is characterized as an ionized gas containing electrons, ions and highly charged massive dust particles. Dusty plasmas differ from the usual multi-component ion plasmas since the ratio between the dust charge and the dust mass is non-uniform and it is considered as a new dynamical variable. The presence of dust particles are found to modify the propagation of waves, wave instabilities etc. in dusty plasmas [1–3].

Highly charged massive dust grains present in a plasma may exhibit charge fluctuations in response to certain types of oscillations incorporated to the plasma. Under this situation the grain charge becomes a time dependent and self-consistent variable. The consequent modifications in the collective properties of a dusty plasma in response to the variation of charge is studied [4–5].

It may be noted that the existence of dust acoustic wave on a very slow time scale of dust dynamics was investigated for the first time by Rao *et al* [6]. They also showed the formation of rarefactive type dust acoustic soliton solution in the above plasma system. Similarly Ma and Liu [7] discussed the existence of rarefactive dust acoustic soliton solution in a plasma in presence of dust charge fluctuations.

In this paper we try to study the existence of double layers solutions associated with dust acoustic waves on a very slow time scale along with the presence of dust charge fluctuations.

It has been known that the particles associated with double layer potential variation may be conveniently divided into four classes: free electrons, free ions, trapped electrons and trapped ions. But in principle, three of these are sufficient to maintain the double layers e.g. one can assume that the free electrons and ion number densities are equal on the low potential side. It has been shown that plasma double layers solution may be possible in an ordinary acoustic branch even in the presence of ions and two species of electrons [8,9].

Our double layers analysis is based on a fluid model as described by Levine and Crawford [10]. In the uniform region on the high potential side of our dust acoustic double layers, the plasma consists of cold negatively charged dusts drifting away from the double layers, ions drifting toward the double layers and electrons are reflected by the double layers.

In § 2 the potential of the double layers associated with dust acoustic mode in presence of dust charge fluctuation is derived. Finally the discussion of our theoretical result is presented in § 3.

2. Derivation of the potential of our double layers

We consider a one-dimensional nonlinear electrostatic wave associated with acoustic speed in dusty plasma containing electrons, ions and negatively charged dust particles. We assume that the dust particles are cold and their dynamics are governed by the usual fluid equations

$$\frac{\partial n_d}{\partial t} + \frac{\partial}{\partial x}(n_d v_d) = 0\,, \tag{1}$$

$$\frac{\partial v_d}{\partial t} + v_d \frac{\partial}{\partial x} v_d = \left(\frac{q_d}{m_d}\right) \frac{\partial \phi}{\partial x}\,, \tag{2}$$

where n_d is the density of dust particle, v_d is the velocity of the dust particle, q_d is the charge of the dust particle and m_d is the mass of the dust particle.

For slow dust time scale, the electrons and ions are in local thermodynamic equilibrium and therefore one can use Boltzmann relation to describe their densities i.e.

$$n_e = n_{e0} \exp\left(\frac{e\phi}{T_e}\right)\,, \tag{3}$$

$$n_i = n_{i0} \exp\left(\frac{-e\phi}{T_i}\right)\,, \tag{4}$$

where $n_{e0}(n_{i0})$ is the electron (ion) equilibrium density, ϕ is the electrostatic potential and $T_e(T_i)$ is the electron (ion) temperature.

The charge of a dust particle is determined by currents collected by the dust particle. The charging equation of the dust particle is given by

$$\frac{\mathrm{d}q_d}{\mathrm{d}t} = I_e + I_i\,, \tag{5}$$

where I_e and I_i are respectively the electron currents and ion currents at the dust particle surface and they are given by

$$I_e = -\pi a_d^2 e \sqrt{\left(\frac{8T_e}{\pi m_e}\right)} n_e \exp\left(\frac{eq_d}{CT_e}\right) \; , \tag{6}$$

$$I_i = \pi a_d^2 e \sqrt{\left(\frac{8T_i}{\pi m_i}\right)} n_i \left(1 - \frac{eq_d}{CT_i}\right) , \tag{7}$$

where $m_e(m_i)$ is the mass of the electrons(ions), a_d is the radius of the dust particle and $C = 4\pi\epsilon_d a_d$. It is seen that on the hydrodynamics time scale the dust charge can reach local equilibrium. Thus we get $I_e + I_i = 0$. And we find a relationship between the local electrostatic potential and the dust charge i.e.

$$\phi = \theta_2[G - q_d + \ln(\theta_1 - q_d)] , \tag{8}$$

where $\theta_1 = T_i/T_e$, $\theta_2 = T_i/(T_i + T_e)$, $G = \ln(\delta\sqrt{m_e T_e/m_i T_i})$ and $\delta = n_{i0}/n_{e0}$. To close the system we use the Poisson's equation

$$\frac{\partial \phi}{\partial x^2} = -4\pi(en_i - en_e - q_d n_d) . \tag{9}$$

For stationary solution we consider a variable $\xi = \lambda_{\rm De}^{-1}(x - ut)$, where u is the double layer velocity and $\lambda_{\rm De}$ is the electron Debye length. For simplicity we have used another dimensionless function to calculate the dust particle density i.e. $\psi = \int_0^\phi q_d \mathrm{d}\phi$. Therefore from eqs (1)–(8) we get

$$n_d = (\delta - 1)\left(1 + \frac{2\psi}{M^2}\right)^{-1/2} , \tag{10}$$

$$n_e = \exp(\phi) , \tag{11}$$

$$n_i = \delta \exp\left(-\frac{\phi}{\theta_1}\right) \tag{12}$$

and

$$\psi = \theta_1(-\phi - (q_d - q_{d0})) - \frac{\theta_2}{2}(q_d^2 - q_{d0}^2) , \tag{13}$$

where $M = u/c_d$ is the Mach number of the double layer, c_d is the acoustic velocity. We have normalized the potential by T_e/e, dust charge by $Z_d e$, x by the electron Debye length, velocities by c_d and densities by the equilibrium electron densities. The dust charge number $z_d = a_d T_e/e^2$ is evaluated at $q_d = 1$. In deriving the above equations we have used the boundary conditions viz. $\phi = 0$, $\mathrm{d}\phi/\mathrm{d}\xi = 0$, $v_d = 0$, $n_d = n_{d0}$ and $q_d = q_{d0}$ at $\xi \to \pm\infty$.

The construction of our double layer solutions which is presented here is similar to Schamel [11]. To do this we use the general method with potential ϕ that varies monotonously between $\phi_{\min} < \phi < \phi_{\max}$, where $\phi_{\min}$ and $\phi_{\max}$ are two extremes of

ϕ. This solution will be valid in between $\xi_{\min}$ and $\xi_{\max}$. Putting the values of n_e, n_i, n_d and q_d in the Poisson's equation (9) and then expanding for small amplitude ($\phi << 1$) we obtain [7,12],

$$\frac{\mathrm{d}^2\phi}{\mathrm{d}\xi^2} = B_1\phi + B_2\phi^2 + B_3\phi^3 = -\frac{\mathrm{d}V(\phi)}{\mathrm{d}\phi}\,, \tag{14}$$

where

$$B_1 = 1 - \frac{\delta}{\theta_1} - (\delta - 1)\left(\frac{1}{M^2} + \frac{\beta}{\theta_2}\right)\,, \tag{15}$$

$$B_2 = \frac{1}{2} + \frac{\delta}{2\theta_1^2} + (\delta - 1)\left(\frac{3}{2M^4} + \frac{\beta}{\theta_2 M^2} - \frac{\beta^2}{2\theta_2^2}\right)\,, \tag{16}$$

$$B_3 = \frac{1}{6} - \frac{\delta}{6\theta_1^3} - (\delta - 1)\left(\frac{5}{2M^6} + \frac{3\beta}{\theta_2 M^4} - \frac{\beta^2}{2\theta_2^2 M^2} - \frac{\beta^3}{3\theta_2^3}\right) \tag{17}$$

and

$$\beta = \frac{1}{\theta_2}\left(1 + \frac{1}{(\theta_1 - q_{d0})}\right)\,. \tag{18}$$

We now introduce the classical potential $V(\phi)$ to point out the analogy with the equation of motion of a classical potential $(\mathrm{d}^2/\mathrm{d}x^2 = \mathrm{d}V/\mathrm{d}x)$. To study the shape of the potential ϕ and the classical potential $V(\phi)$ we use the 'energy law' which we can obtain by integrating eq. (14) and is given by

$$\frac{1}{2}\left(\frac{\mathrm{d}\phi}{\mathrm{d}\xi}\right)^2 + V(\phi) = 0\,, \tag{19}$$

where

$$-V(\phi) = \frac{1}{2}B_1\phi^2 + \frac{1}{3}B_2\phi^3 + \frac{1}{4}B_3\phi^3\,. \tag{20}$$

Now using boundary conditions $\mathrm{d}V(\phi)/\mathrm{d}\phi = 0$ at $\phi = \phi_0$ (or quasi neutral condition $n_e + z_d n_d = n_i$ exists at $\phi = \psi$) in eq. (14) and $V(\phi) = 0$ at $\phi = \phi_0$ (i.e. vanishing electric field at $\phi = \psi$) in (20) and then integrating (19) we find [8]

$$\phi = \frac{\phi_0}{2}\left(1 - \tanh(K\xi)\right)\,, \tag{21}$$

where

$$K = \sqrt{\left(\frac{B_3}{8}\right)}\phi_0\,. \tag{22}$$

From eq. (21) it is clear that the double layer solution exists provided $B_3 > 0$. It is seen from eq. (17) that the value B_3 will be positive only if the positive ion concentration (δ) is smaller than one.

Similarly if we retain unto upto ϕ^2 and using the boundary condition viz. $V(\phi) = 0$ at $\phi = \psi$, the integration becomes

$$\phi = -\psi \operatorname{sech}^2 \left(\sqrt{\frac{B_2 \psi}{6}} \xi \right) . \tag{23}$$

Equation (23) represents a soliton solution provided $B_2 > 0$. However it is seen that only negative solution exists for such a type of dust acoustic waves since B_2 is positive, which agree with Ma and Liu [7].

3. Discussion and conclusion

In this paper we have discussed the formation of double layer solution in a plasma containing electrons, ions and negatively charged massive dust particles. Considering the dust particle motion on a very slow time scale along with the dust charge fluctuations we have shown the existence of both soliton and double layer solution. To get these solutions positive ion concentration should be less than unity. Potential profile of the double layers and the soliton are shown in figures 1, 3 and 4 respectively. Similarly the variation of classical potential with respect to the potential for double layers (soliton) is shown in figure 2 (figure 3).

Our theoretical model is similar to Ma and Liu [7]. However they have shown the existence of soliton solution in a plasma consists of electrons, ions and positively charged dust particles. They have found that the soliton solution exists in such a plasma provided the positive ion concentration is higher than unity.

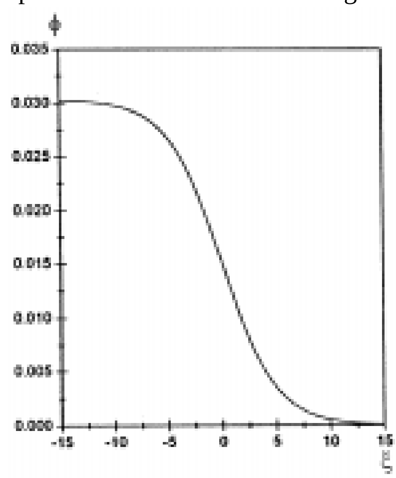

Figure 1. The double layers potential profile versus ξ for $\delta = 0.9$, $M = 1.2$ and $\theta_1 = 1$.

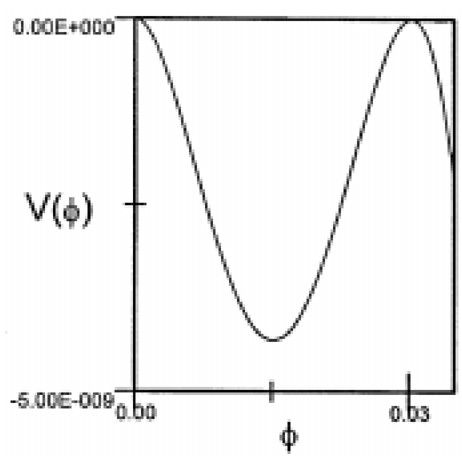

Figure 2. The Sagdeev potential (double layers) $V(\phi)$ versus ϕ for $\delta = 0.9$, $M = 1.2$ and $\theta_1 = 1$.

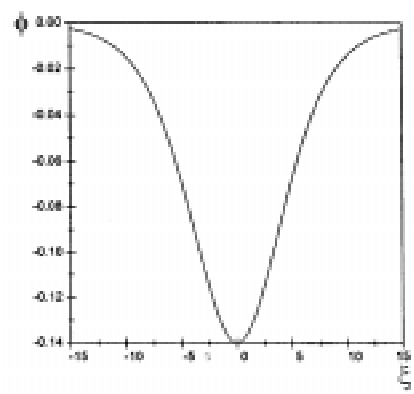

Figure 3. The potential profile of the soliton versus ξ for $\delta = 0.9$, $M = 1.2$ and $\theta_1 = 1$.

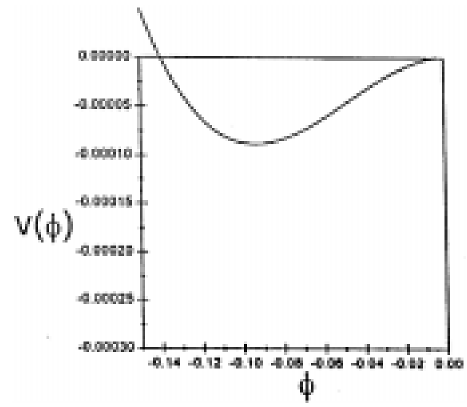

Figure 4. The Sagdeev potential (soliton) $V(\phi)$ versus ϕ for $\delta = 0.9$, $M = 1.2$ and $\theta_1 = 1$.

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