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Bulk viscosity of strange quark matter in density dependent quark mass model

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Abstract. We have studied the bulk viscosity of strange quark matter in the density dependent quark mass model (DDQM) and compared results with calculations done earlier in the MIT bag model where u, d masses were neglected and first order interactions were taken into account. We find that at low temperatures and high relative perturbations, the bulk viscosity is higher by 2 to 3 orders of magnitude while at low perturbations the enhancement is by 1–2 order of magnitude as compared to earlier results. Also the damping time is 2–3 orders of magnitude lower implying that the star reaches stability much earlier than in MIT bag model calculations.

Keywords. Bulk viscosity; radial damping; stability of strange stars; density dependent masses.

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1. Introduction

Some years ago it was suggested that the quark matter composed of comparable numbers of u, d and s quarks may be the true ground state of matter which is stable at zero pressure and temperature [1–3], in which case some or all neutron stars can turn out to be so-called strange stars [1–5]. If on the other hand strange matter is only metastable, the high pressure in the central regions of neutron stars may lead to formation of hybrid stars, having strange matter cores.

Observationally, it is not easy to distinguish strange stars, hybrid and ordinary neutron stars, since in the observed mass region of around 1.4 times the solar mass, the three have similar radii, as at these masses they are bound mainly by gravity, not by strong interactions. Neutrino cooling, once thought to be one of the best discriminating criterion, have lost much of its glamour since the revival of ordinary URCA processes in nuclear matter has narrowed down the difference in the expected neutrino cooling rates from two types of matter [6,7].

Another, perhaps the best way to distinguish between these three kinds of stars could be through the studies related to the damping of vibrations and secular instability of rapidly rotating neutron stars [8–12]. It has been pointed out by Wang and Lu [8] that the stellar pulsations will be highly damped in quark stars. The time scales for damping of vibrational and gravitational radiation reaction instability leads to a limit on the maximum rotation rate of the star. These are determined by the bulk viscosity of quark matter which arises mainly due to variation of relative concentration of strange quark and down quark through the non-leptonic weak process

$$u + d \longleftrightarrow s + u\,. \tag{1}$$

The source of bulk viscosity of quark matter is the deviation from β-equilibrium and the resulting non-equilibrium reactions implied by the compressions and rarefactions of matter in pulsating quark star; thus they are driven by the non-zero value of $\delta\mu = \mu_s - \mu_d$ away from β-equilibrium. The dissipation caused by bulk viscosity will be substantial as long as the reaction rate of (1) is comparable to the rate of density change. On the other hand, if the weak reaction rates are small, the dissipation will not be significant as the quark concentrations would remain frozen during the faster density changes. If the reaction rates of weak interactions are faster than pulsations, the system will remain in equilibrium and again there would be little dissipation.

The chemical equilibrium is also achieved in quark matter through the leptonic processes

$$\begin{aligned} u + e &\longleftrightarrow s(d) + \nu_e; \\ s(d) &\longrightarrow u + e + \bar{\nu}_e. \end{aligned} \tag{2}$$

The rates of these processes are highly suppressed under the conditions that prevail in the core of the strange quark star. In fact for non-interacting massless quarks energy-momentum conservation forces the matrix element to vanish altogether. But for quarks with masses, the rates are not negligible and in fact provide the main source of energy depletion through neutrino emission.

So far most of the calculations of quark star viscosity have been done in the usual MIT bag model [8,9]. An alternative description of quark matter in which the confinement is treated by assuming a baryon density dependence of the quark masses was introduced by Fowler, Raha and Weiner [13]. In this model the quark masses are taken as

$$m_u = m_d = \frac{C}{3n_B}, \quad m_s = m_{s0} + \frac{C}{3n_B}, \tag{3}$$

where m_{s0}, the strange quark current mass and C a constant, are free parameters to be constrained by stability argument.

Later on, the model was reformulated [14,15] to show that properties of SQM in this model are quite similar to those predicted by the MIT bag model. It effectively includes the first order QCD coupling correction and yet is much easier to work with. However in some respects this density dependent quark mass [DDQM] model is quite different from the MIT bag model. In the low pressure region, the speed of sound is much larger than the relativistic $c/\sqrt{3}$ of the bag model, and it agrees remarkably well with the phenomenologically extracted values from hadronic collisions over a large range of temperatures [13]. It is the aim of this work to generalize the results of [15] for the study of the bulk viscosity of SQM in the DDQM model. In this model, since all the quark masses are finite, the contribution from the leptonic processes is expected to be quite significant for some densities and temperatures. In fact in our study we find that for temperatures higher than 3 MeV the contribution from leptonic processes begins to dominate over non-leptonic processes in the linear regime.

In § 2 we review very briefly the density dependent quark mass model. In § 3 we derive the expressions for the bulk viscosity of the quark matter for both the leptonic and non-leptonic processes. Section 4 deals with results and discussion.

2. Density dependent quark mass (DDQM) model and thermodynamics

We assume SQM to be a free gas of u, d, s quarks, antiquarks, electrons and positrons. The quark mass dependence on the baryon number density is given by eq. (3). The thermodynamical potential for the system is

$$\Omega = \sum_i \Omega_i = -\sum_i \frac{g_i T}{(2\pi)^3} \int \mathrm{d}^3 p \ln(1 + \mathrm{e}^{-\beta(\varepsilon_i - \mu_i)}) - \frac{8}{45}\pi^2 T^4, \tag{4}$$

where $i = u, \bar{u}, d, \bar{d}, s, \bar{s}, e, \bar{e}$ and g_i is the degeneracy factor ($g_i = 2 \times 3 = 6$ for quarks and antiquarks, $g_i = 2$ for electrons and positrons). The last term is the contribution from gluons. From eq. (4), we obtain using the usual definition the total pressure, the energy density, the net number density of each particle:

$$P = \sum_i \frac{g_i}{(2\pi)^3} \int \frac{\mathrm{d}^3 p}{(p^2 + m_i^2)^{1/2}} \left[\frac{p^2}{3} - \frac{C m_i}{n_B}\right] (f_i(T) + \bar{f}_i(t)) + \frac{8}{45}\pi^2 T^4, \tag{5}$$

$$E = \sum_i \frac{g_i}{(2\pi)^3} \int \frac{\mathrm{d}^3 p}{(p^2 + m_i^2)^{1/2}} \left[(p^2 + m_i^2) + \frac{C m_i}{3 n_B}\right] \times (f_i(T) + \bar{f}_i(T)) + \frac{8}{45}\pi^2 T^4 \tag{6}$$

$$n_i = -\left.\frac{\partial \Omega_i}{\partial \mu_i}\right|_{T, n_B}. \tag{7}$$

Here $f_i(T)$ are the usual Fermi distribution functions. The total baryon number density is

$$n_B = \frac{n_u + n_d + n_s}{3}. \tag{8}$$

Assuming that the neutrinos leave freely the beta equilibrium condition gives

$$\mu_s = \mu_d \quad \text{and} \quad \mu_s = \mu_u + \mu_e, \tag{9}$$

and the charge neutrality condition implies

$$2n_u - n_d - n_s - 3n_e = 0. \tag{10}$$

The parameters (C, m_{s0}) are constrained by requiring that for a given T,

(a) $E/n_B < 930$ MeV for SQM,
(b) $E/n_B > 940$ MeV for two flavor quark matter.

In fact Benvenuto and Lugones [15] have calculated the stability window for the SQM. They found that the range for C is from 70 to 110 MeV/fm^3 and for m_{s0} from 50 to 180 MeV. For given values of n_B, T, C and m_{s0} the above equations can be solved self-consistently to give the values of μ_s, μ_d, μ_u, μ_e, P and E.

3. Rederivation of the bulk viscosity

The bulk viscosity of SQM was earlier studied by Wang and Lu [8] and Sawyer [9] in the linear region where $\delta\mu \ll T$ and as a result the net reaction rate is proportional to $\delta\mu$. Madsen [11] and Goyal *et al* [12] included the nonlinear terms in the derivation of the bulk viscosity. In the above analysis the mass of the strange quark was taken to be finite but the u, d quarks were taken massless. However, in the DDQM model the u, d quark masses can also become appreciable at low densities. We follow the approach of Madsen and Goyal *et al* for the derivation of viscosity except for the modification due to the finite (u, d) quark masses. Assuming a periodic fluctuation in the specific volume v, of the star according to the relation

$$v(t) = v_0 + \Delta v \sin(2\pi t/\tau) = v_0 + \delta v(t), \tag{11}$$

where v_0 is the equilibrium volume and Δv is the amplitude of perturbation, the mean dissipation rate of energy per unit mass can be expressed as

$$\left\langle \frac{\mathrm{d}W}{\mathrm{d}t} \right\rangle = -\frac{1}{\tau}\int_0^\tau P(t)\left(\frac{\mathrm{d}v}{\mathrm{d}t}\right)\mathrm{d}t. \tag{12}$$

The change in pressure arises due to both change in volume and concentration of various species of particles. Thus we can write

$$P(t) = p_0 + \left(\frac{\partial P}{\partial v}\right)_0 \delta v + \left(\frac{\partial P}{\partial n_d}\right)_0 \partial n_d + \left(\frac{\partial P}{\partial n_s}\right)_0 \delta n_s, \tag{13}$$

where

$$\partial n_d = -\partial n_s = \int_0^t \frac{\mathrm{d}n_d}{\mathrm{d}t}\mathrm{d}t. \tag{14}$$

The bulk viscosity (ζ) is related to the mean dissipation rate of energy by

$$\zeta = \frac{2\langle\frac{\mathrm{d}W}{\mathrm{d}t}\rangle}{v_0}\left(\frac{v_0}{\Delta v}\right)^2\left(\frac{\tau}{2\pi}\right)^2. \tag{15}$$

Substituting for $P(t)$ from (13) into (12) and using the thermodynamical relation

$$\left(\frac{\partial P}{\partial n_i}\right)_0 = -\left(\frac{\partial \mu_i}{\partial v}\right)_0, \tag{16}$$

we get

$$\left\langle \frac{\mathrm{d}W}{\mathrm{d}t} \right\rangle = \frac{1}{\tau}\int_0^\tau \delta\tau \sum_i \left(\frac{\partial \mu_i}{\partial v}\right)\left(\int_0^t \frac{\mathrm{d}n_i}{\mathrm{d}t}\mathrm{d}t\right). \tag{17}$$

Physically a change in the specific volume leads to a small change in the chemical potentials from their equilibrium values, which is turn leads to small changes in the reaction rates of the two processes: $u + d \longleftrightarrow u + s$. This leads to change in the concentration of quark species via the relation

$$\delta\mu(t) = \left(\frac{\partial(\delta\mu)}{\partial v}\right)_0 \delta v + \sum_i \frac{\partial(\delta\mu)}{\delta n_i}\delta n_i. \tag{18}$$

The quantities $(\partial\delta\mu/\partial v)_0$ and $(\partial\delta\mu/\partial n_i)_0$ can be found from the equilibrium number densities n_i per unit mass and knowing the rates of various reactions, bulk viscosity can now be determined.

Contribution of the process $u + s \longleftrightarrow d + u$

To obtain the expression for bulk viscosity from the above process, we need the net transition rate per unit volume of strange quark to down quark, generated by the chemical potential difference

$$\delta\mu = \mu_s - \mu_d. \tag{19}$$

In the standard model the rate per unit volume is given by

$$\Gamma_{s\to d} = \int \left[\prod_i \frac{\mathrm{d}^3 p_i}{2E_i(2\pi)^3}\right] (2\pi)^4 \delta^4(p_u + p_s - p_d - p_{u'})|M|^2 \\ f_u f_s (1 - f_d)(1 - f_{u'}), \tag{20}$$

where $i = u,\ s,\ d,\ u'$ and $|M|^2$ is the matrix elements squared and summed over initial and final spins and colour degrees of freedom. f_i's are the Fermi distribution functions for various species. The rate for inverse process is obtained by interchanging momenta and chemical potentials appropriately. The net rate of conversion of s quark to d quark is then obtained by taking the difference of the two rates:

$$\frac{\mathrm{d}n_s}{\mathrm{d}t} = \Gamma_{s\to d} - \Gamma_{d\to s} = 2\delta\mu(\partial\Gamma/\partial\mu). \tag{21}$$

To simplify the integration procedure we assume that the quarks are highly degenerate and at low temperatures their momenta can be replaced by their respective Fermi momenta. This leads to the following expression for the net reaction rate:

$$\frac{\mathrm{d}n_d}{\mathrm{d}t} = \frac{3}{\pi^3} G_F^2 \sin^2\theta \cos^2\theta_c J T^2 \left(1 + \frac{\delta\mu^2}{4\pi^2 T^2}\right)\delta\mu, \tag{22}$$

where J is given in [7]. For the sake of completeness, we reproduce the rather complicated expression for the angular integral J in the Appendix.

Finally, (19) for $\delta\mu$ can be rewritten as

$$\frac{\partial\delta\mu}{\partial t} = \frac{\omega}{\pi} B \cdot \left(\frac{\Delta v}{v_0}\right) \cos\omega t - CAf(\delta\mu(t)), \tag{23}$$

where

$$A = \frac{3}{\pi^3} G_F^2 \cos^2\theta_c \sin^2\theta_c J T^2, \tag{24}$$

$$B = (m_s^2 - m_d^2)/3\mu_d \tag{25}$$

$$C = \frac{\pi^2}{3\mu_d} \cdot \frac{(\mu_s^2 - m_s^2)^{1/2} + (\mu_d^2 - m_d^2)^{1/2}}{(\mu_s^2 - m_s^2)^{1/2}(\mu_d^2 - m_d^2)^{1/2}}. \tag{26}$$

Note that for $m_d = 0$, these expressions for B and C go over to the expressions given in [12]. The bulk viscosity can now be calculated numerically by using eqs (15)–(26). For small perturbations such that $\delta\mu \ll 2\pi T$, the rate given by (22) is linear in $\delta\mu$ and the bulk visocity can be found analytically by using eqs (15)–(26). We find

$$\zeta = \frac{AB^2}{w^2 + (AC)^2}\left[1 - \frac{\omega AC}{\pi} \cdot \frac{1 - \mathrm{e}^{-2\pi AC/\omega}}{(AC)^2 + \omega^2}\right]. \tag{27}$$

However for small T the nonlinear terms in $\mathrm{d}n_d/\mathrm{d}t$ have to be taken into account and the bulk viscosity can be calculated only numerically

Contribution of the process $s \to u + e + \bar{\nu}_c$

The net transition rate of the conversions of $s \to u$ in the above process is generated by the chemical potential difference

$$\delta\mu = \mu_s - \mu_u - \mu_e, \tag{28}$$

and is given by [16,17]

$$\Gamma_{s\to u} = 24G_F^2 \sin^2\theta_c \int \left[\prod_i \frac{\mathrm{d}^3 p_i}{E_i}\right] \delta^4(p_s - p_u - p_e - p_\nu) \\ \cdot p_s \cdot p_\nu p_u \cdot p_e f_s(1 - f_u)(1 - f_e). \tag{29}$$

For low temperatures and degenerate quarks, the integrations in eq. (29) can be performed and we get

$$\Gamma_{s\to u} = \frac{3G_F^2 \sin^2\theta_c}{2\pi^5}\mu_S(m_s^2 - m_u^2 - m_e^2)T^5 \\ \cdot \int_0^\infty \frac{\mathrm{d}x \cdot x^2 \left\{\pi^2 + \left(x - \frac{\delta\mu}{T}\right)^2\right\}}{1 + \mathrm{e}^{-\delta\mu/T + x}}. \tag{30}$$

The net rate of conversion of s to u quark when $\delta\mu/T \ll 1$ is

$$\frac{\mathrm{d}n_s}{\mathrm{d}t} = \frac{17}{40\pi}G_F^2 \sin^2\theta_c \mu_s(m_s^2 - m_u^2 - m_e^2)T^4\delta\mu \equiv A'\delta\mu. \tag{31}$$

This leads to the same equation for viscosity as (27) with A replaced by A'.

4. Results and discussion

In §3 we have derived analytic expression for the bulk viscosity of quark matter in the linear case: $\delta\mu/T \ll 1$ (eq. (27) for non-leptonic process and the corresponding equation with A replaced by A' for the leptonic process.) For a given baryon density and temperature, the chemical potentials are evaluated by demanding charge neutrality and β-equilibrium condition. For the general case bulk viscosity has to be computed numerically. For this purpose we compute $\delta\mu$ as a function of T by solving eq. (23) numerically, using eqs (15)–(26). The bulk viscosity can now be determined and by solving numerically the integral involved in (17).

In figure 1 we have plotted viscosity vs. relative perturbation $\Delta v/v$. As discussed in the introduction, we have taken quark masses to be density dependent. We have studied the bulk viscosity for two sets of parameters: (i) $C = 75$ MeV fm^{-3}, $m_{s0} = 140$ MeV and (ii) $C = 95$ MeV fm^{-3}, $m_{s0} = 55$ MeV, $n_B = 0.4$ fm^{-3} and $\tau = 10^{-3}$. We see that at low perturbations the bulk viscosity of the SQM is an order of magnitude higher for the former choice of C and m_{s0}. However for large perturbation and small temperatures ($T < 10^{-2}$ MeV) differences becomes even two orders of magnitude.

In figure 2 we have plotted the bulk viscosity vs. relative perturbation for $c = 75$ MeV fm^{-3} and $m_s = 140$ MeV at $n_B = 1.36$ fm^{-3} and $\tau = 10^{-3}$ s, and compared our results with those of Goyal *et al* [12] for $m_s = 80$ MeV, $\alpha_c = 0.1$ and $\tau = 10^{-3}$ s. We notice that at low temperatures and high relative perturbations our results are higher by 2 to 3 orders of magnitude, while at low perturbations the enhancement is by 1–2 orders of magnitude.

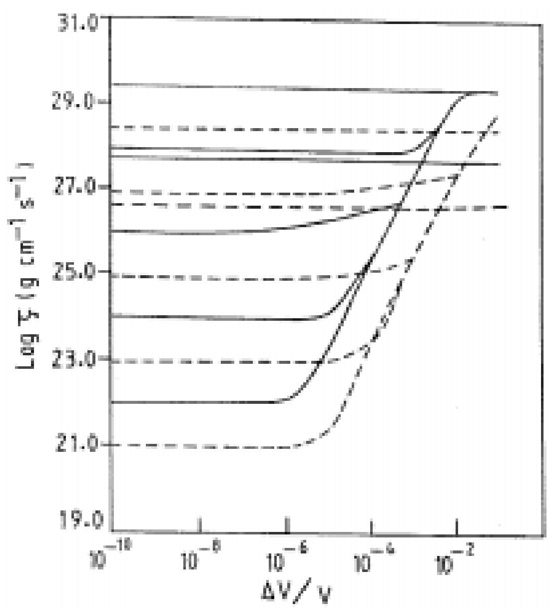

Figure 1. Bulk viscosity as a function of relative perturbation $(\Delta v/v)$ for $n_B = 0.4$ fm^{-3}, $\tau = 10^{-3}$ s for temperatures $10^{-5}, 10^{-4}, 10^{-3}, 1, 10^{-2}, 10^{-1}$ from bottom to top. Solid curves are for $c = 75.0$ MeV fm^{-3}, $m_{s0} = 14.0$ MeV and dashed curves are for $c_s = 95$ and $m_{s0} = 55$.

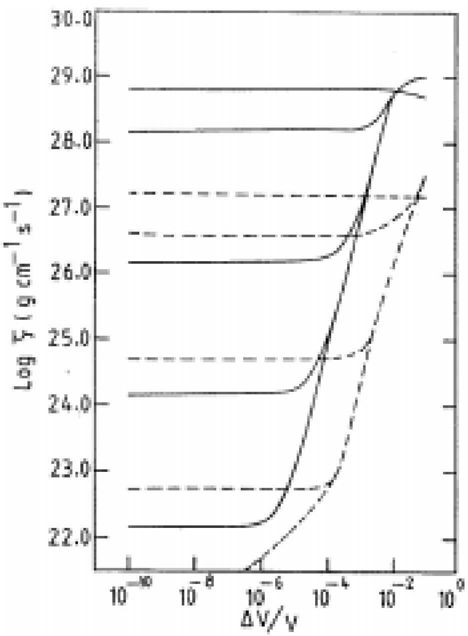

Figure 2. Graph of viscosity vs. $(\Delta v/v)$. Solid curves have the same meaning as in figure 1 for temperatures $10^{-5}, 10^{-4}, 10^{-3}, 10^{-2}$ and 10^{-1} MeV from bottom to top. The dashed lines are from Ashok Goyal *et al* [12] for $m_s = 80$, $\alpha_c = 0.1$ and $\tau = 10^{-3}$ for same temperatures from bottom to top.

In figure 3 we have shown the variation of viscosity as a function or baryon density at two different temperatures namely $T = 10^{-2}$ MeV and $T = 1.0$ MeV for the case when $\delta\mu$ can be taken to be small as compared to temperature. For comparison we have also plotted the bulk viscosity (dotted graph) calculated by earlier authors [12] which was obtained by taking into account the effect of temperature and quark-gluon coupling perturbatively to first order in the chemical composition of the quark matter. At low temperatures the bulk viscosity increases with baryon density where as reverse is the case when temperature rises to about 1 MeV and above. Furthermore, the effect of the density-dependence of quark masses is to increase the viscosity. In fact as the reaction rate increases with temperature and density it approaches the rate of density change at low temperatures and the dissipation increases resulting in the increase of viscosity with temperature and density. At higher temperatures the reaction rate exceeds the rate of density change and dissipation decreases and a further increase in temperature and/or density results in lowering of viscosity.

The contribution of β-decay process (2) to the bulk viscosity is given in (31) and depends strongly on temperature and difference in masses of u, d and s quarks as compared to non-leptonic processes (22). In figure 4, we have plotted viscosity for both leptonic (solid lines) and non-leptonic processes (dashed lines) vs. the baryon density for various temperatures. We observe that at lower temperatures the contribution from leptonic processes is negligible as compared to that from non-leptonic processes. But as the temperature increases the contribution from the two processes becomes comparable. Lattimer *et al* [6] and Gupta *et al* [18] have obtained a similar behaviour for bulk viscosity of normal nuclear matter

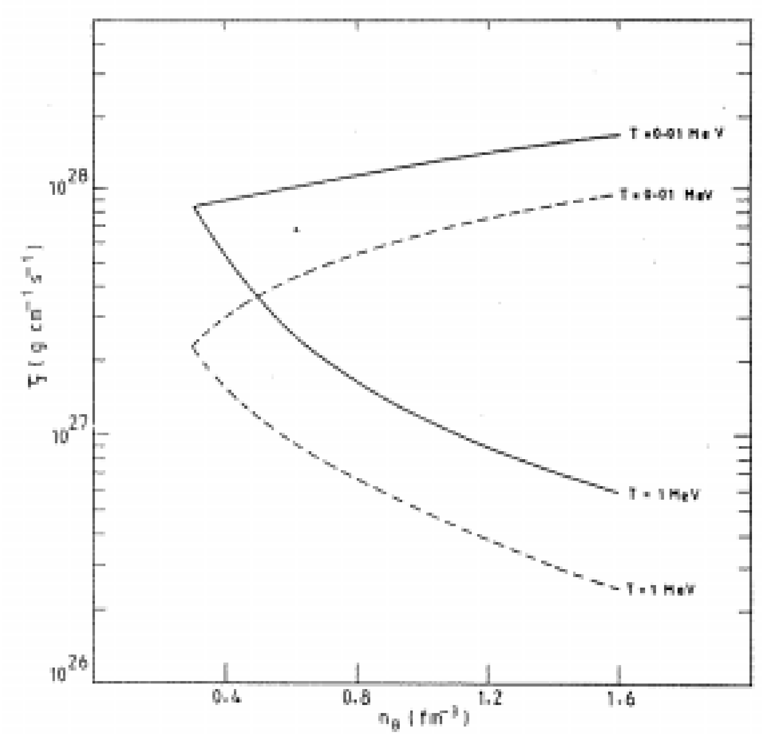

Figure 3. Bulk viscosity as a function of baryon density at $T = 10^{-2}$ and $T = 1$ MeV. Solid curves are for $c = 75, m_{s0} = 140, \tau = 10^{-3}$ s. The dashed curves are from ref. [12].

for direct URCA processes $n \rightarrow p + e + \bar{\nu}_c$ and $p + e \rightarrow n + \nu_e$ for proton fraction exceeding a certain critical value.

As discussed in the introduction the motivation for studying the bulk viscosity of SQM is to estimate the damping time of vibration of a strange star or a neutral star with quark core. A rough estimate of the damping time can be made in the manner of Sawyer [9] and used by Madsen [11] by estimating the kinetic energy per unit volume of the star and obtaining the damping for the mean rate of dissipation. For a typical star of 1.4$M_\odot$ mass, the damping time is given by

$$\tau_D = 30^{-1} \rho R^2 \zeta^{-1},$$

where ρ and R are corresponding density and radius which have been obtained by us by studying the mass-radius relationship in DDQM model (to appear in the July 1, 2000 Issue of *Astrophys. J.*).

Here the oscillation time is taken as 10^{-3} s, which is typical for the fundamental mode. In figure 5 we have plotted the damping time vs. temperature. As mentioned earlier (see figure 2), the bulk viscosity in DDQM model is on the average higher by about two orders of magnitude, the damping time will be lower by the same order implying that the radial pulsations will be damped more efficiently in the DDQM model.

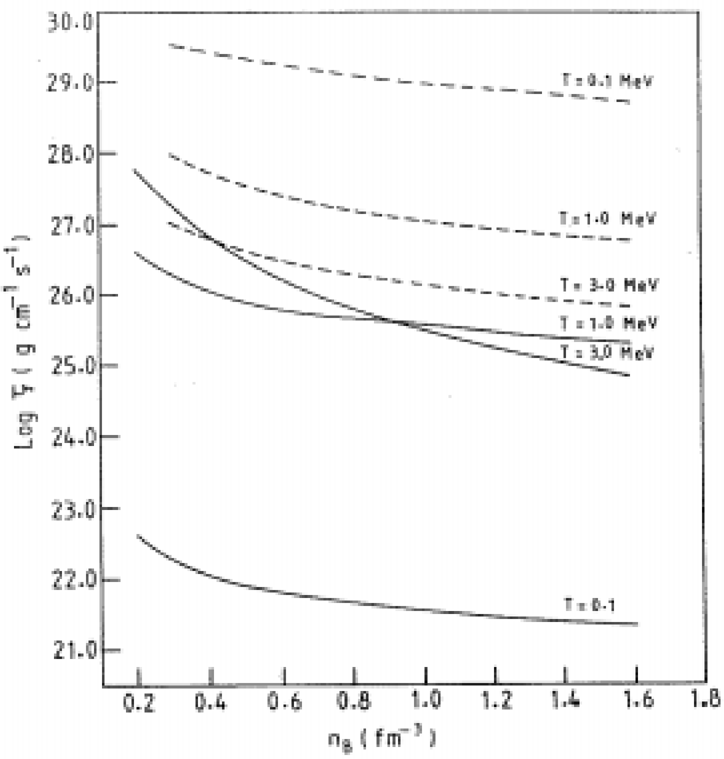

Figure 4. Bulk viscosity as a function of baryon density for $c = 75.0, m_{s0} = 140.0, \tau = 10^{-3}$ for leptonic (solid) and non-leptonic (dashed) processes.

In figure 6, we have studied the variation of bulk viscosity with temperature at $n_B = 0.4$ and 1 fm^{-3} and for $C = 75$ MeV fm^{-3} and $m_{s0} = 140$ MeV. We observe that initially the bulk viscosity increases reaching a maximum at temperature 0.75 MeV for $n_B = 0.4$ and 0.625 MeV for $n_B = 1.0$ fm^{-3}, and finally the bulk viscosity falls steeply with further increase of temperature. We see that as the star cools the viscosity increases rapidly; consequently the damping time decreases making the star more stable. However as the star cools further, the viscosity decreases and the star becomes unstable once again.

Appendix

Here we reproduce the expression for the angular integral part, J, from our earlier paper [7]:

$$J(E_1, E_2, E_3, E_4) = (1 + g_A)^2 J_{12} + (1 - g_A)^2 J_{23} + m_n m_p (g_A^2 - 1) J_{24},$$

where

$$J_{12} = (p_1 p_2 p_3 p_4/16\pi^3) \int \left[\prod_i \mathrm{d}\Omega_i\right] (\mathbf{p}_1 \cdot \mathbf{p}_2)(\mathbf{p}_3 \cdot \mathbf{p}_4)\delta(\vec{p}_1 + \vec{p}_2 - \vec{p}_3 - \vec{p}_4),$$

$$J_{23} = (p_1 p_2 p_3 p_4/16\pi^3) \int \left[\prod_i \mathrm{d}\Omega_i\right] (\mathbf{p}_1 \cdot \mathbf{p}_4)(\mathbf{p}_2 \cdot \mathbf{p}_3)\delta(\vec{p}_1 + \vec{p}_2 - \vec{p}_3 - \vec{p}_4),$$

$$J_{24} = (p_1 p_2 p_3 p_4/16\pi^3) \int \left[\prod_i \mathrm{d}\Omega_i\right] (\mathbf{p}_2 \cdot \mathbf{p}_4)\delta(\vec{p}_1 + \vec{p}_2 - \vec{p}_3 - \vec{p}_4).$$

These integrals have been evaluated exactly [7] and the results are

$$J_{12} = k_+^{12} k_+^{34} I_1(1,2;3,4) - (1/6)(k_+^{34} + k_+^{12}) I_2(1,2;3,4) + (1/20) I_3(1,2;3,4),$$

$$J_{23} = k_+^{23} k_+^{14} I_1(2,3;1,4) + (1/6)(k_-^{23} + k_+^{14}) I_2(2,3;1,4) - (1/20) I_3(2,3;1,4),$$

$$J_{24} = k_+^{24} I_1(2,4;1,3) - (1/6) I_2(2,4;1,3),$$

where

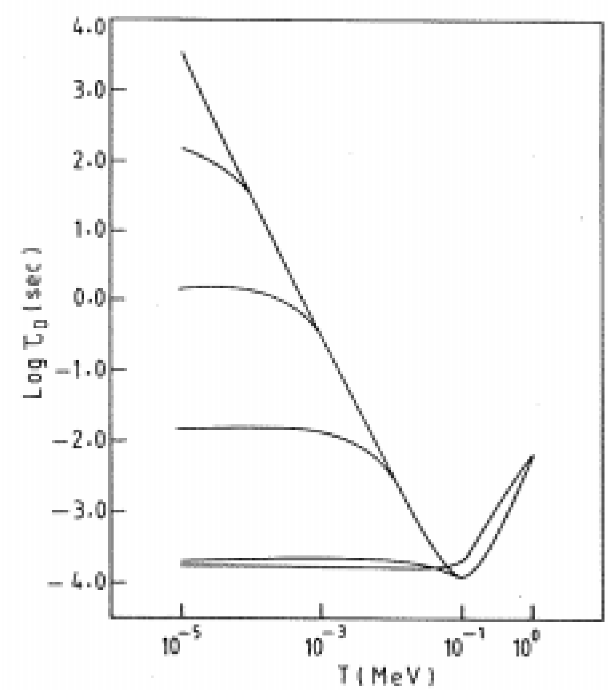

Figure 5. Plot of damping time vs. temperature for a 1.4 $M_\odot$ strange star for $c = 75$ MeV fm^{-3}, $m_{s0} = 140.0$ MeV, $\tau = 10^{-3}$ s. Curves from bottom to top correspond to perturbation amplitudes $(\Delta v/v) = 10^{-1}, 10^{-2}$, ending at 10^{-6}.

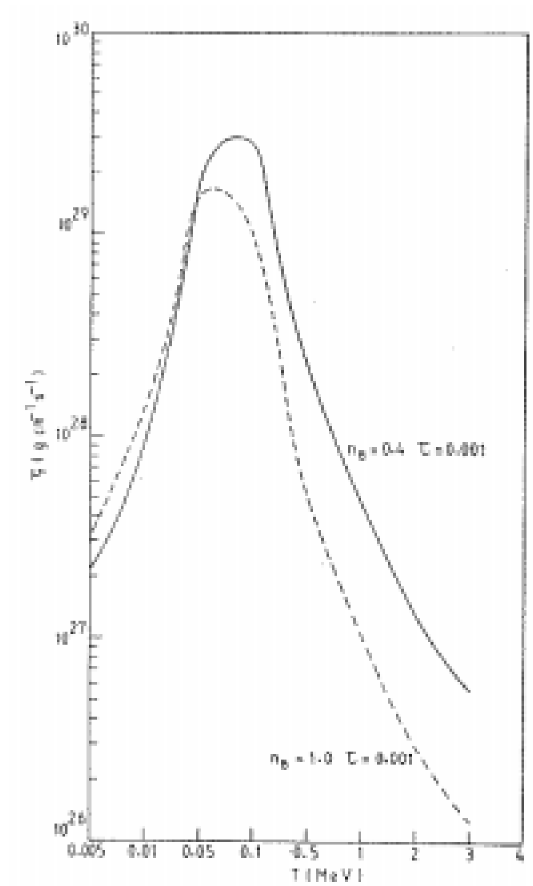

Figure 6. Variation of bulk viscosity with temperature for $c = 75$, $m_{s0} = 140, \tau = 10^{-3}$. The solid curve is for $n_b = 0.4$ fm^{-3} and the dashed curve is for $n_b = 1.0$ fm^{-3}.

$$
\begin{aligned}
k_{\pm}^{ij} &= E_i E_j \pm 0.5(p_i^2 + p_j^2), \\
I_n(i,j;k,l) &= (P_{ij}^{2n-1} - p_{ij}^{2n-1})\theta(P_{kl} - P_{ij})\theta(p_{ij} - p_{kl}) \\
&\quad + (P_{ij}^{2n-1} - p_{kl}^{2n-1})\theta(P_{kl} - P_{ij})\theta(p_{kl} - p_{ij})\theta(P_{ij} - p_{kl}) \\
&\quad + (P_{kl}^{2n-1} - p_{kl}^{2n-1})\theta(P_{ij} - P_{kl})\theta(p_{kl} - p_{ij}) \\
&\quad + (P_{kl}^{2n-1} - p_{ij}^{2n-1})\theta(P_{ij} - P_{kl})\theta(p_{ij} - p_{kl})\theta(P_{kl} - p_{ij})
\end{aligned}
$$

and

$$P_{ij} = p_1 + p_j, \quad p_{ij} = |p_i - p_j|.$$

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