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Collapse of a charged radiating star with shear

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Abstract. The junction conditions for a magnetohydrodynamic fluid sphere undergoing dissipative gravitational collapse in the form of a radial heat flux with shear are obtained. These conditions extend particular results of earlier treatments. We demonstrate that the pressure is proportional to the magnitude of the heat flux as is the case in shear-free models. However in our case the gravitational potentials must be solutions of the Einstein–Maxwell system of equations. The mass function $m(v)$ is increased by a factor related to the charge Q of the radiating star. Physical quantities relating to the local conservation of momentum and surface redshift are obtained.

Keywords. Gravitational collapse; radiating stars.

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1. Introduction

The problem of gravitational collapse has many interesting applications in astrophysics where the formation of compact stellar objects such as white dwarfs and neutron stars are usually preceded by a period of radiative collapse. The problem of gravitational collapse was first investigated by Oppenheimer and Snyder [1] in which they investigated the contraction of a spherically symmetric dust cloud. Here the exterior spacetime is described by the exterior Schwarzschild solution and the interior spacetime is represented by a Friedmann-like solution. Vaidya [2] derived the line element which describes the exterior gravitational field of a spherically symmetric radiating mass. It then became possible to model the interior of radiating stars by matching such solutions to the exterior spacetime (de Oliveira *et al* [3–5], Kramer [6] and Govender *et al* [7]). The junction conditions for a spherically symmetric radiating star was completely derived by Santos [8]. The crucial result that follows from Santos is that the pressure on the boundary of a radiating sphere is nonvanishing in general. These results have subsequently been generalized to include the electromagnetic field for the shear-free case (de Oliveira and Santos [9] and Tikekar and Patel [10]).

The aim of this paper is to generalize these results to include the effects of shear. In §2 we present the relevant background material and the line elements for the interior and exterior spacetimes. The junction conditions are derived in detail in §3. We show that our results reduce to that of Santos in the relevant limit. A physical interpretation of the main result obtained is given in terms of conservation of momentum and an expression is found for the surface redshift. We briefly discuss the significance of our results in §4 and we consider some general aspects of the thermodynamics of our model. It is well known that the standard Eckart formalism of thermodynamics is noncausal and the theory predicts unstable equilibrium states. We employ a heat transport equation of Maxwell–Cattaneo form which respects causality as a vehicle to obtain the temperature profile. Recently there has been a huge effort in constructing stellar models of radiative collapse with causal heat flux (Di Prisco *et al* [11], Govender *et al* [12], Herrera and Santos [13] and Martínez [14]).

On physical grounds it may be argued that an additional spacetime region is required for the presence of the radiation field (generated by the heat flux) between the interior of the star and the Vaidya exterior. This approach would need the application of the junction conditions at two surfaces with qualitatively different characteristics. This is a more complex and difficult problem which is outside the scope of this paper. For our purposes we are taking the point of view that there are only two regions: the interior of the star and the Vaidya exterior. The outgoing heat flux is converted to null radiation at the boundary of the star and, consequently, matching is required at only one surface. We should add that this approach is the one followed by other treatments in gravitational collapse for a magnetohydrodynamic fluid with outgoing dissipation in the form of radial heat flow. For details of particular models the reader is referred to the works of de Oliveira and Santos [9] and Tikekar and Patel [10], amongst others.

2. Interior and exterior spacetimes

The boundary of a collapsing star divides spacetime into two distinct regions, the interior region and the exterior region. The interior spacetime is described by the most general spherically symmetric line element

$$\mathrm{d}s^2 = -A^2\mathrm{d}t^2 + B^2\mathrm{d}r^2 + Y^2(\mathrm{d}\theta^2 + \sin^2\theta\mathrm{d}\phi^2), \tag{1}$$

where A, B and Y are functions of the coordinates t and r. The interior matter distribution is given by

$$T_{ab} = (\rho + p)u_a u_b + p g_{ab} + q_a u_b + q_b u_a + E_{ab}, \tag{2}$$

where ρ denotes the energy density, p is the isotropic pressure, $u^a = (1/A)\delta^a_0$ is a timelike four-velocity vector, q_a is a radial heat flux vector and E_{ab} represents the electromagnetic contribution to the energy–momentum tensor. The heat flow vector satisfies the condition $q^a u_a = 0$ relative to the fluid four-velocity u^a. The energy–momentum tensor for the electromagnetic field is

$$E_{ab} = F_a^c F_{bc} - \tfrac{1}{4} g_{ab} F^{cd} F_{cd}$$

and we express the electromagnetic field tensor as

$$F_{ab} = \phi_{b;a} - \phi_{a;b},$$

where the scalar ϕ is the four-potential. Maxwell's equations, governing the behaviour of the electromagnetic field, are

$$F^{ab}_{;b} = J^a, \tag{3}$$
$$F_{[ab;c]} = 0. \tag{4}$$

In the above $J^a = \mu u^a$ is the four-current density and μ is the proper charge density.

Choosing the four-potential as

$$\phi_a = [\Phi(t,r), 0, 0, 0]$$

which is a simple form consistent with spherical symmetry, we obtain

$$F_{01} = -F_{10} = -\Phi_r$$

as the nonzero components of F_{ab}. Then Maxwell's equation (3) yields the following two equations

$$\Phi_{rr} + \left(-\frac{A_r}{A} - \frac{B_r}{B} + 2\frac{Y_r}{Y}\right)\Phi_r = \mu A B^2, \tag{5}$$

$$\left(\frac{1}{A^2B^2}\Phi_r\right)_t + \frac{1}{A^2B^2}\left(\frac{A_t}{A} + \frac{B_t}{B}\right)\Phi_r + \frac{2}{A^2B^2}\frac{Y_t}{Y}\Phi_r = 0. \tag{6}$$

Equation (4) is identically satisfied. On integration of (5) and (6) we obtain

$$\Phi_r = \frac{AB}{Y^2}l,$$
$$l(r) = \int_0^r \mu Y^2 B \mathrm{d}r,$$

where $l(r)$ represents the charge distribution within the radius r. These expressions are similar to those of de Oliveira and Santos [9] for the shear-free case. The electric field intensity is given by

$$E = \frac{l}{Y^2}$$

which is time-dependent in the interior of the star. This is generated by the four-potential $\phi_a = [\Phi(t,r), 0, 0, 0]$ which is clearly dependent on time.

The coupled Einstein–Maxwell field equations for the interior matter distribution become

$$\rho + \frac{1}{2}\frac{l^2}{Y^4} = \frac{2}{A^2}\frac{B_t}{B}\frac{Y_t}{Y} + \frac{1}{Y^2} + \frac{1}{A^2}\frac{{Y_t}^2}{Y^2} - \frac{1}{B^2}\left(2\frac{Y_{rr}}{Y} + \frac{{Y_r}^2}{Y^2} - 2\frac{B_r}{B}\frac{Y_r}{Y}\right), \tag{7}$$

$$p - \frac{1}{2}\frac{l^2}{Y^4} = \frac{1}{A^2}\left(-2\frac{Y_{tt}}{Y} - \frac{{Y_t}^2}{Y^2} + 2\frac{A_t}{A}\frac{Y_t}{Y}\right) + \frac{1}{B^2}\left(\frac{{Y_r}^2}{Y^2} + 2\frac{A_r}{A}\frac{Y_r}{Y}\right) - \frac{1}{Y^2}, \tag{8}$$

$$p + \frac{1}{2}\frac{l^2}{Y^4} = -\frac{1}{A^2}\left(\frac{B_{tt}}{B} - \frac{A_t}{A}\frac{B_t}{B} + \frac{B_t}{B}\frac{Y_t}{Y} - \frac{A_t}{A}\frac{Y_t}{Y} + \frac{Y_{tt}}{Y}\right) + \frac{1}{B^2}\left(\frac{A_{rr}}{A} - \frac{A_r}{A}\frac{B_r}{B} + \frac{A_r}{A}\frac{Y_r}{Y} - \frac{B_r}{B}\frac{Y_r}{Y} + \frac{Y_{rr}}{Y}\right), \tag{9}$$

$$q = -\frac{2}{AB}\left(-\frac{Y_{rt}}{Y} + \frac{B_t}{B}\frac{Y_r}{Y} + \frac{A_r}{A}\frac{Y_t}{Y}\right), \tag{10}$$

for the line element (1). The heat flux has the form $q_a = q(t,r)n_a$ where n_a is a unit radial vector, so that q is a covariant scalar measure of the heat flux ($q^2 = q^a q_a$). The field equations (7)–(10) describe the gravitational interaction of a shearing matter distribution with heat flux in the presence of an electromagnetic field $(l \neq 0)$. On equating (8) and (9) we generate the condition of pressure isotropy

$$\frac{1}{B^2}\left[\frac{Y_{rr}}{Y} + \frac{A_{rr}}{A} - \frac{A_r}{A}\frac{B_r}{B} - \frac{Y_r}{Y}\left(\frac{Y_r}{Y} + \frac{A_r}{A} + \frac{B_r}{B}\right)\right] + \frac{1}{Y^2} - \frac{l^2}{Y^4} + \frac{1}{A^2}\left[\frac{Y_{tt}}{Y} - \frac{B_{tt}}{B} + \frac{A_t}{A}\frac{B_t}{B} + \frac{Y_t}{Y}\left(\frac{Y_t}{Y} - \frac{A_t}{A} - \frac{B_t}{B}\right)\right] = 0 \tag{11}$$

which contains the metric functions A, B and Y.

The exterior gravitational field of a charged, radiating star is described by

$$\mathrm{d}s^2 = -\left(1 - \frac{2m(v)}{\mathsf{r}} + \frac{Q^2}{\mathsf{r}^2}\right)\mathrm{d}v^2 - 2\mathrm{d}v\mathrm{d}\mathsf{r} + \mathsf{r}^2\left(\mathrm{d}\theta^2 + \sin^2\theta\mathrm{d}\phi^2\right) \tag{12}$$

in coordinates $(v, \mathsf{r}, \theta, \phi)$. The mass function $m(v)$ is assumed to be positive and Q represents the total charge as viewed by an observer at infinity. The line element (12) is referred to as the Vaidya–Reissner–Nordström spacetime.

Note that the treatment of Lindquist *et al* [15] demonstrates that geodesics are incomplete in this spacetime. An extension of the manifold is necessary to investigate the final stages of gravitational collapse. Fayos *et al* [16] have studied this problem extensively.

3. Junction conditions

The boundary of a radiating star divides spacetime into two distinct regions, the interior spacetime ($\mathcal{M}^-$) and the exterior spacetime ($\mathcal{M}^+$). Each of these regions is described by a distinct smooth four-dimensional manifold containing Σ as its boundary, a timelike three-space. We assume that Σ is endowed with an intrinsic metric $g_{\alpha\beta}$ so that

$$\mathrm{d}s^2_\Sigma = g_{\alpha\beta}\mathrm{d}\xi^\alpha\mathrm{d}\xi^\beta.$$

It follows that the intrinsic coordinates to Σ are given by ξ^α where $\alpha = 1, 2, 3$. The line elements in the regions $\mathcal{M}^\pm$ assume the form

$$\mathrm{d}s^2_\pm = g_{ab}\mathrm{d}\mathcal{X}^a_\pm\mathrm{d}\mathcal{X}^b_\pm.$$

The coordinates in $\mathcal{M}^{\pm}$ are $\mathcal{X}^a_{\pm}$ where $a = 0,1,2,3$. When approaching Σ from the exterior $\mathcal{M}^+$ or the interior spacetime $\mathcal{M}^-$ we demand

$$(\mathrm{d}s^2_-)_\Sigma = (\mathrm{d}s^2_+)_\Sigma = \mathrm{d}s^2_\Sigma, \tag{13}$$

where $(\ )_\Sigma$ represents the value of $(\ )$ on Σ. Consequently the coordinates of Σ in $\mathcal{M}^{\pm}$ are given by $\mathcal{X}^a_{\pm} = \mathcal{X}^a_{\pm}(\xi^\alpha)$. Continuity of the intrinsic metrics across Σ generates the first junction condition. The second junction condition is obtained by requiring continuity of the extrinsic curvature of Σ across the boundary. This yields

$$K^+_{\alpha\beta} = K^-_{\alpha\beta}, \tag{14}$$

where

$$K^{\pm}_{\alpha\beta} \equiv -n^{\pm}_a \frac{\partial^2 \mathcal{X}^a_{\pm}}{\partial\xi^\alpha \partial\xi^\beta} - n^{\pm}_a \Gamma^a_{cd} \frac{\partial \mathcal{X}^c_{\pm}}{\partial\xi^\alpha} \frac{\partial \mathcal{X}^d_{\pm}}{\partial\xi^\beta} \tag{15}$$

and $n^{\pm}_a(\mathcal{X}^b_{\pm})$ are the components of the vector normal to Σ. A comprehensive and complete treatment of junction conditions for boundary surfaces and surface layers in general relativity is provided by Lake [17].

The intrinsic metric to Σ is given by

$$\mathrm{d}s^2_\Sigma = -\mathrm{d}\eta^2 + \mathcal{Y}^2(\mathrm{d}\theta^2 + \sin^2\theta \mathrm{d}\phi^2) \tag{16}$$

with coordinates $\xi^\alpha = (\eta, \theta, \phi)$ and $\mathcal{Y} = \mathcal{Y}(\eta)$. Note that the time coordinate η is defined only on the surface Σ. We use comoving coordinates and we take the interior spacetime $\mathcal{M}^-$ to be described by the line element (1). The boundary of the interior matter distribution is given by

$$f(r,t) = r - r_\Sigma = 0,$$

where r_Σ is a constant. The vector with components $\partial f/\partial \mathcal{X}^a_-$ is orthogonal to Σ. Hence the unit vector normal to Σ must be of the form

$$n^-_a = [0, B(r_\Sigma, t), 0, 0]. \tag{17}$$

The first junction condition (13), for the metrics (16) and (1), yields the restrictions

$$A(r_\Sigma, t)\dot{t} = 1, \tag{18}$$

$$Y(r_\Sigma, t) = \mathcal{Y}(\eta), \tag{19}$$

where dot represent differentiation with respect to η. The extrinsic curvature $K^-_{\alpha\beta}$ can be calculated using (1), (15) and (17). The nonzero components are given by

$$K^-_{\eta\eta} = \left(-\frac{1}{B}\frac{A_r}{A}\right)_\Sigma, \tag{20}$$

$$K^-_{\theta\theta} = \left(\frac{YY_r}{B}\right)_\Sigma, \tag{21}$$

$$K^-_{\phi\phi} = \sin^2\theta\, K^-_{\theta\theta}, \tag{22}$$

valid on the surface Σ.

The defining equation for the surface Σ in $\mathcal{M}^+$ is given by

$$f(\mathsf{r}, v) = \mathsf{r} - \mathsf{r}_\Sigma(v) = 0.$$

It follows that the vector orthogonal to Σ is

$$\frac{\partial f}{\partial \mathcal{X}^a_+} = \left(-\frac{\mathrm{d}\mathsf{r}_\Sigma}{\mathrm{d}v}, 1, 0, 0\right).$$

Hence the unit normal to Σ can be cast into the following form

$$n^+_a = \left(1 - \frac{2m}{\mathsf{r}_\Sigma} + 2\frac{\mathrm{d}\mathsf{r}_\Sigma}{\mathrm{d}v} + \frac{Q^2}{{\mathsf{r}_\Sigma}^2}\right)^{-1/2} \left(-\frac{\mathrm{d}\mathsf{r}_\Sigma}{\mathrm{d}v}, 1, 0, 0\right). \tag{23}$$

For $\mathcal{M}^+$ the first junction condition (13), for the line elements (16) and (12), generates the equations

$$\mathsf{r}_\Sigma(v) = \mathcal{Y}(\eta), \tag{24}$$

$$\left(1 - \frac{2m}{\mathsf{r}} + 2\frac{\mathrm{d}\mathsf{r}}{\mathrm{d}v} + \frac{Q^2}{\mathsf{r}^2}\right)_\Sigma = \left(\frac{1}{\dot{v}^2}\right)_\Sigma. \tag{25}$$

With the use of (25) we can rewrite the unit normal vector (23) as

$$n^+_a = (-\dot{\mathsf{r}}, \dot{v}, 0, 0). \tag{26}$$

After a lengthy calculation the nonvanishing components of the extrinsic curvature tensor for the exterior spacetime assume the following form

$$K^+_{\eta\eta} = \left[\frac{\ddot{v}}{\dot{v}} - \dot{v}\frac{m}{\mathsf{r}^2} + \frac{Q^2}{\mathsf{r}^3}\dot{v}\right]_\Sigma, \tag{27}$$

$$K^+_{\theta\theta} = \left[\dot{v}\left(1 - \frac{2m}{\mathsf{r}} - \frac{Q^2}{\mathsf{r}^2}\right)\mathsf{r} + \mathsf{r}\dot{\mathsf{r}}\right]_\Sigma, \tag{28}$$

$$K^+_{\phi\phi} = \sin^2\theta\, K^+_{\theta\theta} \tag{29}$$

valid on the surface Σ.

The first junction condition (13) corresponds to eqs (18), (19), (24) and (25). Note that η was defined only as an intermediate variable. On eliminating η from these equations we find that the necessary and sufficient conditions on the spacetimes for the first junction condition (13) to be valid are that

$$A(r_\Sigma, t)\mathrm{d}t = \left(1 - \frac{2m}{\mathsf{r}_\Sigma} + 2\frac{\mathrm{d}\mathsf{r}_\Sigma}{\mathrm{d}v} + \frac{Q^2}{{\mathsf{r}_\Sigma}^2}\right)^{1/2}\mathrm{d}v, \tag{30}$$

$$Y(r_\Sigma, t) = \mathsf{r}_\Sigma(v) \tag{31}$$

as required.

By equating the appropriate extrinsic curvature components (20) and (27) we generate the second set of junction conditions (14). These are given by

$$\left(-\frac{1}{B}\frac{A_r}{A}\right)_\Sigma = \left[\frac{\ddot{v}}{\dot{v}} - \dot{v}\frac{m}{\mathsf{r}^2} + \frac{Q^2}{\mathsf{r}^3}\dot{v}\right]_\Sigma, \tag{32}$$

$$\left(\frac{YY_r}{B}\right)_\Sigma = \left[\dot{v}\left(1 - \frac{2m}{\mathsf{r}} - \frac{Q^2}{\mathsf{r}^2}\right)\mathsf{r} + \mathsf{r}\dot{\mathsf{r}}\right]_\Sigma. \tag{33}$$

An expression for the mass function in terms of the metric functions can be obtained after eliminating r, $\dot{\mathsf{r}}$ and $\dot{v}$. This eventually leads to

$$m(v) = \left[\frac{Y}{2}\left(1 + \frac{{Y_t}^2}{A^2} - \frac{{Y_r}^2}{B^2} + \frac{Q^2}{Y^2}\right)\right]_\Sigma. \tag{34}$$

We may interpret $m(v)$ as representing the total gravitational mass within the surface Σ. The expression (34) corresponds to the mass function of Cahill and McVittie [18] for spheres of radius r inside Σ. From (18) and (24) we can write

$$\dot{\mathsf{r}}_\Sigma = \left(\frac{Y_t}{A}\right)_\Sigma.$$

On using this expression for $\dot{\mathsf{r}}_\Sigma$ and substituting (34) in (33) we obtain

$$\dot{v}_\Sigma = \left(\frac{Y_t}{A} + \frac{Y_r}{B}\right)^{-1}_\Sigma. \tag{35}$$

If we now differentiate (35) with respect to η and make use of (18) we can write

$$\ddot{v}_\Sigma = \left[-\frac{1}{A}\left(\frac{Y_t}{A} + \frac{Y_r}{B}\right)^{-2}\left(\frac{Y_{rt}}{B} - \frac{B_tY_r}{B^2} - \frac{A_tY_t}{A^2} + \frac{Y_{tt}}{A}\right)\right]_\Sigma. \tag{36}$$

Then substituting (19), (24), (34), (35) and (36) into (32) leads to

$$\left(-\frac{1}{B}\frac{A_r}{A}\right)_\Sigma = \left[\left(-\frac{Y_{rt}}{B} + \frac{B_tY_r}{B^2} + \frac{A_tY_t}{A^2} - \frac{Y_{tt}}{A} - \frac{{Y_t}^2}{2AY}\right.\right.$$
$$\left.\left. + \frac{A}{2Y}\left(\frac{{Y_r}^2}{B^2} + \frac{Q^2}{Y^2} - 1\right)\right) \times \left(\frac{Y_t}{A} + \frac{Y_r}{B}\right)^{-1}\right]_\Sigma.$$

On multiplying the above equation by $((Y_t/A) + (Y_r/B))$ and simplifying we obtain the following result

$$(p)_\Sigma = (q)_\Sigma,$$

where we have utilized the field equations (8) and (10). Hence we have established that the necessary and sufficient conditions on the spacetimes for the second junction condition (14) to be valid are that

$$m(v) = \left[\frac{Y}{2}\left(1 + \frac{{Y_t}^2}{A^2} - \frac{{Y_r}^2}{B^2} + \frac{Q^2}{Y^2}\right)\right]_\Sigma , \tag{37}$$

$$(p)_\Sigma = (q)_\Sigma \tag{38}$$

as required.

The important result $(p)_\Sigma = (q)_\Sigma$, relating the isotropic pressure p to the heat flow q, was first established by Santos [8] for shear-free spacetimes. Our treatment involves the general spherically symmetric line element (nonzero shear) and a nonvanishing electromagnetic field. Even though the form of (37) is similar to the result of Santos, note that the charge Q contributes to the mass function $m(v)$. In addition the explicit form of the metric function B will be different as the spacetime is shearing – this requires an explicit solution of the Einstein–Maxwell system (7)–(10). The first attempt to generalize the above junction conditions to include shear for neutral matter was carried out by Glass [19]. In order to obtain a complete solution of radiative gravitational collapse in the present scheme one has to solve the pressure isotropy condition (11) together with the junction condition (38) for a particular choice of the line element (1). These equations with all the associated quantities need to be carefully checked for consistency in order to obtain a physically reasonable model. The paper by Chan [20] neglected to perform a proper analysis and the resulting model was unphysical as observed by Govinder *et al* [21]. As far as we are aware an explicit solution with shear and nonvanishing eletromagnetic field, has not been given before. This is an area for further investigation. In contrast a variety of shear-free models, with different forms of the interior spacetime (1) have been analysed.

The equations (30), (31), (37) and (38) are the most general matching conditions for the spherically symmetric spacetimes $\mathcal{M}^+$ and $\mathcal{M}^-$. The pressure p_Σ on the boundary can only be zero when q_Σ becomes zero. In this case there is no radial heat flow and the exterior spacetime consequently is not the Vaidya–Reissner–Nordström spacetime but is the exterior Reissner–Nordström spacetime. Note that the result (38) has been established in general for spherically symmetric, shearing spacetimes without assuming any particular forms for the metric functions. In the past some authors have erroneously assumed that for isotropic collapsing fluids with radial heat flow $p_\Sigma = 0$. For an example of a treatment that makes such an assumption see Glass [22]. We should point out that the junction conditions for shearing spacetimes for the special case with geodesic motion have been obtained by Tomimura and Nunes [23]. The shear-free case with a nonvanishing electromagnetic field was investigated by de Oliveira and Santos [13]. The charged shear-free case with an anisotropic energy–momentum tensor was considered by Tikekar and Patel [10]. We regain the Santos [8] junction conditions

$$A(r_\Sigma, t)\mathrm{d}t = \left(1 - \frac{2m}{\mathsf{r}_\Sigma} + 2\frac{\mathrm{d}\mathsf{r}_\Sigma}{\mathrm{d}v}\right)^{\frac{1}{2}} \mathrm{d}v,$$

$$r_\Sigma B(r_\Sigma, t) = \mathsf{r}_\Sigma(v),$$

$$m(v) = \left(\frac{r^3 B}{2A^2}{B_t}^2 - r^2 B_r - \frac{r^3}{2B}{B_r}^2\right)_\Sigma ,$$

$$(p)_\Sigma = (q)_\Sigma ,$$

for the shear-free line element

$$\mathrm{d}s^2 = -A^2\mathrm{d}t^2 + B^2\left[\mathrm{d}r^2 + r^2\left(\mathrm{d}\theta^2 + \sin^2\theta\mathrm{d}\phi^2\right)\right]$$

from our general equations (30), (31), (37) and (38). This special case is the one which has been most extensively applied in relativistic astrophysics (Bonnor *et al* [24]).

We can give a physical interpretation to (38) by considering the radial momentum flux across the boundary. As the expression (34) also gives the total energy for a sphere of radius r within Σ we can write $m(v) = m(t,r)$. On differentiating partially with respect to t we obtain

$$\left(\frac{\partial m}{\partial t}\right)_\Sigma = \left[Y_t\left(\frac{Y_{tt}Y}{A^2} + \frac{{Y_t}^2}{2A^2} - \frac{{Y_r}^2}{2B^2} - \frac{A_tY_tY}{A^3} - \frac{Q^2}{2Y^2} + \frac{1}{2}\right)\right.$$
$$\left. - \frac{Y_rY_{rt}Y}{B^2} + \frac{B_t{Y_r}^2Y}{B^3}\right]_\Sigma .$$

Then on using the field equations (8) and (10) we obtain

$$\left(\frac{\partial m}{\partial t}\right)_\Sigma = \left[-\frac{AY^2}{2}\left(\frac{Y_t}{A} + \frac{Y_r}{B}\right)p\right]_\Sigma . \tag{39}$$

Since the radial coordinate is comoving with respect to Σ we can write

$$\left(\frac{\partial m}{\partial t}\right)_\Sigma = \left(\frac{\mathrm{d}m}{\mathrm{d}t}\right)_\Sigma = \left(\frac{\dot{v}}{\dot{t}}\frac{\mathrm{d}m}{\mathrm{d}v}\right)_\Sigma \tag{40}$$

and by considering (18), (24), (39) and (40) we obtain

$$\left(-\frac{2}{\mathsf{r}^2}\frac{\mathrm{d}m}{\mathrm{d}v}\dot{v}^2\right)_\Sigma = p_\Sigma . \tag{41}$$

The radial flux of momentum of the radiation on both sides of Σ is given by

$$F^\pm = e_0^{\pm a} n^{\pm b} T^\pm_{ab},$$

where

$$e_0^{+a} = \left(1 - \frac{2m}{\mathsf{r}_\Sigma} + 2\frac{\mathrm{d}\mathsf{r}_\Sigma}{\mathrm{d}v} + \frac{Q^2}{{\mathsf{r}_\Sigma}^2}\right)^{-1/2}\left(\delta_0^a + \frac{\mathrm{d}\mathsf{r}_\Sigma}{\mathrm{d}v}\delta_1^a\right),$$
$$e_0^{-a} = {A_\Sigma}^{-1}\delta_0^a,$$

are the unit tangent vectors in the η-direction of Σ. For details of this result, without the electromagnetic field, see Lindquist *et al* [15]. Then it is easy to show that

$$F^+ = \left[\frac{2}{\mathsf{r}^2}\frac{\mathrm{d}m}{\mathrm{d}v}\dot{v}^2\right]_\Sigma ,$$
$$F^- = [-q]_\Sigma ,$$

so that $F^+ = F^-$ which is equivalent to the junction condition (38). Therefore the result (38) corresponds to the continuity of the radial flux of momentum of the radiation across the surface Σ, i.e. it expresses the local conservation of momentum in the presence of an electromagnetic field.

The total luminosity for an observer at rest at infinity is given by

$$L_\infty(v) = -\frac{\mathrm{d}m}{\mathrm{d}v} = \lim_{\mathsf{r}\to\infty} 4\pi\mathsf{r}^2\epsilon, \tag{42}$$

where $\mathrm{d}m/\mathrm{d}v \leq 0$ since L_∞ is positive. An observer with four-velocity $v^a = (\dot{v}, \dot{\mathsf{r}}, 0, 0)$ located on Σ has proper time η related to the time t by $\mathrm{d}\eta = A\mathrm{d}t$. The radiation energy density that this observer measures on Σ is

$$\epsilon_\Sigma = \frac{1}{4\pi}\left(-\frac{\dot{v}^2}{\mathsf{r}^2}\frac{\mathrm{d}m}{\mathrm{d}v}\right)_\Sigma$$

and the luminosity observed on Σ can be written as

$$L_\Sigma = 4\pi\mathsf{r}^2\epsilon_\Sigma.$$

The boundary redshift z_Σ of the radiation emitted by the star is given by

$$1 + z_\Sigma = \frac{\mathrm{d}v}{\mathrm{d}\eta} \tag{43}$$

which can be used to determine the time of formation of the horizon. The above expressions allow us to write

$$1 + z_\Sigma = \left(\frac{L_\Sigma}{L_\infty}\right)^{1/2}$$

which relates the luminosities L_Σ to L_∞ via the surface redshift. Note that our expressions, with nonzero shear and electromagnetic field, have the same form as the expressions in Lindquist *et al* [15]. However the contribution of the shear (σ_{ab}) and the electromagnetic field (E_{ab}) is introduced via the metric functions in the definition of the mass function m in (34).

4. Discussion

In this paper we have found the junction conditions for a shearing spacetime with an electromagnetic field that matches smoothly to the Vaidya–Reissner–Nordström solution. This work extends and generalizes previous treatments. Investigations in the past focussed primarily on shear-free models for the interior spacetime. We have generated the general framework for studying particular models with shear and charge. Note that the pressure is proportional to the magnitude of the heat flow, as is the case in shear-free flows. However the shear changes the form of the mass function $m(v)$ (of course the metric functions are different from the shear-free case) and the charge Q appears explicitly as a new, additional factor in $m(v)$.

One of the important reasons for generating the junction conditions at the stellar surface is to study the temperature profile and the thermodynamical evolution of the radiating star. In the past the approach for studying thermodynamical behaviour in general relativity was to utilize the Eckart formalism which is a first order theory. In this theory the temperature T is governed by Fourier's law of heat conduction

$$q^a = -\kappa h^{ab} \left(T_{,b} + T\dot{u}_b \right), \tag{44}$$

where κ is the thermal conductivity, $\dot{u}_b = u_{b;c}u^c$ and $h_{ab} = g_{ab} + u_a u_b$ is the projection tensor. The equation (44) has been used extensively to study radiating stars in general relativity (Grammenos [25,26], Grammenos and Kolassis [27], Martínez and Pavón [28]). The temperature obtained in this approach provides a reasonable approximation when the fluid is close to equilibrium. However at later stages of collapse the fluid is far from equilibrium and (44) has to be modified.

The Eckart formulation of relativistic thermodynamics is unsatisfactory for a number of reasons as observed by Jou *et al* [29] and Maartens [30]. The theory is noncausal since it gives rise to superluminous propagation velocities for the dissipative signals. To generate a consistent theory of irreversible thermodynamics second order effects in the dissipative fluxes have to be included. The entropy flux vector S^a is defined by

$$S^a = Snu^a + \frac{\mathcal{R}^a}{T}, \tag{45}$$

where S is the specific entropy, n is the particle number density and $\mathcal{R}^a$ represents dissipation. In the Eckart theory $\mathcal{R}^a$ in (45) is an algebraic function of the particle four-current $n^a = nu^a$ and the energy-momentum tensor **T**. Note that $\mathcal{R}^a$ vanishes in equilibrium. In order to restore causality and stability the algebraic form of $\mathcal{R}^a$ must at least be second order in the dissipative fluxes in extended irreversible thermodynamics. This generates a system of transport equations which govern the behaviour of the dissipative fluxes and their associated quantities. Assuming there is no viscous/heat coupling we have the following relationship for the temperature

$$\tau h^{ab} u^c q_{b;c} + q^a = -\kappa h^{ab} \left(T_{,b} + T\dot{u}_b \right) \tag{46}$$

which is the covariant relativistic Maxwell–Cattaneo heat transport equation in the truncated Israel–Stewart theory (Israel and Stewart [31]). In the above τ is the relaxation time and when $\tau = 0$ we regain the Fourier equation (44).

For the line element (1) the causal transport equation (46) becomes

$$\tau q_{,t} + Aq = -\kappa \frac{(AT)_{,r}}{B} \tag{47}$$

which governs the behaviour of the temperature. The above form of the causal transport equation must be utilized to construct physically viable models of radiating stars. The analysis of the temperature profile in (47) is difficult; however progress has been made in particular models (Govender *et al* [7] and Maharaj and Govender [32]). Our aim is to produce a physically reasonable model under the assumptions of this paper, with a plausible causal temperature satisfying (47). To proceed further it is necessary to make a choice for κ based on physical grounds. For a physically reasonable model it is possible to use the thermodynamic coefficients for radiative transfer outlined in Martínez [14]. For the situation where energy is carried away from the stellar core by massless particles, that are thermally generated with energies of the order of kT, the thermal conductivity has the form

$$\kappa = \gamma T^3 \tau_{\rm c}, \tag{48}$$

where γ (≥ 0) is a constant and $\tau_{\rm c}$ is the mean collision time between the massless and massive particles. For thermally generated neutrinos

$$\tau_{\rm c} \propto T^{-3/2}$$

to a good approximation. Based on this treatment we may assume the power-law behaviour

$$\tau_{\rm c} = \left(\frac{\alpha}{\gamma}\right) T^{-\omega}, \tag{49}$$

where α (≥ 0) and ω (≥ 0) are constants. With $\omega = 3/2$ we regain the case of thermally generated neutrinos in neutron stars. The mean collision time decreases with growing temperature, as expected, except for the special case $\omega = 0$, when it is constant. This special case can only give a reasonable model for a limited range of temperature.

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