PRAMANA — journal of physics
© Indian Academy of Sciences
Vol. 54, No. 4
April 2000
pp. 611–622

The solar neutrino problem

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Abstract. I review the solar neutrino problem and what it has taught us about the Sun and fundamental physics.

Keywords. Solar; neutrinos; Bayesian; statistics.

PACS Nos 26.65; 96.60.J; 02.50

1. Introduction

The solar neutrino problem, the longstanding disagreement between the measured and predicted neutrino flux from the Sun, has moved from being a curiosity of solar physics to a research problem that now commands the attention of a large number of physicists who have at their disposal impressive experimental resources. The change in fortune of this problem owes much to the persistence of the pioneers in this field and the mounting evidence that the deficit is real. This article is divided into two parts. In the first, I outline the history of the solar neutrino problem and the status of the standard solar model [1,2]. In the second half, I give an appraisal of what we have learned from it. A more complete review of the solar neutrino problem can be found in ref. [3].

1.1 *A brief history*

For a thorough account of the history of solar neutrino physics, the article [4] by two of its founders, John Bahcall and Ray Davis, is highly recommended. Here, I shall touch upon the highlights and refer the reader to the extensive bibliography in ref. [4].

The question 'How does the Sun shine?' presented the scientists of the 19th century with a great puzzle: there appeared to be no physically plausible mechanism that could account for the Sun's luminosity of nearly 400 trillion trillion Watts. The difficulty was not only to explain how to generate so a prodigious a power, but also to explain how it could be maintained for hundreds of millions of years. To illustrate the magnitude of the problem, in 1871 Hermann von Helmholtz computed that this power output is equivalent to that produced by burning 6 metric tons of coal per hour for every square meter of the Sun's photosphere! Various attempts were made to explain the origin of sunlight, however, the world had to wait another half-century before a plausible mechanism was found.

A plausible mechanism, fusion, was first suggested by the British astronomer Sir Arthur Eddington in 1919, the same year he confirmed Einstein's prediction of the bending of light by gravity. However, although Eddington made the crucial suggestion that the fusion of hydrogen to helium could provide sufficient energy to account for the Sun's power he did not perform detailed calculations. The development of the fusion theory was initiated some twenty years later by Bethe and Critchfield on the eve of Europe's descent into a period of unfettered carnage. Other important contributors were Fowler, Gamov, Vogt and von Weizsäcker.

In the earliest papers on stellar energy generation neutrinos were not mentioned explicitly. But in 1948, Crane noted that the Sun should be a copious producer of these particles. The first detailed calculations of solar neutrino rates were presented in 1963 by Bahcall, Fowler, Iben and Sears. A year later, John Bahcall and Raymond Davis pointed out that 100,000 gallons of tetrachloroethylene would be sufficient to measure the neutrino capture rate on ^{37}Cl. The idea that ^{37}Cl might be a good neutrino absorber was suggested by Pontecorvo as early as 1946. In 1966 Davis and collaborators completed the construction of the chlorine experiment in the Homestake mine of South Dakota and two years after that the first comparison between theory and experiment was made.

Bahcall and Shaviv predicted a rate of 7.5 ± 3 SNU [5]. Davis reported a rate of less than 3 SNU. The discrepancy between the predicted and measured solar neutrino rates, which persists to this day, has come to be known as the *solar neutrino problem*. For about two decades, the Davis experiment was the only one reporting results and it was easy to dismiss the solar neutrino problem on the grounds that there was insufficient compelling evidence of a discrepancy, given the presumed uncertainties in the calculation of solar neutrino rates and the extreme difficulty of the experiment.

The situation today is rather different. Four other experiments, the Russian–American experiment SAGE, which uses gallium as an absorber, the European gallium experiment GALLEX, the Japanese experiment Kamiokande and, more recently, the Japanese–American experiment super-Kamiokande, each using a different experimental technique, have confirmed the solar neutrino deficit. Indeed, when analyzed together, these experiments have rendered the solar neutrino problem more acute. Three decades after the problem arose it has finally reached the center stage in fundamental physics.

The pioneers of solar neutrino physics saw their work as a way of testing the fusion theory of stellar energy production. The direct observation of neutrinos from the Sun, with a measured rate in qualitative agreement with the fusion theory, is an impressive theoretical and experimental achievement, which may one day garner a Nobel prize. Today, however, the focus has shifted away from solar physics and towards understanding the neutrinos themselves.

Before we examine what precisely can be deduced from the solar neutrino problem it is helpful to be reminded of some aspects of the theory of the Sun, called the standard solar model (SSM) [1,2]. Actually, there are several such models; however, all use the same basic physics and all are in good agreement with each other. Therefore, for concreteness I shall restrict my discussion to the model associated with John Bahcall and collaborators.

1.2 *Sunshine and neutrinos*

Solar neutrinos arise from the nuclear reactions that power the Sun. The principal reactions are thought to be those of the *proton–proton chain*, listed in table 1. According to the standard solar model, these reactions account for 98% of the solar luminosity. There are three branches, each giving rise to an electron neutrino. The dominant branch is the first, which occurs 91% of the time; branches II and III occur approximately 9% and 0.1% of the time, respectively. Branches I and III produce continuous neutrino energy spectra, characteristic of beta decay, while branch II produces two discrete neutrino lines at 0.862 MeV and 0.383 MeV, with intensities in the ratio of 9 to 1. The neutrino end point energy of branch I is 0.420 MeV. However, the neutrinos produced in branch III, via the boron 8 reaction are much more energetic and have an end point energy just over 14 MeV. The neutrinos from branches I, II and III are called, respectively, the *pp*, ^{7}Be and ^{8}B neutrinos, reflecting the reactions that produce them. The neutrino spectrum, as predicted by the 1998 SSM of Bahcall and Pinsonneault, is shown in figure 1.

An important point to note about this spectrum, especially the part due to the boron 8 reaction, is that it is largely unaffected by the solar plasma [6]. Firstly, because the latter is very cold relative to the neutrinos and, secondly, because the interaction between the neutrinos and the solar plasma is exceedingly weak, due to the small neutrino cross section. The predicted temperature of the solar core is 1.5×10^7 K. This corresponds to an energy of the order of 1 keV, which is more than hundred times smaller than the lowest energy solar neutrinos. The distortion of the boron 8 spectrum, due to the solar plasma, is less than one part in 100,000 [6,7]. Therefore, even if the standard solar model were wrong in detail about the nature of the solar core, and in its prediction of the neutrino fluxes, the predicted *shape* of the spectrum would still be that dictated by electroweak theory, assuming that the set of reactions given in table 1 are indeed the ones that occur at the Sun's core.

Table 1. Main reactions of the proton–proton chain, according to the Standard Solar Model. Branch I occurs about 91% of the time, while branches II and III occur about 9% and 0.1% of the time, respectively.

Branch I
$E_\nu^{\max}$ = 0.420 MeV
$p + p \to d + e^+ + \nu$
$p + d \to {}^3\text{He} + \gamma$
${}^3\text{He} + {}^3\text{He} \to {}^4\text{He} + p + p$

Branch II	Branch III
E_ν = 0.86 MeV (90%), 0.38 MeV (10%)	$E_\nu^{\max}$ = 14.06 MeV
${}^3\text{He} + {}^4\text{He} \to {}^7\text{Be} + \gamma$	${}^7\text{Be} + p \to {}^8\text{B} + \gamma$
${}^7\text{Be} + e^- \to {}^7\text{Li} + \nu$	${}^8\text{B} \to {}^8\text{B}^* + e^+ + \nu$
${}^7\text{Li} + p \to {}^4\text{He} + {}^4\text{He}$	${}^8\text{B}^* \to {}^4\text{He} + {}^4\text{He}$

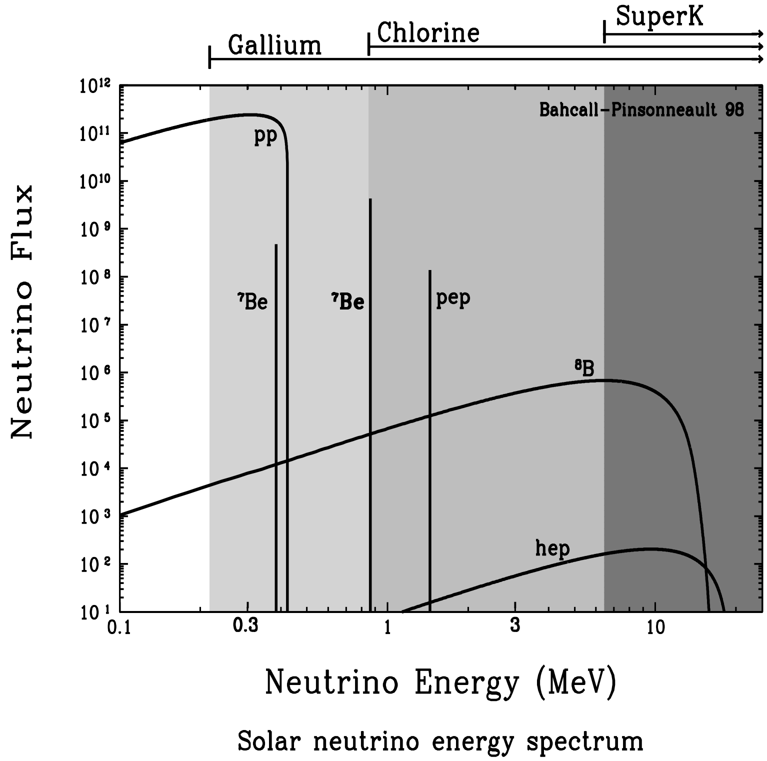

Figure 1. Solar neutrino energy spectrum as predicted by the Bahcall–Pinsonneault 1998 Standard Solar Model. Shown also, are the energy thresholds for different solar neutrino experiments. Courtesy J N Bahcall.

The calculations leading to the SSM are extremely complicated and involve a very wide range of physics [7]. What confidence do we have that they reflect reality? The most striking evidence that the SSM is a precise theory of the Sun is the exquisite agreement between the solar oscillation modes predicted on the basis of the SSM and the modes measured using helioseismology [8]. In figure 2, we reproduce a plot from ref. [8], which shows the fractional difference between the predicted and measured solar sound velocities as a function of the distance from the Sun's core.

From the fact that theory and experiment agree so well we may conclude that the temperature profile predicted by the SSM is substantially correct and, more importantly, we may take at face value its prediction for the core temperature. This is important because it is the core temperature that governs the nuclear reaction rates. The rate for the ^{7}Be reaction is predicted to depend upon the core temperature T, like T^{10}, while that for the boron 8 reaction varies as T^{25}! Clearly, even a small error in the core temperature could drastically alter the predicted neutrino flux.

The most up to date version of the SSM is the 1998 model [1] of Bahcall and Pinsonneault. Its predictions for the solar neutrino rates and how they compare with the data presented at the *Neutrino 98* conference [9] are shown in figure 3. Given the strong evidence in support of the standard solar model, and the absence of any evidence that the

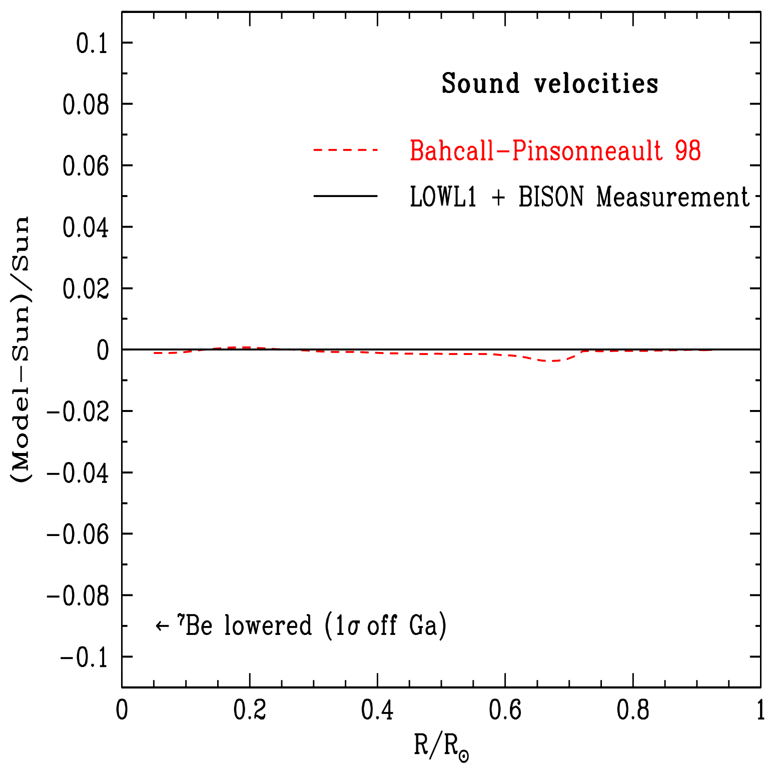

Figure 2. Fractional difference between predicted and measured solar sound velocities as a function of the distance from the solar core. Courtesy J N Bahcall.

experiments are in error, it would be reasonable to conclude that the discrepancies depicted in figure 3 are real. There have been several attempts to construct non-standard solar models [10], but none is wholly successful. On the other hand, the discrepancies are readily explained [11–13] as consequences of new physics such as neutrino oscillations in vacuum [14] or in matter [15].

2. What have we learnt?

The answer to that question depends upon one's attitude towards the standard solar model. I shall assume the SSM to be correct within its known uncertainties. Given this assumption, the inevitable conclusion is that, somehow, a fraction of the electron neutrinos created within the solar core are lost before they reach the solar neutrino detectors on Earth. The electron neutrino loss is described by the *neutrino survival probability*, $P(E_\nu)$, which is the probability that an electron neutrino created within the Sun's core arrives at the Earth.

Most analyses of the solar neutrino problem posit a particular loss mechanism, based on some assumed new physics; for example, loss due to the conversion of electron neutrinos

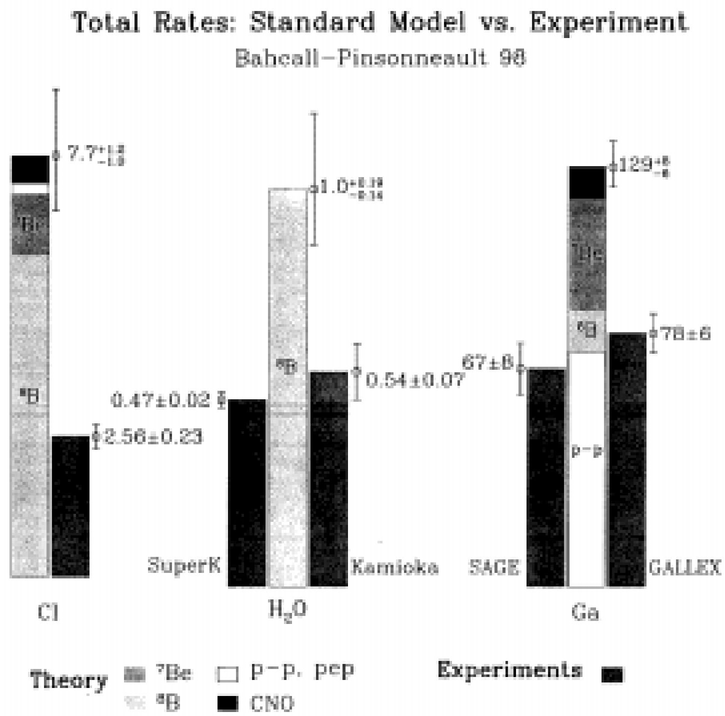

Figure 3. Predictions of the 1998 Standard Solar Model of Bahcall and Pinsonneault relative to data presented at *Neutrino 98*. Courtesy J N Bahcall.

to less readily observed states such as muon, tau or sterile neutrinos [14]. They then proceed to determine the model parameters that provide the best fit to data [11]. To the degree that a fit is good it provides evidence in favor of the particular new physics that has been assumed. Therefore, from this perspective, solar neutrino physics is an avenue for probing physics beyond the standard electroweak theory.

But what if none of these explanations is correct? Can we still learn something from the solar neutrino problem? The answer is yes, but one must take a somewhat different tack to answer it [16]. Rather than consider particular models, we ask the following question: can we extract the neutrino survival probability independently of any particular model, and if so, what do the data tell us about it? The answer is relevant because it allows us to judge to what degree the solar neutrino data can really constrain the models that seek to explain the neutrino deficit.

2.1 *The neutrino survival probability*

Almost all analyses of solar neutrino data make use of χ^2 methods to extract information about the model parameters. However, a more powerful way to extract information from data is to use Bayes' theorem [17]

$$P(H|D,I) = \frac{P(D|H,I)P(H|I)}{\int_H P(D|H,I)P(H|I)}, \tag{1}$$

where $P(H|D,I)$ is the probability of hypothesis H, given measured quantities D and prior information I; $P(D|H,I)$ is the probability assigned to data D and $P(H|I)$ is the prior probability assigned to hypothesis H. The integration in the denominator is over all hypotheses of interest. We shall use Bayes' theorem to extract the neutrino survival probability in a manner that provides quantitative information about how well it is known, given current data.

The solar neutrino capture rate S_i for the chlorine and gallium experiments is given by

$$S_i = \sum_j \Phi_j \int_{E_{\min_i}} \sigma_i(E_\nu)\phi_j(E_\nu)P(E_\nu)\mathrm{d}E_\nu, \tag{2}$$

where Φ_j is the total flux from neutrino source j, ϕ_j is the corresponding normalized neutrino energy spectrum, σ_i is the cross-section for the ith experiment, $E_{\min_i}$ is its threshold energy (see figure 1) and $P(E_\nu)$ is the neutrino survival probability.

The super-Kamiokande experiment [9] is sensitive to the high energy neutrinos, in particular, to the ^{8}B neutrino flux. The latter is inferred by measuring the scattering of neutrinos off electrons. In practice, super-Kamiokande measures the electron recoil spectrum. For our analysis, we use the electron recoil spectrum reported at *Neutrino 98*. That spectrum spans the range 6.5 to 20 MeV. Since super-Kamiokande is sensitive to (but does not distinguish between) all neutrino flavors we must consider two possibilities. It could be that the ν_e deficit is caused by ν_e conversions to ν_x, where x is either μ or τ. In this case, we must take account of the fact that super-Kamiokande can record all these neutrino flavors, albeit with different sensitivities, and the measured flux would then be the sum of these flavors. If, however, the ν_e are lost through a mechanism that does not result in a detectable signal, for example because of conversion into sterile neutrinos, then the measured rate would be due to the ν_e flux only. Both possibilities are considered.

The *measured* electron recoil spectrum $N(T)$ is given by

$$\begin{aligned} N(T) = N_0 &\int_0^{T^{\max\prime}(E_\nu^{\max})} \mathrm{d}T' R(T|T') \\ &\times \int_{E_\nu^{\min}(T')}^{E_\nu^{\max}} \mathrm{d}E_\nu \phi_B(E_\nu)[P(E_\nu)\sigma_e(T', E_\nu) \\ &+(1-P(E_\nu))\sigma_\mu(T', E_\nu)], \end{aligned} \tag{3}$$

where $R(T|T')$ is the super-Kamiokande resolution function (which can be approximated by a Gaussian with mean T' and standard deviation $1.5\sqrt{(T'/10\,\mathrm{MeV})}$ [18]), $T = E_e - m_e$ and T' are the measured and true electron kinetic energies, respectively, with E_e and m_e the electron energy and mass. The quantity ϕ_B is the normalized neutrino energy spectrum from the ^{8}B reaction, and σ_e and σ_μ are the ν_e and ν_μ differential electron scattering cross-sections [19], respectively. For a fixed neutrino energy E_ν, the recoiling electron can have any kinetic energy up to a maximum of $T^{\max\prime}(E_\nu) = 2E_\nu^2/(2E_\nu + m_e)$, while for a fixed electron kinetic energy T' the minimum neutrino energy is given by $E_\nu^{\min}(T') = [T' + \sqrt{(T'(T'+2m_e))}]/2$. We denote by $E_\nu^{\max}$ the maximum possible neutrino energy, which we take to be 20 MeV. The constant N_0 is a normalization factor that depends on the units used for the event rate.

Equation (3) assumes that ν_e are lost through conversion to active neutrinos, for example to ν_μ. If the measured flux is due to ν_e only, however, the recoil spectrum is given by eq. (3) with the term proportional to σ_μ omitted. The event rate S_i in the ith electron energy bin is $N(T)$ integrated over that bin.

Our ability to extract information about the neutrino survival probability depends both on the shape of the solar neutrino spectrum and on the experimental sensitivity to it. Each experiment is sensitive to different parts of the neutrino energy spectrum as is evident from the (normalized) plots of $\sigma(E_\nu)\sum_j \Phi_j \phi_j(E_\nu)$ (or $\phi_B(E_\nu) \times \int_{6.5-m_e}^{20-m_e} \mathrm{d}T \int_0^{T(E_\nu)} \mathrm{d}T' R(T|T')\, \sigma_e(T', E_\nu)$ for super-Kamiokande) shown in figure 4. We also note the existence of regions where the sensitivity is essentially zero. We should not expect to learn much about the survival probability in these regions.

The probability assigned to the data, that is, the likelihood, $P(D|H, I)$ is assumed to be a Gaussian $g(D|S, \Sigma)$, where $D \equiv (D_1, \ldots, D_{19})$ represents the 19 data—3 rates from the chlorine and gallium experiments plus 16 rates from the binned super-Kamiokande electron recoil spectrum; Σ denotes the 19×19 error matrix for the experimental data and $S \equiv (S_1, \ldots, S_{19})$ represents the predicted rates. The error matrix is constructed from the data of ref. [18].

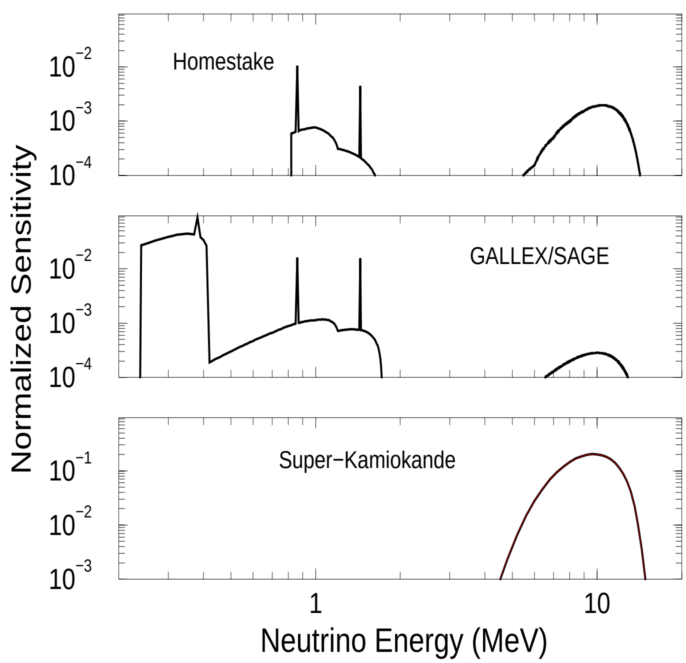

Figure 4. Spectral sensitivity as a function of the neutrino energy.

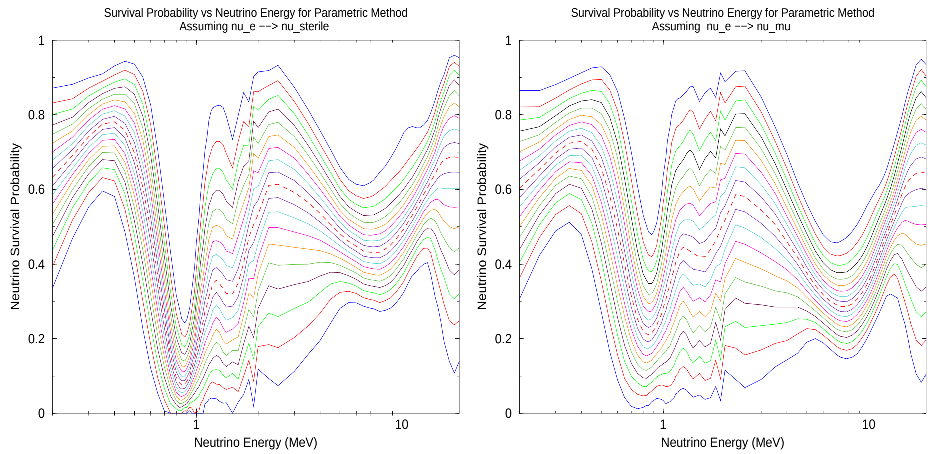

Figure 5. Survival probability vs neutrino energy assuming the neutrino flux consists of ν_e only (left plot) and ν_e to active neutrinos (right plot).

In any Bayesian analysis one must confront the issue of prior probabilities. They are a necessity, indeed an important part of the Bayesian method. However, this is not the place to get distracted by the intricacies of the prior probability debate. We short-circuit discussion by taking, as a matter of convention, the prior probability to be constant. In practice any other plausible choice makes very little difference to our conclusions.

Given our desire to be noncommittal with respect to models, yet being mindful of previous analyses [11,12,20], we are motivated to write the survival probability as a sum of two finite Fourier series:

$$\mathcal{P}(E_\nu|a) = \sum_{r=0}^{7} a_{r+1}\cos(r\pi E_\nu/L_1)/(1+\exp[(E_\nu - L_1)/b]) \\ + \sum_{r=0}^{3} a_{r+9}\cos(r\pi E_\nu/L_2). \tag{4}$$

The first term in eq. (4) is defined in the interval 0.0 to L_1 MeV—and suppressed beyond L_1 by the exponential, while the second term covers the interval 0.0 to L_2 MeV. The function is divided this way to accommodate a survival probability that varies rapidly in the interval 0.0 to L_1 and less rapidly elsewhere. Holding the parameters L_1, L_2 and b fixed at 1.0, 15.0 and 0.1 MeV, respectively, we compute the posterior probability $P(a|D,I)$ for the parameters $a \equiv (a_1,\ldots,a_{12})$. The hypothesis here is that the vector of parameters a assume specified values.

The theoretical uncertainties can be incorporated by treating the fluxes $\Phi \equiv (\Phi_1,\ldots)$ as parameters with an associated prior probability, $P(\Phi|I)$, that encodes the flux predictions and the correlations that exist amongst them. We represent our knowledge of the fluxes by a multivariate Gaussian prior probability $P(\Phi|I) = g(\Phi|\Phi^0,\Sigma_\Phi)$, where $\Phi^0 \equiv (\Phi_1^0,\ldots,\Phi_8^0)$ is the vector of flux predictions and Σ_Φ is the corresponding error matrix [1].

The posterior probability (using a constant prior probability for the parameters a) is given by

$$P(a,\Phi|D,I) = \frac{P(D|a,\Phi,I)P(\Phi|I)}{\int_{a,\Phi} P(D|a,\Phi,I)P(\Phi|I)}, \tag{5}$$

which when marginalized (that is, integrated) with respect to Φ gives $P(a|D,I)$. Finally, the survival probability is estimated using

$$P(E_\nu|D,I) = \int_a P(E_\nu|a)P(a|D,I). \tag{6}$$

Figure 5 shows our results for the survival probability, which includes both experimental and theoretical uncertainties. The form obtained agrees with the inferences from previous analyses [11,12]. But unlike these analyses the one described here provides detailed information about the survival probability and its uncertainties as a function of the neutrino energy.

As one might have anticipated, given the sensitivity plot in figure 4, our knowledge of the survival probability is very uncertain between 1 and 5 MeV. In fact, the survival probability is tightly constrained in only two narrow regions: in the ^{7}Be region just below 1 MeV and another at around 8 MeV, near the peak of the ^{8}B neutrino spectrum. For neutrino energies above 12 MeV or so, the survival probability is unconstrained by current data.

The key feature of figure 5, which is well known, is the marked reduction in the ^{7}Be flux, relative to the prediction of the SSM [1]. This is extremely difficult to explain by any plausible modification of the SSM [10]. Because of its strong temperature dependence, it is easy to reduce the ^{7}Be neutrino flux by slightly reducing the core temperature. But, the temperature dependence of the ^{8}B neutrino flux is considerably stronger, T^{25} compared to T^{10} for ^{7}Be. Therefore, the ^{8}B neutrino flux would be reduced by a much greater factor than that of the ^{7}Be neutrinos. Moreover, the ^{8}B reaction is a daughter reaction of the ^{7}Be reaction. Thus the ^{7}Be and ^{8}B neutrino fluxes are reduced, relative to the predictions, in a manner contrary to what one would expect from any plausible lowering of the core temperature. This is the strongest evidence that a non-solar explanation must be sought to explain the loss of electron neutrinos from the Sun.

3. Summary

There is strong evidence that the solar neutrino deficit is real. In particular, the marked suppression of the flux of ^{7}Be neutrinos is very difficult to explain, given the standard solar model. If the SSM is correct, then we are inevitably led to conclude that electron neutrinos are getting lost en route to Earth. What we should infer from this, however, is less clear. Currently, the favored hypothesis, rendered more probable by the super-Kamiokande atmospheric neutrino results [21], invokes some form of neutrino oscillations. In particular, the MSW model [15] provides a good description of the data, which is clearly a strong point in its favor. But our extraction of the neutrino survival probability suggest that the data could accommodate a wider range of models than have been considered. Therefore, at present, it is not possible to come to a definite conclusion regarding the solar neutrino problem. However, we can look forward, in the very near future, to results from SNO [22], which should allow us to make some headway.

Acknowledgement

The author would like to thank the organizers of the DAE Symposium for hosting a memorable and enjoyable event. The author also thanks his colleagues Chandra Bhat, Pushpa Bhat and Marc Paterno with whom this work was done, John Bahcall for providing the latest theoretical information and Robert Svoboda for providing the latest super-Kamiokande data. This work was supported in part by the U.S. Department of Energy.

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